SHAPE OF DIPOLE RESONANCE IN DEFORMED NUCLEI

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Submitted to JETP editor June 27, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 2188—2193 (December, 1962)

The single-particle excitation model in which residual interactions in the dipole state are taken into account is used to analyze the giant resonance in strongly deformed axially-symmetric nuclei. It is shown that such a model leads to conclusions (which are qualitatively in accordance with the experiments) concerning the relation between the center of gravity and characteristics of the energy spread of the longitudinal and transverse dipole maxima.

1. INTRODUCTION

IT was shown as a result of recent investigations [1-3] that the shell model, previously used to describe the ground and weakly-excited states of the atomic nuclei can be successfully extrapolated to the region of high excitations. Calculations of giant dipole resonance for the doubly-magic nuclei based on this model have led to very good agreement with experiments [1-3]. It is natural to attempt to generalize the method of this model to non-magic nuclei, particularly to deformed nuclei.

The investigation of giant resonance on strongly deformed nuclei is of greatest interest, since it has the largest number of global characteristics (the so-called parameters of optical anisotropy) [4]. The comparison between theory and experiment should in this case be more critical to details of the model than in the case of spherical nuclei.

As is well known, the cross section of dipole photoabsorption in strongly deformed nuclei is split into two resonant peaks, corresponding to the excitation of the dipole oscillations along and transverse to the symmetry axis of the nucleus [5,6]. The frequencies of the longitudinal and transverse maxima are inversely proportional to the lengths of the corresponding semiaxes of the nucleus. The width of the transverse maximum is 1.5 – 2 times larger than the width of the longitudinal one [6].

These properties of the optical-anisotropy parameters are in the remarkable agreement with the predictions of the simple single-particle of Mottelson and Nilsson [7]. At the same time, this model gives an absolute value of the energy of the dipole resonance which is approximately half that obtained in experiment. An account of the residual interactions in the dipole state should bring this value into agreement with experiment [1-3]. The photoabsorption curve obtained with allowance for the residual interaction can differ greatly from the single-particle curve [1], so that it is not clear beforehand whether the deductions of the single-particle model, which agree with the experimental estimates, apply in this case.

The purpose of the present work is to consider in general outline the effect of residual interaction in the problem of the photoabsorption of strongly deformed nuclei, the single-particle variant of which was formulated by Nilsson and Mottelson. It will be shown that the inclusion of the residual interaction in this scheme does not change the deductions of the single-particle model concerning the ratio of the frequencies and characteristics of the energy scatter of the longitudinal and transverse maxima.

2. POSITION OF DIPOLE MAXIMA

Let us consider dipole excitation of a nucleus, the ground state of which will be described by a wave function of particles moving independently in an ellipsoidal axially-symmetrical potential.[1)] The average value of the energy in the dipole state is [2,3]

$$E = \sum_i (d|i)\,\varepsilon_i\,(i|d) + \sum_{i,j} (d|i)\,(i|V|j)\,(j|d), \quad (1)$$

where $|i)$ is the particle-hole state, V the potential of effective residual interaction, and ϵ_i the single-particle energy.

We write the self-consistent potential in the form

[1)] The problem is considered in a coordinate system rigidly fixed in the nucleus.

$$U = \tfrac{1}{2} m [\omega_\perp^2 (x^2 + y^2) + \omega_z^2 z^2]$$

$$+ alS + \beta v\{[m\omega_\perp^2 (x^2+y^2)+m\omega_z^2 z^2]/\hbar\omega_0\}, \quad (2)$$

where $\omega_0 = \sqrt[3]{\omega_\perp^2 \omega_z}$, and we assume that the following condition holds true

$$\frac{\alpha, \beta}{(\omega_\perp - \omega_z)/\omega_0} \ll 1. \quad (3)$$

In this case the single-particle state can be described by the asymptotic wave function [7] $|n_z, n_\perp, \Lambda, \sigma)$, where n_z and $n_\perp$ are the oscillator quantum numbers, while Λ and σ are the projections of the orbital and spin momenta on the symmetry axis of the nucleus.

In the asymptotic approximation, the transitions excited by the longitudinal (z) and transverse components of the dipole moment obey the following respective selection rules:

$$\Delta n_z = 1, \quad \Delta n_\perp = \Delta\Lambda = \Delta\sigma = 0, \quad (4)$$

$$\Delta n_z = 0, \quad \Delta n_\perp = 1, \quad \Delta\Lambda = 1, \ \Delta\sigma = 0, \quad (5a)$$

$$\Delta n_z = 0, \quad \Delta n_\perp = 1, \quad \Delta\Lambda = -1, \ \Delta\sigma = 0. \quad (5b)$$

If the residual forces are central, the states formed by transitions (4), (5a), and (5b) are mutually orthogonal.

For simplicity we put N = Z, and also assume that for given n_z and $n_\perp$ all the states with possible Λ and σ are filled.

Putting

$$V_{ij} = -V_0 \delta(\mathbf{r}_i - \mathbf{r}_j) P(\tau_i, \tau_j),$$

we have respectively for the displacements of the z- and x, y-maxima due to the residual interactions

$$\Delta_z = \frac{V_0 F}{\Sigma (n_z + 1)} \sum\sum \int \psi^*_{n_z+1, n_\perp, \Lambda} \psi^*_{n_z', n_\perp', \Lambda'} \sqrt{(n_z' + 1)(n_z + 1)}$$
$$\times \psi_{n_z'+1, n_\perp', \Lambda'} \psi_{n_z, n_\perp, \Lambda} d\tau, \quad (6)$$

$$\Delta_\perp = \frac{V_0 F/2}{\Sigma (n_\perp + \Lambda + 1)} \sum\sum \int \psi^*_{n_z, n_\perp+1, \Lambda+1} \psi^*_{n_z', n_\perp', \Lambda'}$$
$$\times \sqrt{(n_\perp' + \Lambda' + 2)(n_\perp + \Lambda + 2)} \psi_{n_z', n_\perp'+1, \Lambda'+1} \psi_{n_z, n_\perp, \Lambda} d\tau. \quad (7)$$

Here F is a factor that includes the spin and charge variables. The summation in (6) is over the coordinate states from which the transitions (4) are possible, while in (7) the summation is over states from which the transitions (5a) are possible.

We note further that the order of level filling in an oscillator of spheroidal potential is determined by the condition

$$\hbar\omega_\perp (n_\perp + 1) + \hbar\omega_z (n_z + 1/2) \leqslant \mathscr{E}_{max} \quad (8)$$

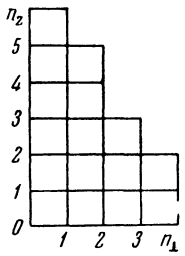
and can be graphically represented as an aggregate of occupied cages in the n_z, $n_\perp$ plane, up to the "Fermi boundary":

$$\hbar\omega_\perp (n_\perp + 1) + \hbar\omega_z (n_z + 1/2) = \mathscr{E}_{max}.$$

Condition (8) denotes a tendency toward a preferred filling of the states with large n_z and small $n_\perp$, so that

$$2 \sum_i (n_z + 1/2)^i \Big/ \sum_i (n_\perp + 1)_i = \omega_\perp / \omega_z \quad (9)$$

($\sum_i$ denotes summation over all the nucleons).



For example, for $n_z \leq 5$ and $\omega_\perp/\omega_z \approx 1.4$–1.5 we have the filling scheme shown in the diagram. It is easily seen from the foregoing that the number of transitions (4) is $\omega_\perp/\omega_z$ times larger than the number of (5a) or (5b) transitions, and that these transitions initiate at different states, by virtue of which $\Delta_\perp$ and Δ_z should differ. Direct calculation for the scheme shown in the figure yields $\Delta_\perp/\Delta_z \approx 1.4$, that is,

$$\Delta_\perp/\Delta_z = \omega_\perp/\omega_z. \quad (10)$$

As $A \to \infty$ (classical limit) relations (10) can be proved in general form. Using the equality

$$|n_z + 1) = \frac{1}{\sqrt{2(n_z+1)}} \left(\zeta - \frac{\partial}{\partial\zeta}\right) |n_z), \quad \zeta = \sqrt{\frac{m\omega_z}{\hbar}}\, z,$$

we rewrite Δ_z in the form

$$\Delta_z = \frac{1}{\Sigma (n_z + 1)} \Sigma (n_z + 1)$$
$$+ [(n_z + 1, n_\perp, \Lambda | U_2 + U_1 | n_z + 1, n_\perp, \Lambda)$$
$$- (n_z, n_\perp, \Lambda | U_2 + U_1 | n_z, n_\perp, \Lambda)]; \quad (11)$$

$$U_1 = -\tfrac{1}{2} F V_0 \sum |\psi_{n_z' n_\perp' \Lambda'}|^2, \qquad \frac{\partial}{\partial\zeta} U_2 = -2\zeta U_1. \quad (12)$$

In the calculation of U_1 the quantities $|\psi|^2$ are replaced by the classical density distribution, for example

$$|\psi_{n_z}|^2 \sim [\hbar\omega_0/(\hbar\omega_z n_z - m\omega_z^2 z^2/2)]^{1/2}, \quad m\omega_z^2 z^2/2 < \hbar\omega_z n_z,$$
$$|\psi_{n_z}|^2 = 0, \quad m\omega_z^2 z^2/2 \geqslant \hbar\omega_z n_z,$$

and the summation is replaced by the corresponding integration. As a result we obtain

$$U_2 = -{}^1/_2 FV_0\rho\,[m\omega_\perp^2(x^2+y^2) + \omega_z^2 z^2]/\hbar\omega_0, \quad (13)$$

where ρ is the nucleon density in the nucleus;

$$U_1 = {}^1/_2\,\hbar\omega_z \partial U_2/\partial \mathscr{E}_{\max}; \quad U_1/U_2 \sim A^{-1/3} \to 0. \quad (14)$$

The arguments for $\Delta_\perp$ are perfectly analogous.

Thus, $\Delta_\perp$ and Δ_z are equal to the average values of transitions (4) and (5) in the ellipsoidal potential (13). From this follows directly

$$\Delta_z/\Delta_\perp = \omega_z/\omega_\perp.$$

Thus, an account of the residual δ-forces in the dipole state does not violate the conclusion of the single-particle model concerning the localization of the z- and x, y-transitions at different energies, and conserves the ratio of these energies.[2)]

3. VARIANCES OF THE DIPOLE MAXIMA

The energy spread of the z- and x, y-transitions will be characterized, as usual, with the aid of the variances

$$D_{z,\perp} = \overline{E_{z,\perp}^2} - \overline{E_{z,\perp}}^2. \quad (15)$$

According to this model we have[2,3]

$$D = \sum (d|i)(i|V|j)(j|V|k)(k|d) + 2\sum (d|i)(i|V|j)\varepsilon_j (j|d) + \sum (d|i)\varepsilon_i^2 (i|d) - \Big[\sum (d|i)(i|V|j)(j|d) + \sum (d|i)\varepsilon_i (i|d)\Big]^2. \quad (16)$$

As was shown by Nilsson and Mottelson, in the case when $V = 0$ the energy variance of transitions (5a) [or (5b)] is three times larger than the energy variance of transitions (4). Let us show that an account of the residual interactions ($V \neq 0$) does not change this result.

When estimating the ratio of the variances we shall neglect the "scale factor," that is, we put $\omega_\perp = \omega_z$. In this case the transitions can be regarded as coming from a filled shell $n_z + n_\perp = n$. Thus, we are estimating the effect connected with the difference in the selection rules for the most intense z- and x, y-transitions.

We shall put in the formulas that follow $\omega_z = \omega_\perp = \hbar = m = 1$. Using a procedure perfectly analogous with that employed in Sec. 2, we write $D_\perp$ and D_z in the form of the variances of transitions (5a) and (4) respectively in a certain effective potential.

Let us consider, for example, the transitions (4). For the residual δ-forces we have

$$(d_z|i)\,\varepsilon_i = \frac{1}{\Sigma(n_z'+1)}\sqrt{n_z+1} + [(n_z+1, n_\perp, \Lambda|U|n_z+1, n_\perp, \Lambda) - (n_z, n_\perp, \Lambda|U|n_z, n_\perp, \Lambda)] = \frac{1}{\Sigma(n_z'+1)}(n_z+1, n_\perp, \Lambda\Big|\frac{1}{\sqrt{2}}\frac{\partial U}{\partial z}\Big|n_z, n_\perp, \Lambda), \quad (17)$$

$$\sum (i|V|j)(j|d) = \frac{-V_0 F}{\Sigma(n_z'+1)}$$

$$\sum_{n_{z'}, n_\perp', \Lambda'} \int \psi^*_{n_z+1, n_\perp, \Lambda} \psi^*_{n_{z'}, n_\perp', \Lambda'} \sqrt{n_z'+1}\, \psi_{n_{z'}+1, n_\perp', \Lambda'} \psi_{n_z, n_\perp, \Lambda}\, d\tau = \frac{1}{\Sigma(n_z'+1)}(n_z+1, n_\perp, \Lambda\Big|\frac{1}{\sqrt{2}}\frac{\partial U^e}{\partial z}\Big|n_z, n_\perp, \Lambda); \quad (18)$$

$$\frac{\partial U^e}{\partial z} = FV_0\left(z - \frac{1}{2}\frac{\partial}{\partial z}\right)\sum |\psi_{n_{z'} n_\perp' \Lambda'}|^2. \quad (19)$$

Substituting (18) in (16), we reduce the problem to the analysis of the variance of single-particle energies of transitions (4) and a potential

$$U' = U + U^e. \quad (20)$$

In the case of residual central forces of arbitrary form, the result is perfectly analogous, except that the "effective potential" U^e can be non-local. We note further that the product of the wave functions entering into the matrix elements (17) and (18) can be written in the form

$$re^{-r^2}\sum_{i=0}^{n} g_i(\Omega)\,\alpha_i F\left(-i, \tfrac{5}{2}, r^2\right),$$

where $g_i(\Omega)$ are functions of the angles and $F(-i, 5/2, r^2)$ are confluent hypergeometric functions which are orthogonal with weight $r^4 \exp(-r^2)$. This means that for the shell n the total form of the anharmonicity of the potential U', which leads to the energy scatter of the dipole transitions, is as follows:

$$u = \int_0^{r^2} dr^2 \sum_{i=1}^{n} \alpha_i F\left(-i, \tfrac{5}{2}, r^2\right), \quad (21)$$

that is, it can be characterized by n coefficients α_i.

For all nuclei $n \leq 6$; putting $n = 4$ (the rare-earth region), we obtain as a result of calculations analogous to those of [7] for arbitrary anharmonicity

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[2)] It is interesting to note that within the framework of the given model the residual dipole-dipole interaction of the type $\tau_1\tau_2 r_1 r_2$ leads to a violation of the optical anisotropy effect, yielding a ratio

$$\Delta_z/\Delta_\perp = \omega_\perp/\omega_z.$$

$$D_\perp = kD_z + (a/2)^2, \qquad (22)$$

where $2 < k \leq 3$. An account of the scale factor increases k by approximately $(\omega_\perp/\omega_z)^2$ times. The result (22) is also close to the corresponding deduction of the single-particle model.

4. CONCLUSIONS

The model considered makes it possible to draw the following conclusions regarding the form of the dipole resonance of axially-symmetrical strongly deformed nuclei.

Transitions excited by the longitudinal and transverse components of the dipole moment are localized at energies inversely proportional to the lengths of the corresponding semi-axes. This result has been obtained for zero-radius forces. The long-range residual forces can lead to a violation of this conclusion and to discrepancy with experiment. For arbitrary central residual interactions, the dispersion of the transitions (5) is approximately three times larger than the dispersion of transitions (4). This circumstance corresponds to some degree to the experimental fact that the second maximum is always broader than the first.

A more detailed comparison with experiment is hardly possible at the present point, since the variance calculated here and the experimentally determined width of the curve at half its height are generally speaking independent characteristics of its shape.

Recognizing that this model leads to a correct estimate of the energy of the entire dipole maximum (this has been proved by numerous calculations for spherical nuclei) [1-3], it can be assumed that the model under consideration affords a description of dipole resonance in strongly deformed nuclei without contradicting the experiment.

The author is grateful to A. M. Baldin for continuous interest in the work.

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Translated by J. G. Adashko
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