CALCULATION OF THE 2p → 1s TRANSITION ENERGY IN μ-MESIC ATOMS

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Submitted to JETP editor June 22, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 2170-2172 (December, 1962)

The 2p → 1s transition energy in μ-mesic atoms is calculated for $13 \leq Z \leq 57$ and various models of charge distribution in the nucleus. When a Fermi-type model is employed in conjunction with the parameters found by Hofstadter et al from electron scattering on nuclei, the calculations are found to be in satisfactory agreement with the experimental data on the radiation from μ-mesic atoms with the indicated values of Z.

In connection with the recent experimental data on μ-mesic atoms,[1,2] we carried out a more accurate calculation of the 2p → 1s transition energy in μ-mesic atoms for a number of elements between $_{13}$Al and $_{57}$La by means of a method developed by us earlier.[3,4] The calculations were performed for a smoothed homogeneous charge distribution (Fermi model):

$$\rho(r) = \rho_0 \{1 + \exp[(r-c)/z]\}^{-1}. \quad (1)$$

The parameters of this distribution (its half-width c and surface layer thickness $t = 2z \ln 9 \approx 4.4z$) were found by Hofstadter and co-workers from high-energy electron scattering on nuclei (see, for example, [5]). For the calculation, we took the average value of the parameters:

$$c = 1.08 A^{1/3} \text{ F and } t = 2.4 \text{ F}, \quad (2)$$

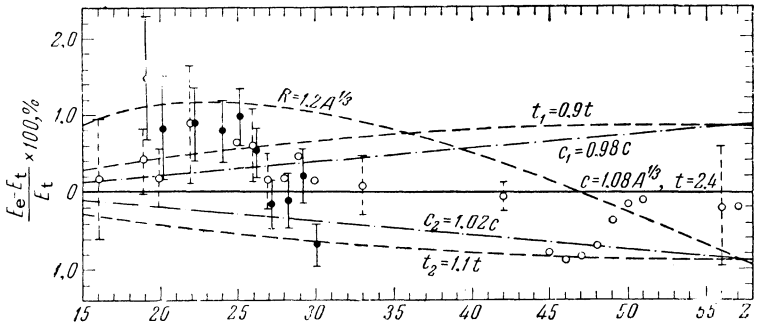
where A is the atomic weight of the natural isotope mixture.

In the final results, we took into account the corrections for the energy levels as a result of the vacuum polarization, with allowance for the finite size of the nucleus.[6] However, we did not take into account corrections for the polarization of the nucleus by the meson. An estimate made by means of the Steinwedel-Jensen model[7] shows that, with the accuracy of our calculations (somewhat greater than 0.1%), these corrections can be omitted. On the other hand, according to other earlier estimates of these quantities, these corrections can amount to 0.5—1% of the energy level for heavy elements.[8] Their exact calculation requires individual consideration not only of each element but also of each isotope, and it is therefore very difficult.

The table lists the calculated energies (in keV) of the $1s_{1/2}$ ($E_{1s_{1/2}}$) level; for the 2p level, the position of the center of gravity ($E_{2p_{c.g.}}$) and the value of the relativistic splitting ($E_{2p_{3/2}} - E_{2p_{1/2}}$) are given.

The figure shows the relative deviation (in percent) of the experimental values of the E_e energy transition from the calculated values $E_t = E_{2p_{c.g.}} - E_{1s_{1/2}}$ (black circles—data from [1], open circles—data from [2]). Shown in the same figure are the relative changes in E_t for changes in the individual parameters of the distribution (c by ±2% and t by ±10%). Moreover, we also show the

Element	$-E_{1s_{1/2}}$	$-E_{2p_{c.g.}}$	$E_{1p_{3/2}} - E_{2p_{1/2}}$	Element	$-E_{1s_{1/2}}$	$-E_{2p_{c.g.}}$	$E_{2p_{3/2}} - E_{2p_{1/2}}$
$_{13}$Al	465	119	0.3	$_{33}$As	2639	773	9.8
$_{16}$S	695	180	0.6	$_{37}$Rb	3209	973	14.8
$_{19}$K	966	255	1.2	$_{42}$Mo	3970	1256	23.5
$_{20}$Ca	1065	282	1.5	$_{45}$Rh	4443	1443	29.8
$_{22}$Ti	1271	342	2.2	$_{46}$Pd	4603	1508	32.4
$_{24}$Cr	1493	407	2.9	$_{47}$Ag	4764	1574	35.1
$_{25}$Mn	1609	442	3.5	$_{48}$Cd	4919	1642	37.8
$_{26}$Fe	1730	479	4.0	$_{49}$In	5084	1711	40.5
$_{27}$Co	1851	516	4.6	$_{50}$Sn	5241	1782	43.4
$_{28}$Ni	1979	555	5.3	$_{51}$Sb	5404	1854	45.5
$_{29}$Cu	2104	596	6.1	$_{56}$Ba	6228	2235	62.2
$_{30}$Zn	2236	638	6.9	$_{57}$La	6401	2314	65.8



difference between the values of the energy of this transition calculated for a homogeneous distribution of nuclear charge, whose radius is $R = 1.2\,A^{1/3}$ F, and values for the distribution (1) with the parameters (2).

For some elements, the values of the parameters c and t were found directly from experiments on electron scattering.[5] However, the results of the calculation with these values of the parameters do not differ essentially from the results of the calculation with the averaged parameters (2), since the deviations of c and t from the mean values in these cases practically offset each other.

As is seen from the figure, the deviations of the experimental data from the calculated values are less than 1% in all cases (except for K, and in this case only for the data from [1]). Since the parameters c and t were determined by Hofstadter with some uncertainty (about ±2% for c and ±10% for t), it can be concluded that the experiments on electron scattering and on the study of μ-mesic atoms are in good agreement with one another. Nevertheless, in the region between Ca and Zn, the nuclei are apparently more compact than should follow from the distribution (1) with the parameters (2).

We also attempted to calculate the charge distribution of the nucleus suggested by Ford and Hill (the so-called family II)[9]:

$$\rho(r) = \rho_0\left[1 - \tfrac{1}{2}\exp(-n)\right]^{-1} \times \begin{cases} 1 - 1/2\exp[-n(1-r/c)], & r \leqslant c \\ 1/2\exp[n(1-r/c)], & r \geqslant c \end{cases} \quad (3)$$

Here c is the half-width of the distribution, while the thickness of the surface layer t is determined by the expression nt = 2c ln 5. It turned out that, for the distribution (3) with the parameters (2), the transition energy agrees with the values obtained for the distribution (1) with the same value of c, but for t increased by 10%. As can be seen from the figure, the experimental data in this case is in poorer agreement with the distribution (3) than with the distribution (1). If it is concluded that the distribution (3) is in agreement with experiments on electron scattering at somewhat higher values of t (on the average, about 2.6 F), then the disagreement with experiment can only increase.

The authors are very grateful to E. P. Hinks and P. Brix for sending preprints.

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Translated by E. Marquit
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