ON THE THEORY OF THE SPONTANEOUS HALL EFFECT IN FERROMAGNETIC SEMICONDUCTORS

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The theory of the spontaneous Hall effect developed previously for metals is applied to semiconductors. For the simplest case (a non-degenerate semiconductor, one type of carrier) calculation predicts that the spontaneous Hall coefficient is proportional to the first power of the electrical resistance. The result obtained is compared with existing experimental data.

1. The spontaneous Hall effect in ferromagnets has been mainly studied in metals, both experimentally [1] and theoretically. [2-4] Only in recent years have several papers appeared concerned with studying this effect in ferrites. [5-8] Meanwhile the study of the spontaneous Hall effect in semiconductors is of great interest in at least two respects.

First, there exist at present two interpretations of the temperature variation of the spontaneous Hall coefficient R_S in ferromagnets, one of which associates R_S with the electrical resistance, and the other with the square of the magnetization: 1) $R_S(T) \sim \rho^n(T)$ (ρ is the electrical resistance, $n = 1, 2$) [2-4] 2) $R_S(T) \sim M_S^2(T)$ (M_S is the spontaneous magnetization of the specimen). [7] Because the temperature variation of the magnetization has the same character both in metallic and in semiconducting ferromagnets, whereas the temperature variation of the electrical resistance is markedly different, comparison of $R_S(T)$ in metals and semiconductors enables the validity of the two interpretations to be tested.

Second, the conduction mechanism in ferrites is not yet clear. The study of the spontaneous Hall effect, together with the electrical conductivity and normal Hall effect, can give valuable information on this question.

In the present paper the temperature variation of $R_S(T)$ in semiconductors is calculated using a method, applied previously to metals [2-4], for scattering impurities and phonons under the assumption that normal transport theory is valid (i.e., the mean free path of the carriers is sufficiently large).

2. In previous papers [2,3] the density matrix method was used to obtain the kinetic equations to high approximations in the scattering interaction for impurity scattering and phonon scattering, respectively. The solutions of these equations were used in the papers referred to in order to calculate the spontaneous Hall coefficient in metals. For semiconductors the kinetic equations have the same form, and we can use the solutions from Eqs. (2) and (3), taking into account only the difference in the statistics of the carriers (nondegenerate semiconductors are considered).

For the impurity case the contribution to the distribution function which causes the spontaneous Hall effect is inversely proportional to the first power of the interaction potential of the carriers with the impurities, and has the form [2] [1)]

$$f_l^{(-1)} = ieF_\alpha \left(\frac{\partial J_\alpha^l}{\partial k_\beta} - \frac{\partial J_\beta^l}{\partial k_\alpha} \right) k_\beta \frac{\partial \rho_l^0}{\partial \varepsilon_l} \frac{\varepsilon_l}{3\bar{\varphi}}, \qquad (1)$$

where F_α, k_α are the α components of the external electrical field and the wave vector of the electron; e is the electronic charge; $\rho_l^0 \equiv \rho_{nk}^0$ and ϵ_{nk} are, respectively, the zero order distribution function, and the electron energy in the band number n; $\bar{\varphi}$ is the mean impurity potential (see [3]), and, finally, the quantity J_α^l related to the spin-orbit interaction is [3]

$$J_\alpha^l = -i \frac{eH^{s0}}{m^2\Delta^2 c M_s^0} [\mathbf{k}\mathbf{M}_s]_\alpha, \qquad (2)^*$$

where c is the velocity of light, m is the electronic mass, M_S^0 and M_S are the spontaneous magnetizations at T = 0°K and at temperature T; Δ is the energy separation between the bands for which the spin-orbit interaction is taken into account, using perturbation theory; H^{S0} is the effective spin-orbit field in gauss.

In the case of phonons the contributions f_l and

[1)] $f(\bar{l}^{-1})$ in [2] differs in sign, since $e > 0$ there.

*$[\mathbf{k}\mathbf{M}_s] = \mathbf{k} \times \mathbf{M}_s$.

$f_{ll'}$ which provide the spontaneous Hall effect are obtained in one order of approximation higher than for the impurity case:

$$f_l^{(0)} = ieF_\alpha J_\alpha^l \partial\rho_l^0/\partial\varepsilon_l; \tag{3}$$

$$f_{ll'}^{(0)} = (\varepsilon_l - \varepsilon_{l'})^{-1}\, ieF_\alpha J_\alpha^{l'l}\delta_{kk'}(\rho_{l'}^0 - \rho_l^0), \tag{4}$$

where the non-diagonal matrix elements $J_\alpha^{l'l}$ can be expressed in terms of the derivatives with respect to k of the J_α^l, which are defined in (2) (see, for example, [3]). Using (1) and (4), it is now easy to calculate for both scattering mechanisms the nondiagonal component of the electrical conductivity tensor which determines the Hall effect:

$$\sigma_{yx} = e\bar{v}_y F_x^{-1} = eF_x^{-1}\Big(\sum_l f_l v_y^l + \sum_{l\neq l'} f_{ll'}v_y^{ll'}\Big), \tag{5}$$

$$v_y^l = \partial\varepsilon_l/\partial k_y, \qquad v_y^{ll'} = -(\varepsilon_l - \varepsilon_{l'})\, J_y^{ll'}\delta_{kk'}. \tag{6}$$

Carrying out the integration over k and the summation over n′ in (6), and assuming the presence of one type of carrier, we obtain the following expressions for the cases of phonon and impurity scattering ($T \gg \Theta_D$):

$$\sigma_{yx}^{ph} = -\frac{E_{s0}}{\Delta^2}\frac{M_s}{M_s^0}\frac{e^2N}{m^*}, \tag{7}$$

$$\sigma_{yx}^{(i)} = \tfrac{5}{3}\sigma_{yx}^{ph}\varkappa T/\bar{\varphi}. \tag{8}$$

Here the spin-orbit energy is $E_{S0} = H^{S0}e/mc = \mu_B H^{S0} < 0$.[3]

When both scattering mechanisms are present we obtain

$$\sigma_{yx} = (1/\sigma_{yx}^{ph} + 1/\sigma_{yx}^{(i)})^{-1} = \sigma_{yx}^{ph}\left(1 + \tfrac{3}{5}\bar{\varphi}/\varkappa T\right)^{-1}. \tag{9}$$

It was shown in [4] that the collision term for scattering by spin inhomogeneities does not contribute to the anomalous Hall effect. The same applies to scattering by optical phonons. Insofar as the mechanisms specified exhaust the principal scattering sources in ferrites, formula (9) can apparently be considered valid in the general case.

By definition the Hall coefficient is

$$R_s = -\sigma_{yx}/4\pi M_s\sigma^2, \tag{10}$$

where the electrical conductivity $\sigma = \sigma_{XX} = eNu$ and u is the mobility defined, for example, in terms of the normal Hall coefficient R_0:

$$u = KR_0\sigma c. \tag{11}$$

The numerical factor $K \sim 1$ depends on the mechanism of scattering.

We obtain from (7) to (11)

$$R_s = -\frac{E_{s0}}{4\pi\Delta^2}\frac{\mu_B u^{-1}c}{1 + {}^3/_5\bar{\varphi}/\varkappa T}\frac{m}{|m^*|}\frac{\rho}{M_s^0}. \tag{12}$$

Equation (12) is highly convenient for analysis and for comparison with experiment: the factor ρ/M_S^0 has the same dimensions as R_S (ohm-cm/gauss), and the remaining quantities in (12) form a dimensionless combination. The most important (exponential) temperature dependence is provided by ρ, whilst the mobility u and (when there is impurity scattering) $\bar{\varphi}/\varkappa$ depend comparatively little on temperature. The sign of R_S is determined by the sign of u, and, consequently, from (11), coincides with the sign of R_0.

3. We now compare our result (12) with experimental data. The theory developed is applicable when the following conditions are satisfied: 1) the kinetic equation is valid (sufficiently long mean free paths), 2) there is only one type of carrier, 3) there is no degeneracy.

From the experimental work known to us [5-8], these conditions are, apparently, satisfied only for single crystals of iron-nickel ferrite [8] and manganese ferrite. [7] The activation energy ΔE determined from the temperature variation of the electrical resistance of the spontaneous Hall effect provides for the iron-nickel ferrite in the temperature range 200—400°K values ΔE_ρ and ΔE_{R_S} of the order 0.06—0.07 eV, and, for the manganese ferrite [2] in the range 400—480°K values $\Delta E_\rho = 0.31$ eV, $\Delta E_{R_S} = 0.26$ eV. In [8] the signs of R_S and R_0 are the same, but in [7] the sign of R_S is not, unfortunately, specified.

The two examples considered indicate in our opinion the qualitative agreement of equation (12) with the experimental data.[3] In this connection we note that attempts [8] to interpret the spontaneous Hall effect in ferrites in the same way as in metals with the formula $R_S = r\rho^2$ (where r is a constant) lead to unsatisfactory results, since r becomes a rapid function of temperature. It should be emphasized that the agreement between the temperature variations of the spontaneous Hall coefficient and the temperature variations of the resistance both in metals (R_S and ρ increase with T) and in semiconductors (R_S and ρ fall with T), while the magnetization in both cases diminishes with increasing temperature, apparently indicates the validity of the first of the interpretations specified at the start of this paper.

In order to compare (12) with experiment quan-

[2] The electrical resistance data are taken from the work of Belov, Popova, and Talalaeva [9] (specimen No. 3).

[3] The experimental verification of the relation $R_S/R_0 \approx u^{-2}(T)$, which follows from (11) and (12) when $\bar{\varphi} \ll \varkappa T$, is interesting; the relation can also be used to determine the temperature variation of the mobility.

titatively, it is necessary to know m* and the ratio E_{S0}/Δ^2 [4)] and also the experimental quantities ρ, M_S^0, R_0. Taking the latter for iron-nickel ferrite at T = 300°K from [8], and putting m* ~ 10 m, we find that, for R_S calculated from (12) to agree with the experimental data, we must assume $E_{S0}/\Delta^2 \sim 10^8$ ergs^{-1}, whilst in metals this quantity is $\sim 10^{11}$ ergs^{-1}. Since it is doubtful that the value of Δ exceeds 10^{-11} ergs, the value of E_{S0} for carriers in iron-nickel ferrite must be $\sim 10^{-14}$ ergs ($H^{S0} \sim 10^6$ gauss), i.e., at least an order smaller than in nickel. It is possible that this peculiarity is associated with the smaller values of momentum $\hbar k$ in nondegenerate semiconductors relative to the Fermi momentum $\hbar k_\Phi$ in metals.

For manganese ferrite, however, a similar calculation gives $E_{S0}/\Delta^2 \sim 10^{10}$ ergs^{-1}, i.e., a value almost the same as in metals (the scatter of Δ being half an order).

Equation (12) is not directly applicable to the remaining ferrites studied experimentally, since in magnetite [6,8] there appears to be degeneracy of the carriers [5)] (ρ increases with temperature), and in polycrystalline manganese and nickel-zinc ferrite [7] the Hall mobility is very small (~ 0.08 cm^2/V-sec). The measured variation of $R_S(T)$ with temperature is not of an exponential type in these cases. Order of magnitude estimates of the quantity E_{S0}/Δ^2 from (12) give for magnetite the value 10^{10} ergs^{-1}, and for the polycrystalline materials 10^6—10^7 ergs^{-1}. Such a marked difference between the Hall effect in polycrystalline specimens and in single crystals (their electrical conductivities are equal in order of magnitude) is not understood.

As is seen from what has been said, the absence of sufficient experimental data on the Hall effect in ferrites at present hinders comparison of the proposed theory with experiment. In addition, the inevitable presence in such a type of theory of the combination of the three quantities E_{S0}, Δ, and m*, the numerical values of which can be determined at best in order of magnitude, requires experimental data to be provided on other effects. Of great interest in this connection are the thermomagnetic effects and the Faraday effect, the spontaneous coefficients of which will contain the same quantities. One of us [10] has calculated the spontaneous Nernst-Ettinghausen effect in ferrites. The estimate of E_{S0}/Δ^2 thus obtained for magnetite, the only case studied experimentally, [6] gives $\sim 10^{10}$ ergs^{-1}, which agrees with the value obtained above from the Hall effect. Further development of theory and experiment in this area can in our opinion lead to great progress in the understanding of the electrical properties and the electronic structure of ferrites.

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[4)] At not very low temperatures $\bar{\varphi}/\varkappa T \leq 1$, and therefore does not affect the calculations in order of magnitude.

[5)] At lower temperatures, however, as follows from the Hall effect measurements in weak fields, [5] there is observed in magnetite a good exponential variation with T of both the Hall coefficient and the resistance. Numerical comparison is difficult because of the inadequacy of the data given in [5].

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