NUCLEON-NUCLEON SCATTERING AMPLITUDES AND COMPLEX SPIN-ORBIT INTERACTION POTENTIAL BETWEEN NUCLEONS AND NUCLEI

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Relations between nucleon-nucleon and nucleon-nucleus scattering amplitudes based on the superposition model are used to determine the values of the 660 MeV forward scattering amplitude for nucleon-nucleon scattering averaged over the isotopic spins. Comparison with the phase shift analysis of the experimental data on nucleon-nucleon scattering shows that the spin-orbit potential of the nucleon-nucleus interaction is complex in the energy region of 40 to 660 MeV.

WATSON and collaborators [1] have established a quantitative relation between the nucleon-nucleon and nucleon-nucleus scattering amplitudes within the direct interaction model. A number of effects are neglected, such as the presence of correlated clusters in nuclei, the motion of the nucleons inside the nucleus, etc., which become less important as the energy E increases and are therefore small at high energies. It is therefore natural to expect that the experimental data on nucleon-nucleus scattering can be used, in the region of high energies, to gain information on the nucleon-nucleon scattering amplitudes. This approach is reasonable at high energies where it is very difficult to carry out a complete set of experiments on the nucleon-nucleon scattering.

In Born approximation the amplitude for nucleon-nucleus scattering on a nucleus with spin 0 is given by the expression

$$f(q) = G(q)\,F(q) + i\sigma_n H(q)\,F(q)\sin\theta, \qquad (1)$$

where $F(q)$ is the nuclear form factor, $q = 2k\sin(\theta/2)$ is the momentum transfer, $\hbar k$ and θ are the momentum and scattering angle in the laboratory system (l.s.), and σ_n is the projection of the spin on the normal to the scattering plane. We write the nucleon-nucleon scattering matrix in the form [2]

$$M_{1,0}(q) = A_{1,0}(q) + B_{1,0}(q)\,\sigma_{1n}\sigma_{2n} + C_{1,0}(q)\,(\sigma_{1n} + \sigma_{2n})\sin\theta + E_{1,0}(q)\,\sigma_{1q}\sigma_{2q} + F_{1,0}(q)\,\sigma_{1p}\sigma_{2p}. \qquad (2)$$

Here the indices 1 and 2 correspond to the isotopic spin states T = 1 and T = 0 of the two-nucleon system. In the superposition model we have then the following relation between the nucleon-nucleus and nucleon-nucleon scattering amplitudes:

$$G(q) = N\,(k/k_0)\left[\tfrac{3}{4}A_1(q) + \tfrac{1}{4}A_0(q)\right],$$
$$H(q) = -iN\,(k/k_0)^2\left[\tfrac{3}{4}C_1(q) + \tfrac{1}{4}C_0(q)\right], \qquad (3)$$

where N is the atomic weight of the nucleus and $\hbar k_0$ is the nucleon momentum in the center of mass system of the two nucleons.

The values of the real and imaginary parts of the Born amplitudes G(0) and H(0) at 660 MeV were determined from the small angle measurements of the differential cross section for the elastic scattering of protons on carbon and of the proton polarization in this process. [3] It follows from these data that the real and imaginary parts, $\bar{A}$ and $\bar{C}$, of the nucleon-nucleon forward scattering amplitude averaged over the isotopic spin states are, in the units 10^{-13} cm,[1)]

$$\bar{A}^R(0) = {}^3/_4\,A_1^R(0) + {}^1/_4\,A_0^R(0) = -0.36 \pm 0.03,$$
$$\bar{A}^I(0) = {}^3/_4\,A_1^I(0) + {}^1/_4\,A_0^I(0) = 0.72 \pm 0.04,$$
$$\bar{C}^R(0) = {}^3/_4\,C_1^R(0) + {}^1/_4\,C_0^R(0) = -0.33 \pm 0.28,$$
$$\bar{C}^I(0) = {}^3/_4\,C_1^I(0) + {}^1/_4\,C_0^I(0) = 0.77 \pm 0.20. \qquad (4)$$

We can draw two conclusions from these results. First, the real part of the spinless amplitude $\bar{A}^R(0)$ has a negative sign, which is apparently due to the effects of a hard repulsive core in the nucleon-nucleon interaction. It should be noted that this amplitude is positive for energies

1) The analysis of the small angle differential cross sections for elastic p–p scattering at 657 MeV, where Coulomb and nuclear amplitudes interfere with each other, showed that $A^R_{pp}(0) = (-0.24 \pm 0.08) \times 10^{-13}$ cm. This implies that $A^R_{np}(0) = (-0.48 \pm 0.19) \times 10^{-13}$ cm.

below or equal to 315 MeV. Second, there are indications that the spin-orbit amplitude $\bar{C}(0)$, corresponding in first Born approximation to the spin-orbit potential of the nucleon-nucleus interaction, is complex, since it appears that the real part of the spin-orbit amplitude $\bar{C}^R(0)$ is different from zero at 660 MeV. There have been earlier indications [1,4] that the spin-orbit potential and, hence, the amplitude $\bar{C}(0)$ should be complex. But in later papers [2,5] these results were subjected to doubt, and now the opinion is prevalent that the amplitude $\bar{C}(0)$ is pure imaginary in the energy region of 40 to 970 MeV.

It is of interest in this connection to compare the values of the nucleon-nucleon scattering amplitudes $\bar{A}(0)$ and $\bar{C}(0)$ (averaged over isotopic spins) at different energies. The experimental data on nucleon-nucleus scattering in the region of 40 to 970 MeV [5] are, within the experimental errors, in agreement with the hypothesis of a pure imaginary amplitude $\bar{C}(0)$. However, the results of the phase analysis of the pp and pn scattering data in the 40 to 310 MeV region, carried out recently by Kazarinov and Silin, [6] indicate that the amplitude $\bar{C}(0)$ and hence, if the superposition model is valid, the spin-orbit potential are complex.

In the figure we show the values of the amplitudes $\bar{A}(0)$ and $\bar{C}(0)$ as functions of the kinetic energy of the nucleon $E_{c.m.}$ in the center of mass system (c.m.s.) of the two nucleons. For energies of 40 to 660 MeV, the energy dependence of the amplitudes can be approximated adequately by the expressions

$$\bar{A}^I(0) = (7.20 \pm 0.20)/E_{c.m.} + (4.68 \pm 0.26)\cdot 10^{-3} E_{c.m.},$$
$$\bar{A}^R(0) = (0.673 \pm 0.03) - (6.88 \pm 0.35)\cdot 10^{-3} E_{c.m.},$$
$$\bar{C}^I(0) = (0.188 \pm 0.038) + (3.86 \pm 0.70)\cdot 10^{-3} E_{c.m.},$$
$$\bar{C}^R(0) = (2.45 \pm 0.42)\cdot 10^{-3} E_{c.m.} - (1.97 \pm 0.84)\cdot 10^{-5} E^2_{c.m.}. \quad (5)$$

Here the energy $E_{c.m.}$ is given in MeV and the value of the amplitude, in 10^{-13} cm. The constant coefficients were found by the method of least squares. The shaded areas in the figure indicate the errors in the functions (5).

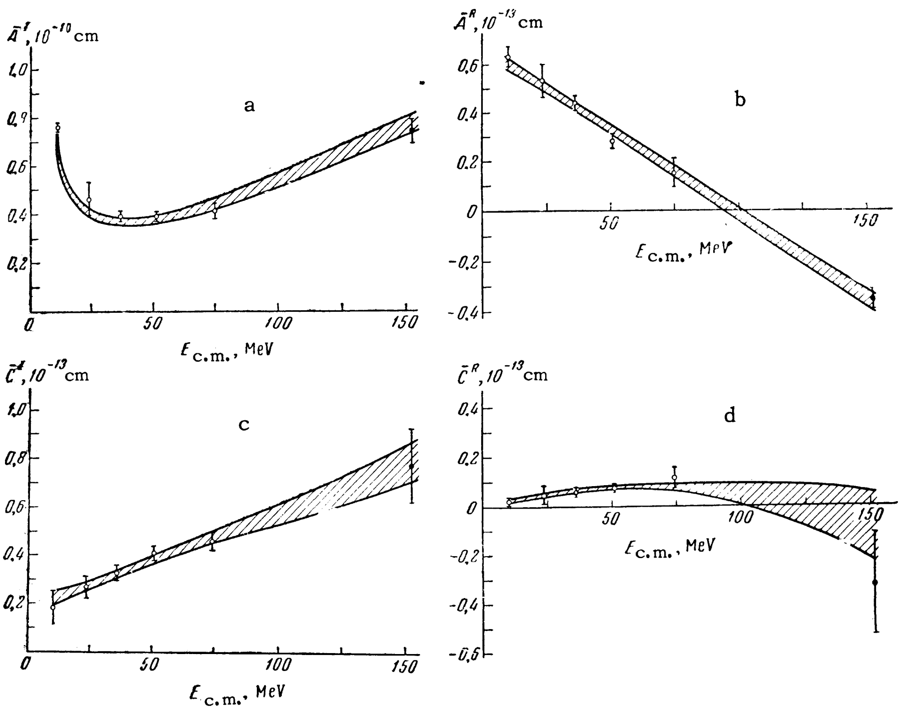
The behavior of $\bar{A}^I$ reflects essentially the variation of the total nucleon-nucleon cross section averaged over isotopic spins, $\bar{\sigma}$, since according to the optical theorem

$$\bar{A}^I(0) = k_0\bar{\sigma}/4\pi = k_0(\sigma_{pp} + \sigma_{np})/8\pi, \quad (6)$$

where σ_{pp} and σ_{np} are the total pp and np cross sections.

The amplitude $\bar{A}^R(0)$ goes through zero at the energy ~ 400 MeV in the laboratory system. Therefore, there is no potential scattering at this energy and we have (except for spin corrections) pure shadow scattering. This leads to a number of conclusions. In particular, the nucleon-nucleus cross section for scattering into the angle 0° must satisfy the relation $\sigma(0) = (k\sigma_t/4\pi)^2$.

As can be seen from Fig. c, the amplitude $\bar{C}^I(0)$ is positive in the energy region under consideration.



Values of the nucleon-nucleon scattering amplitudes averaged over isotopic spin states, $\bar{A}(0)$ and $\bar{C}(0)$. Black circles – results of the present paper; white circles – results of the phase shift analysis of the experimental data on nucleon-nucleon scattering.[6] The dashed areas indicate the errors.

This means that up to 660 MeV the real part of the spin-orbit potential V_{SR} of the nucleon-nucleus interaction has the same sign as in the shell model. This is also the case at 970[7] and at 1700 MeV.[8]

It is seen from Fig. d that the value of $\bar{C}^R(0)$, although small, is nevertheless determined with high accuracy in the region of 40 to 310 MeV. The errors in the figure are somewhat exaggerated since no account was taken of the correlations between the phases in the analysis.[9] However, even within the errors indicated, the real part of the spin-orbit amplitude $\bar{C}^R(0)$ is different from zero and positive in this energy region. Apparently, it goes through zero at ~ 420 MeV and is negative at 660 MeV.

For the determination of the optical potential parameters for nucleon-nucleus scattering from the data on nucleon-nucleon scattering we used the following relations derived within the framework of the superposition model:[10]

$$V_C = \frac{2\pi\hbar^2 c^2 k}{\Omega_1 E k_0} N\bar{A}, \quad V_S = -i\frac{2\pi\hbar^2 c^2}{\Omega_2 E\,(k_0\hbar/\mu c)^2} N\bar{C}\ . \qquad (7)$$

Here E is the total energy of the incident nucleon in the l.s. and Ω_1 and Ω_2 are the normalization volumes for the central and spin-orbit potentials. It is assumed that Ω_1 and Ω_2 are independent of the energy and have the values $\Omega_1 = 45.99 \times 10^{-39}$ cm^3 and $\Omega_2 = 68.64 \times 10^{-39}$ cm^3, as found earlier.[3]

In the table we give the optical potential parameters as calculated from the nucleon-nucleon data. For the energy 660 MeV these parameters were determined from the data on nucleon-nucleus scattering.

Optical potential parameters

E, MeV	V_{CR}, MeV	V_{CI}, MeV	V_{SR}, MeV	V_{SI}, MeV
40	82±6	99±3	8.6±2.9	−1.14±0.36
90	65±9	57±9	5.0±0.9	−0.85±0.56
147	52±4	46±3	3.8±0.4	−0.65±0.09
210	33±4	46±3	3.1±0.2	−0.58±0.07
310	17±7	43±3	2.2±0.2	−0.56±0.19
660	−33±3	67±4	1.3±0.3	0.55±0.48

The energy dependence of the central potential V_C, as obtained from nucleon-nucleon scattering experiments using the superposition model of direct interaction, is in agreement with the energy dependence found directly from the experimental nucleon-nucleus scattering data. The fact that the values of V_C in the table are somewhat too high at small energies is apparently to be explained by the circumstance that the radius of the central potential is not constant, as assumed by us, but increases as the energy goes down.[5] This implies a larger normalization volume Ω_1 and correspondingly smaller values of V_{CR} and V_{CI}, which leads to a better agreement between the values of the parameters found by the two different methods.

The following should be noted in connection with these results. First, the data on nucleon-nucleon scattering imply, as do the nucleon-nucleus scattering data, a decrease in the real part of the spin-orbit potential V_{SR} as the energy increases. Second, the nucleon-nucleon experiments indicate that the imaginary part of the spin-orbit potential is different from zero. As is known, it has not been possible earlier to detect the potential V_{SI} in nucleon-nucleus experiments.[5] There are apparently two reasons for this. Up to now, the experiments on nucleon-nucleus scattering, in particular those measuring the differential cross section, the polarization, and, at some energies, the parameter $R(\theta)$, have been rather inaccurate in the region of small scattering angles. Moreover, these parameters are practically insensitive to the value of V_{SI}, since the parameter $A(\theta)$, which is most sensitive to the value of V_{SI}, has not yet been measured at any energy.

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