RADIATION OF LOW FREQUENCY WAVES BY IONS AND ELECTRONS IN A MAGNETOACTIVE PLASMA

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The radiation of ion-cyclotron and magnetohydrodynamic waves by the ions and electrons of a high-temperature plasma in a magnetic field is considered. The emission and absorption coefficients of the plasma and the equilibrium intensity of the radiation are determined.

1. INTRODUCTION

AN investigation of the study of ion-cyclotron ($\omega \sim \omega_{Hi}$) and magnetohydrodynamic ($\omega \ll \omega_{Hi}$) waves by ions and electrons moving along a helix in a magnetoactive plasma is of interest for many problems in plasma physics. An experimental investigation of the thermal radiation of plasma in the frequency region $\omega \lesssim \omega_{Hi}$ enables us to determine the temperature of the ion and electron gases and the plasma density. The radiation of ion-cyclotron waves was observed in the "Ogra" apparatus on passage of fast ions through a plasma [1], and in investigations of the low-frequency radiation due to solar corpuscular streams passing through the upper atmosphere of the earth [2].

Cyclotron radiation of "fast" ions ($|\mathbf{v}| \gg v_{Te}$) which move along a helix in an unbounded plasma was considered by Pistunovich and Shafranov [3]. In the present work we consider cyclotron radiation of ions having a velocity $|\mathbf{v}|$ on the order of the mean thermal velocity of the plasma ions v_{Ti}. In addition, we consider the Cerenkov radiation of the electron in the low frequency region [1)]. The radiation intensity is determined with account of the cyclotron absorption of the radiated waves by the plasma ions and of Cerenkov absorption by the plasma electrons. The radiating and absorbing capacity of a plasma and the equilibrium radiation intensity are also determined for the investigated frequency regions.

The expressions obtained for the radiation intensity of the individual particle can be used also to estimate the radiation intensity of clusters of charged particles passing through a plasma. If the cluster dimension is smaller than the length of the radiated wave $\lambda\!\!\!^{-} = c/\omega n$, where n is the refractive index, then the radiation intensity is proportional to the square of the total charge of the cluster $(Ne)^2$, i.e., to the square of the number of particles in the cluster N^2 (coherent radiation of the cluster). In the low-frequency case under consideration, the wavelength is large and therefore the radiation can be coherent even at relatively large cluster dimensions.

2. CYCLOTRON RADIATION OF IONS

An ion moving along a helix produces in a plasma a current whose density is equal to

$$\mathbf{j} = e\mathbf{v}_i\,\delta(\mathbf{r} - \mathbf{r}_i),$$

where

$$\mathbf{v}_i(t) = d\mathbf{r}_i/dt,$$

$$\mathbf{r}_i(t) = (r_0 \cos\omega_{Hi}t,\ -r_0 \sin\omega_{Hi}t,\ v_\parallel t);$$

$v_\parallel$ and $v_\perp$ are the components of the ion velocity $\mathbf{v}_i$ parallel and perpendicular, respectively, to the external magnetic field $\mathbf{H}_0$; $r_0 = v_\perp/\omega_{Hi}$; $\omega_{Hi} = eH_0/Mc$ is the gyrofrequency of the ion. To obtain the intensities of the electric and magnetic fields produced by such a current we can use the previously obtained expressions (2.10) from [5], where cyclotron radiation of the electrons was considered. For this purpose it is necessary to make in the indicated expressions the substitution

$$\omega_H \to \omega_{Hi}, \qquad v_\parallel \to -v_\parallel,$$

$$\delta(\omega - s\omega_H - k_\parallel v_\parallel) \to \delta(\omega + s\omega_{Hi} - k_\parallel v_\parallel)$$

(the substitution $v_\parallel \to -v_\parallel$ is not made under the δ-function sign).

As noted earlier [5], integration over $k_\parallel$ and $k_\perp$ (the components of the wave vector $\mathbf{k}$ parallel and perpendicular, respectively, to $\mathbf{H}_0$) is possible only

[1)]Low frequency Cerenkov radiation of a "fast" charge moving along the axis of a plasma-filled waveguide was investigated by Gorbatenko, Kurilko, and Faĭnberg[4].

in the case of weakly damped radiated waves. It is known[6,7] that in the frequency region $\omega \lesssim \omega_{Hi}$ the damping of the normal waves (ordinary and extraordinary) is small if the frequency of the wave is not too close to the gyrofrequency ω_{Hi} and if in addition the pressure of the ion gas $p_i = n_0 T_i$ is much smaller than the magnetic pressure $p_H = H_0^2/8\pi$. If ω is close to ω_{Hi}, then when $p_H \gg p_i$ only one wave propagates; the propagation of the other wave is impossible because of the strong cyclotron damping. When $p_H \sim p_i$, the propagation of both waves with frequency $\omega \sim \omega_{Hi}$ is impossible because of the strong Cerenkov damping in the ion gas and the cyclotron damping at both the first harmonic and at the multiple harmonic. On the other hand, if $p_H \gg p_i$, then the phase velocity of the waves is much larger than the mean thermal velocity of the ions, and therefore Cerenkov damping of waves with frequency $\omega \sim \omega_{Hi}$ is exponentially small. Cyclotron damping at $p_H \gg p_i$ is always small, even if ω is close to $s\omega_{Hi}$ ($s = 2, 3, \ldots$). On the other hand, if ω is not close to $s\omega_{Hi}$, then the cyclotron damping is exponentially small. In this case it is necessary to take into account only the Cerenkov absorption of the waves by the plasma electrons. We shall assume below that the condition $p_H \gg p_i$ is satisfied. The absorption of waves due to short-range collisions will not be taken into account.

We consider first cyclotron radiation of the ions, assuming that the radiated frequency is not very close to ω_{Hi}, so that the inequality $|\omega - \omega_{Hi}| \gg k_\perp v_{Ti}$ is satisfied. In this case the coefficients a, b, ..., which determine the dielectric tensor of the plasma

$$\varepsilon_{ij} = a\delta_{ij} + b\varkappa_i\varkappa_j + ch_i h_j + d\varepsilon_{ijk} h_k + e(\varkappa_i \varepsilon_{jkl}\varkappa_k h_l - \varkappa_j \varepsilon_{ikl}\varkappa_k h_l) + f\varepsilon_{ikl}\varkappa_k h_l \varepsilon_{jmn}\varkappa_m h_n$$

(where $\boldsymbol{\kappa} = \mathbf{k}/k$ and $\mathbf{h} = \mathbf{H}_0/H_0$) have the form

$$a = \varepsilon_1 + 2i\sigma, \quad b = 0, \quad c = -a + 2v_i z_e^2 q(z_e)\, M/m,$$

$$d = i\varepsilon_2 - 2\bar{\sigma} - i\,\mathrm{tg}^2\theta\,\frac{v_i\omega}{\omega_{Hi}}\, q(z_e), \quad e = -i\,\frac{v_i\omega}{\omega_{Hi}\cos^2\theta}\, q(z_e),$$

$$f = i\sqrt{\pi}\,\frac{mv_i}{Mu_i}\,\frac{w(z_e)}{z_e\cos^2\theta}. \qquad (1)^*$$

We assume here the following notation:

$$\varepsilon_1 = 1 - \frac{v_i}{1-u_i}, \quad \varepsilon_2 = -\frac{v_i}{\sqrt{u_i}(1-u_i)}, \quad \sigma = \sum_{l=-\infty}^{\infty}\sigma_l,$$

$$\bar{\sigma} = \sum_{l=-\infty}^{\infty}\bar{\sigma}_l,$$

$$\sigma_l = \frac{l}{|l|}\bar{\sigma}_l = \sqrt{\frac{\pi}{8}}\,\frac{v_i l^2 (l\beta n\sin\theta)^{2|l|-2}}{2^{|l|}|l|!\,\beta n|\cos\theta|}\exp(-z_l^2),$$

$$z_l = \frac{\omega - l\omega_{Hi}}{\sqrt{2}k_\parallel v_{Ti}}, \quad v_i = \frac{\Omega_i^2}{\omega^2}, \quad u_i = \frac{\omega_{Hi}^2}{\omega^2}, \quad \Omega_i^2 = \frac{4\pi e^2 n_0}{M},$$

$$q(z) = 1 + i\sqrt{\pi}zw(z),$$

$$w(z) = e^{-z^2}\left(\mathrm{sign}\, k_\parallel + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right), \quad z_e = \frac{\omega}{\sqrt{2}k_\parallel v_{Te}},$$

$$v_{Te} = \sqrt{\frac{T_e}{m}}, \quad v_{Ti} = \sqrt{\frac{T_i}{M}}, \quad \beta = \frac{v_{Ti}}{c}, \quad n = \frac{kc}{\omega},$$

$$n_\parallel = \frac{k_\parallel c}{\omega} = n\cos\theta, \quad n_\perp = \frac{k_\perp c}{\omega} = n\sin\theta,$$

T_i and T_e are the temperatures of the ion and electron gases, m is the electron mass, M is the ion mass, m_0 is the electron density which is equal to the ion density.

Recognizing that c is appreciably larger than a, d, e, f, and $|n|^2$, we obtain the solution of the dispersion equation for $n_\parallel$:

$$n_\parallel = n_{\parallel j}(n_\perp) + in'_{\parallel j}(n_\perp), \qquad (2)$$

where

$$n_{\parallel j}^2 = \tfrac{1}{2}\left(2\varepsilon_1 - n_\perp^2 \pm \sqrt{n_\perp^4 + 4\varepsilon_2^2}\right); \qquad (3)$$

$$n'_{\parallel j} = n'_{ej} + \sum_{l=-\infty}^{\infty} n'_{lj}; \qquad (4)$$

$$n'_{ej} = \frac{\sqrt{\pi}\,mv_i\sin^2\theta Q}{4Mz_e n_\parallel\cos^2\theta\,(2n_\parallel^2 + n_\perp^2 - 2\varepsilon_1)}\exp(-z_e^2),$$

$$Q = \frac{n_\parallel^2 - \varepsilon_1}{u_i} + \frac{n_\parallel^2(\varepsilon_1 n^2 - \varepsilon_1^2 + \varepsilon_2^2)}{v_i^2|q(z_e)|^2}; \qquad (5)$$

$$n'_{lj} = \sigma_l\,\frac{2n_\parallel^2 + n_\perp^2 - 2\varepsilon_1 + 2(l/|l|)\varepsilon_2}{n_\parallel(2n_\parallel^2 + n_\perp^2 - 2\varepsilon_1)}. \qquad (6)$$

The attenuation coefficient n'_e is brought about by the Cerenkov absorption of the waves by the plasma electrons. The attenuation coefficients n'_l ($l \neq 0$) are due to the presence of cyclotron absorption by the plasma ions under the conditions of normal ($l \geq 1$) and anomalous ($l \leq -1$) Doppler effects. (Expressions for the imaginary part of the refractive index, corresponding to (5) and (6), were derived earlier[6].)

The refractive index of the electromagnetic waves in the region $\omega \lesssim \omega_{Hi}$ (ω not very close to ω_{Hi}) has an order of magnitude

$$n \sim n_A = c/v_A \gg 1,$$

where $v_A = H_0/\sqrt{4\pi n_0 M}$ is the Alfven velocity. We assume that $v_A \ll c$, for when $v_A \lesssim c$ the cyclotron radiation and absorption effects are exceedingly small. The Cerenkov attenuation coefficient is

$$n'_e \sim n_A\, mv_{Te}/Mv_A$$

*tg = tan.

for $v_{Te} \lesssim v_A$. When $v_A \gg v_{Te}$, the coefficient n'_e is exponentially small. The cyclotron attenuation coefficients n'_l are generally speaking different in order of magnitude, therefore it is necessary to maintain in the sum (4) only the largest term. When $|z_l| \lesssim 1$ we have in order of magnitude $n'_l \sim n_A(\beta n_A)^{2l-3}$. The Cerenkov absorption of the waves by the plasma ions is always exponentially small, so that

$$|z_0| \sim (\beta n_A)^{-1} \sim \sqrt{p_H/p_i} \gg 1.$$

If $\omega \to \omega_{Hi}$, we obtain from (3) for a weakly damped wave

$$n_\parallel^2 = \tfrac{1}{2}(n_A^2 - n_\perp^2). \quad (7)$$

$$n'_e = \frac{\sqrt{\pi} m n_A^2 \sin^2\theta}{8 M z_e n_\parallel \cos^2\theta}\left(1 + \frac{n_\parallel^4}{n_A^4 |q(z_e)|^2}\right)\exp(-z_e^2), \quad (8)$$

$$n'_1 = \sqrt{\frac{\pi}{2}}\,\frac{n_\perp^4 (1-u_i)^2}{32\beta n_\parallel^2 n_A^2}\exp(-z_1^2). \quad (9)$$

Inasmuch as $\omega \approx \omega_{Hi}$, we have

$$|n'_1| \gg |n'_l| \; (l \neq 1).$$

The damping coefficient (9) is exponentially small by virtue of the assumed condition

$$|z_1| \sim |1 - \omega_{Hi}/\omega|(\beta n_A)^{-1} \gg 1.$$

For the other wave

$$n_\parallel^2 \sim n_\perp^2 \sim |\varepsilon_1|, \qquad n'_1 \sim (1-u_i)^2 \exp(-z_1^2)/\beta,$$

where

$$z_1 \sim (1 - \omega_{Hi}/\omega)^{3/2}(\beta n_A)^{-1} \gg 1,$$

i.e., the cyclotron damping for this wave is much larger.

In the low-frequency region ($\omega \ll \omega_{Hi}$) expressions (3) simplify to

$$n_\parallel^2 = n_A^2 - n_\perp^2, \quad (10)$$

$$n_\parallel^2 = n_A^2\left(1 + \frac{n_A^2}{u_i n_\perp^2}\right) \quad \left(\sin^2\theta \gg \frac{1}{\sqrt{u_i}}\right). \quad (11)$$

The first of these expressions determines the refractive index of the "fast" magnetic-sound wave, and the second determines the refractive index of the Alfven wave. In this case the damping coefficients of these waves are respectively

$$n'_\parallel = \sqrt{\frac{\pi}{8}}\left(\frac{m}{M}\right)\beta_e n_A^2 \operatorname{tg}^2\theta \exp(-z_e^2),$$

$$z_e = \frac{1}{\sqrt{2}\,\beta_e\sqrt{n_A^2 - n_\perp^2}}, \quad (12)$$

$$n'_\parallel = \sqrt{\frac{\pi}{8}}\,\frac{m\omega^2\beta_e n_A^4}{M\omega_{Hi}^2 n_\perp^2}(1+\xi)\exp(-z_e^2), \quad z_e = \frac{1}{\sqrt{2}\,\beta_e n_A}, \quad (13)$$

where

$$\xi = n_\perp^4/n_A^4 |q(z_e)|^2.$$

Cyclotron damping of magnetohydrodynamic waves and Cerenkov damping in an ion gas are exponentially small. The expressions for the imaginary part of the refractive index of a magnetic-sound wave, corresponding to (12), were obtained in [8-10] (we note that a factor $\sqrt{T_e/T_i}$ was omitted in the expression for the damping coefficient in [10], and a factor $1+\xi$ was omitted in the expression for the damping coefficient of the Alfven wave in [8]).[2)]

The expressions for the electric and magnetic field intensities in the wave zone, produced by the moving ion, have the form

$$\mathbf{E} = \sum_{j}\sum_{s=-\infty}^{\infty}\mathbf{E}_s^{(j)}, \qquad \mathbf{H} = \sum_{j}\sum_{s=-\infty}^{\infty}\mathbf{H}_s^{(j)}; \quad (14)$$

$$E_{\varphi s}^{(j)} = P_{sj}\sin\Psi_{sj}\,[(\varepsilon_1 - n_{\parallel j}^2)J'_{|s|} - s\varepsilon_2 J_{|s|}/k_\perp r_0],$$

$$E_{\chi s}^{(j)} = -P_{sj}\cos\chi\cos\Psi_{sj}\,[\varepsilon_2 J'_{|s|} - s(\varepsilon_1 - n^2)J_{|s|}/k_\perp r_0],$$

$$H_{\varphi s}^{(j)} = -P_{sj}\, n\cos\theta\cos\Psi_{sj}\,[\varepsilon_2 J'_{|s|} - s(\varepsilon_1 - n^2)J_{|s|}/k_\perp r_0],$$

$$H_{\chi s}^{(j)} = -P_{sj}\, n\cos(\chi-\theta)\sin\Psi_{sj}\,[(\varepsilon_1 - n_{\parallel j}^2)J'_{|s|} - s\varepsilon_2 J_{|s|}/k_\perp r_0]; \quad (15)$$

$$P_{sj} = \frac{2(-1)^{j-1}e\omega\beta_\perp}{cR|1 - \beta_\parallel d\omega n_{\parallel j}/d\omega|} \times \sqrt{\frac{n_\perp}{\sin\chi|\cos\chi\cdot d^2 n_\parallel/dn_\perp^2|}\,\frac{\exp(-\varkappa_{sj}R)}{n_{\parallel j}(n_{\parallel 1}^2 - n_{\parallel 2}^2)}}. \quad (15')$$

Here and below the argument of the Bessel functions $J_{|s|}$ and their derivatives $J'_{|s|}$ is $k_\perp r_0$. The phase Ψ_{sj} and the damping coefficient κ_{sj} are

$$\Psi_{sj} = k_{\perp j}R\sin\chi + k_{\parallel j}(k_{\perp j})R\cos\chi - \omega_{sj}t - s\pi/2 - \pi/4 - s\varphi + (\pi/4)\operatorname{sgn}(\cos\chi\, d^2 n_{\parallel j}/dn_\perp^2),$$

$$\varkappa_{sj} = c^{-1}\omega_{sj} n'_{\parallel j}\cos\chi, \quad (16)$$

where $n'_{\parallel j}$ is determined by formulas (4)–(6). The connection between $n_{\perp j}$ and the angle χ is determined by the expression

$$\frac{dn_{\parallel j}}{dn_{\perp j}} = -\frac{n_{\perp j}(\sqrt{n_{\perp j}^4 + 4\varepsilon_2^2} \mp n^2{}_{\perp j})}{2n_{\parallel j}\sqrt{n_{\perp j}^4 + 4\varepsilon_2^2}} = -\operatorname{tg}\chi. \quad (17)$$

[2)] In this connection it is necessary to add to formula (36) of [8]

$$q''' = -\beta_i^2 n_A^2 \operatorname{tg}^2\theta / 2u_i(1 + T_i/T_e - I_i - I_e T_i/T_e),$$

after which formula (40) of [4] becomes

$$\left(\frac{\gamma}{\omega}\right)_1 = \frac{\beta_i^3 n_A^3}{u_i\sqrt{8\pi}}\left(\cot^2\theta - 3 + \tfrac{17}{4}\operatorname{tg}^2\theta\right).$$

The frequency of the radiated wave $\omega = \omega_{sj}$ is determined from the radiation condition

$$\omega_{sj} = s\omega_{Hi} + k_{\parallel j}(k_{\perp j})\, v_{\parallel}. \quad (18)$$

Using (14) and (15) for the fields, we obtain the following expression for the intensity of the ion radiation at frequency ω_{sj} per unit solid angle:

$$w_{sj} = w_0 \left(\frac{\omega_{sj}}{\omega_{Hi}}\right)^2 U_{sj} \exp(-2\varkappa_{sj} R),$$

$$U_{sj} = \sin\theta \{[\varepsilon_2 J_s' - s(\varepsilon_1 - n_j^2) J_s / k_\perp r_0]^2 \cos\chi \cos\theta + [(\varepsilon_1 - n_{\parallel j}^2) J_s' - s\varepsilon_2 J_s / k_\perp r_0]^2 \cos(\chi - \theta)\} \times \left\{\sin\chi \cos^2\theta\, (n_{\parallel 1}^2 - n_{\parallel 2}^2)^2 \left|\cos\chi \frac{d^2 n_{\parallel j}}{dn_{\perp j}^2}\right| \times \left(1 - \beta_\parallel \frac{d\omega n_{\parallel j}}{d\omega}\right)\right\}^{-1}; \quad (19)$$

where $w_0 = e^2\omega_{Hi}^2\beta_\perp^2/2\pi c$ and $w = 4\pi w_0/3$ is the summary ion radiation intensity in vacuum. Formula (19) with $s = 1, 2, 3, \ldots$ determines the ion radiation under the conditions of the normal Doppler effect, and when $s = -1, -2$, and $-3, \ldots$ — under the conditions of the anomalous Doppler effect. Expression (19) for the cyclotron radiation $(s \neq 0)$ is valid when $|\pi/2 - \theta|^2 \gg m/M$ for both fast ions $(v_\parallel \gg v_{Te})$ and slow ions $(v_\parallel \lesssim v_{Te}$, and also $v_\parallel \lesssim v_{Ti}$ at $s \neq 1)$. When $|\pi/2 - \theta|^2 \gg m/M$, formula (19) coincides with the expression for the intensity of cyclotron radiation of fast ions, obtained by Pistunovich and Shafranov [3]. This agreement is connected with the fact that the expressions for the refractive indices in the frequency region $\omega \sim \omega_{Hi}$ of a cold plasma (fast ions) and of a hot plasma (slow ions) coincide, in spite of the fact that the dielectric tensor is appreciably different for cold and for hot plasma.

In the case $s = 0$ (Cerenkov radiation) formula (19) can be used only for fast ions $v_\parallel \gg v_{Ti}$.

Figures 1 and 2 show schematically plots of the functions $n_{\parallel j}^2(n_\perp^2)$ and $dn_{\parallel j}(n_\perp)/dn_\perp$ (for $n_\parallel > 0$) respectively for the cases $\omega < \omega_{Hi}$ $(u_i > 1)$ and $\omega > \omega_{Hi}$ $(u_i < 1)$. The curve $dn_{\parallel 1}/dn_\perp$ on Fig. 1 has a minimum when $n_\perp < n_A$ if $u_i > 7/3$. When $u_i < 7/3$ this minimum lies in the region $n_\perp > n_A$. When $n_\parallel < 0$ $(\tan\chi < 0)$ the function $dn_\parallel/dn_\perp$ reverses sign.

The solutions of (17) correspond to the point of intersection of the curve $y = dn_\parallel/dn_\perp$ with the line $y = -\tan\chi$. As can be seen from Fig. 1, when $\chi > \chi_{max}$ radiation of one wave with $n_\parallel = n_{\parallel 2}$ is possible. When $\chi < \chi_{max}$ three waves are radiated (one wave with refractive index $n_{\parallel 2}$ and two waves with $n_\parallel = n_{\parallel 1}$; these waves have phase velocities

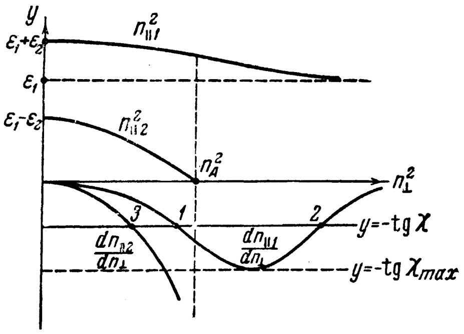

FIG 1

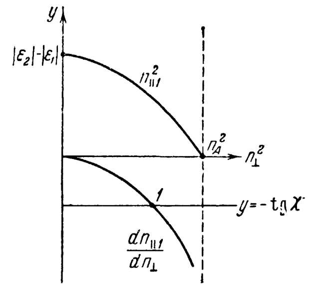

FIG. 2

that differ in value and direction). The angle $\chi = \chi_{max}$ is determined from the equations*

$$dn_{\parallel 1}/dn_\perp = -\text{tg}\,\chi_{max}, \qquad d^2 n_{\parallel 1}/dn_\perp^2 = 0.$$

When $\omega > \omega_{Hi}$, as can be seen from Fig. 2, radiation of only one wave is possible (the radiation of the other wave for which $n^2 < 0$, is impossible). From this and from the radiation condition (18) it follows that in the case of the normal Doppler effect one wave is radiated "forward" $(\cos\theta > 0$, $\omega > \omega_{Hi})$ and either one or three waves can be radiated "backward" $(\cos\theta < 0$, $\omega < \omega_{Hi})$. For the emission of slow particles, the Doppler effect is always normal, so that only one wave is radiated "backward" when $s = 2, 3, \ldots$, since $\omega_s > \omega_{Hi}$.

Let us estimate the order of magnitude of the intensity of ion cyclotron radiation for the first harmonics. If $v_\parallel \sim v_\perp \sim v_A$ (fast ions), then $w_{sj} \sim w_0 n_A$. If $v_\parallel \sim v_\perp \ll v_A$, then $w_{sj} \sim w_0 n_A (v_\perp/v_A)^{2s-2}$ for $s \neq 1$.

When $s = 1$ the intensity of radiation of the wave with refractive index (7) decreases with increasing particle velocity $(v_\parallel \ll v_A$, $v_\perp \ll v_A)$:

$$w_1 = \frac{w_0}{\sqrt{2}} \left(\frac{v_\parallel}{v_A}\right)^2 n_A \frac{\text{tg}^5\chi}{\sin\chi \cos^2\chi\,(1 + 2\,\text{tg}^2\chi)^{3/2}} e^{-2\varkappa_1 R}, \quad (20)$$

where the damping coefficient κ_1 is determined by formulas (16), (8), and (9). $(z_1 = v_\parallel/\sqrt{2}\, v_{Ti} \gg 1$,

*tg = tan

$\tan\theta = 2\tan\chi$). In this case we have in order of magnitude $w_1 \sim w_0 n_A (v_\parallel / v_A)^2 \ll w_0 n_A$.

The radiation intensity of the other wave, for which the refractive index tends to infinity when $\omega \to \omega_{Hi}$ [$n^2 \sim \omega n_A^2/(\omega_{Hi} - \omega)$], to the contrary, increases with decreasing particle velocity:

$$w_1 \sim w_0 n \sim w_0 c\, (v_\parallel v_A^2)^{-1/3}. \tag{21}$$

It must be noted that expression (19) can be used in this case only for fast ions ($z_1 \sim v_\parallel / v_{Ti} \gg 1$). However, the estimate (21) is correct in order of magnitude also when $v_\parallel \sim v_{Ti}$. When $v_\parallel \lesssim v_{Ti}$ the electromagnetic wave under consideration attenuates strongly:

$$\mathrm{Re}\, n_\parallel \sim \mathrm{Im}\, n_\parallel \sim c\,(v_\parallel v_A^2)^{-1/3}.$$

The radiation field of this field can therefore not be represented in the form of a spherical wave.

Using (19) for the radiation intensity of the individual ion, we obtain the contribution to the radiating ability of the plasma for the frequency interval ω, $\omega + d\omega$ per unit solid angle, due to the radiation of the s-th harmonic by the ions:

$$\eta_{sj}(\chi, \omega) = 2\pi n_0 \int_0^\infty w_{sj}(v_\perp, v_\parallel, \chi)\,|_{R=0}\, f_0(v_\parallel, v_\perp)\, v_\perp \left|\frac{dv_\parallel}{d\omega}\right| dv_\perp, \tag{22}$$

where $v_\parallel$ as a function of the angle χ and the frequency ω is determined by (18); $f_0(v_\parallel, v_\perp)$ is the ion velocity distribution function. From (22) we get

$$\eta_{sj}(\chi, \omega) = \frac{e^2 \omega n_0 s^2 \beta\, (s\beta n_\perp)^{2s-2} \sin\theta\, [(\varepsilon_1 - \varepsilon_2 - n_j^2)^2 \cos\chi \cos\theta + (\varepsilon_1 - \varepsilon_2 - n_{\parallel j}^2)^2 \cos(\chi - \theta)]}{(2\pi)^{3/2} c 2^s s!\, n_\parallel \cos^2\theta \sin\chi\, |\cos\chi\, d^2 n_{\parallel j} / dn_\perp^2|\, (n_{\parallel 1}^2 - n_{\parallel 2}^2)^2} \exp(-z_s^2). \tag{23}$$

Inasmuch as $\beta n_\parallel \ll 1$, the quantities (23) are exponentially small if ω is not close to $s\omega_{Hi}$ ($s = 2, 3, \ldots$; when $s = 0$ and 1 expression (23) can be used only if $|z_s| \gg 1$). The contribution of the ion radiation in the frequency interval ω, $\omega + d\omega$ turns out to be noticeable only if ω is close to $s\omega_{Hi}$, so that $z_s \lesssim 1$. With increasing number s, the values of η_{sj} for $z_{sj} \lesssim 1$ decrease rapidly:

$$\eta_{sj} \sim e^2 n_0 \omega\, (v_{Ti}/c^2)\, (v_{Ti}/v_A)^{2s-2}.$$

The equilibrium radiation intensity $I_j(\chi, \omega) = \eta_{sj}/\alpha_{sj}$, where $\alpha_{sj} = 2\kappa_{sj}$ is the absorbing ability of the plasma, coincides with the Rytov formula

$$I_j = I_{RJ}\, n \sin\theta \sqrt{n^2 + n'^2}\, |d\cos\chi / d\theta|^{-1}. \tag{24}$$

Here

$$I_{RJ} = \omega^2 T / 8\pi^3 c^2; \quad n' = dn/d\theta.$$

The refractive index $n(\omega, \theta)$ has according to (3) a value

$$n^2 = \{\varepsilon_1 (1 + \cos^2\theta) \pm \sqrt{\varepsilon_1^2 (1 + \cos^2\theta)^2 - 4(\varepsilon_1^2 - \varepsilon_2^2)}\} / 2\cos^2\theta$$

(see also [7]). The connection between the angles θ and χ is given by (17), which can be represented in the form

$$\frac{1}{n}\frac{dn}{d\theta} = -\,\mathrm{tg}\,(\chi - \theta). \tag{25}$$

In order of magnitude we have in the frequency region under consideration $I \sim I_{RJ} n_A^2$. Near resonance $\omega = \omega_{Hi}$ for the wave with refractive index

$$n^2 \approx \varepsilon_1 (1 + \cos^2\theta) / \cos^2\theta \gg n_A^2 \tag{26}$$

the radiation intensity decreases sharply:

$$I = I_{RJ} \frac{\omega_{Hi} n_A^2}{\omega_{Hi} - \omega} f(\theta), \tag{27}$$

where $f(\theta) \sim 1$. It must be noted, however, that expression (24) can be used in this case only if

$$|1 - \omega_{Hi}/\omega|^{3/2} \gg v_{Ti}/v_A.$$

In the region

$$|1 - \omega_{Hi}/\omega|^{3/2} \lesssim v_{Ti}/v_A$$

this wave attenuates strongly, so that the notion of the equilibrium energy flux cannot be employed. The maximum value to which expression (27) tends when the frequency approaches ω_{Hi} has an order of magnitude $I \sim I_{RJ} n_A^2 (v_A / v_{Ti})^{2/3}$.

Let us examine expressions (19), (16), and (24) for the particular cases when the angle χ is close to zero or to $\pi/2$.

If $\chi \to 0$, then $n_{\parallel 1}$ for $u_i < 1$ and $n_{\parallel 2}$ for $u_i > 1$ have the form

$$n_\parallel^2 = n_A^2 \frac{\sqrt{u_i}}{1 + \sqrt{u_i}} - \frac{1}{2} n_\perp^2 + \frac{1 - u_i}{8 n_A^2 \sqrt{u_i}} n_\perp^4, \tag{28}$$

$$n_\perp = 2 n_A \left(\frac{\sqrt{u_i}}{1 + \sqrt{u_i}}\right)^{1/2} \mathrm{tg}\,\chi \ll n_\parallel.$$

The radiation intensity and the attenuation coefficient tend in this case to zero:

$$w_{sj} = w_0 \frac{\omega_{sj}^2}{\omega_{H_i}^2} \frac{2 s^{2s} \beta_\perp^{2s-2} n_A^{2s-1} (1 - u_i)^2}{(s!)^2 u_i} \left(\frac{\sqrt{u_i}}{1 + \sqrt{u_i}}\right)^{s + 3/2} \chi^{2s+2} \times \exp(-\alpha_{sj} R), \tag{29}$$

$$\alpha_{sj} = \frac{\omega_{Hi}^2}{c\omega_{sj}} \sqrt{\frac{\pi}{2}} \frac{s^{2s}\beta^{2s-3} n_A^{2s-2} (1-u_i)^2}{2s! u_i} \left(\frac{\sqrt{u_i}}{1+\sqrt{u_i}}\right)^s \chi^{2s+2}$$
$$\times \exp(-z_s^2). \quad (30)$$

The equilibrium radiation intensity is equal in this case to

$$I = I_{RJ} \frac{4n_A^2 \sqrt{u_i}}{1+\sqrt{u_i}}. \quad (31)$$

If $\chi \to \pi/2$, then $n_{\|1}$ for $u_i < 1$ and $n_{\|2}$ for $u_i > 1$ are determined by the expressions*

$$n_{\|} = \frac{u_i n_A}{1+u_i} \operatorname{ctg} \chi \ll n_A, \qquad n_\perp^2 = n_A^2 \left(1 - \frac{u_i}{1+u_i} \operatorname{ctg}^2 \chi\right).$$

In this case the radiation intensity, the attenuation coefficient, and the equilibrium radiation intensity are

$$w_{sj} = w_0 \frac{\omega_{sj}^2}{\omega_{Hi}^2} \frac{s^{2s}\beta_\perp^{2s-2} n_A^{2s-1} (1-\sqrt{u_i})^2}{2^{2s}(s!)^2(1+u_i)} \exp(-\alpha_{sj} R), \quad (32)$$

$$\alpha_{sj} = \frac{\omega_{Hi}^2}{c\omega_{sj}} \sqrt{\frac{\pi}{2}} \frac{s^{2s}\beta^{2s-3} n_A^{2s-2} (1+u_i)(1-\sqrt{u_i})^2}{2^s s! u_i^2} \exp(-z_s^2), \quad (33)$$

$$I = I_{RJ} \frac{u_i n_A^2}{1+u_i}. \quad (34)$$

If $u_i > 1$, then $n_{\|1}$ for $\chi \to 0$ has two solutions corresponding to the points of intersection 1 and 2 of Fig. 1. The solution $n_{\|1}$ corresponding to the point 1 has for $\chi \to 0$ the form

$$n_{\|1}^2 = n_A^2 \frac{\sqrt{u_i}}{\sqrt{u_i}-1} - \frac{1}{2} n_\perp^2,$$

where

$$n_\perp = 2n_A \left(\frac{\sqrt{u_i}}{\sqrt{u_i}-1}\right)^{1/2} \operatorname{tg} \chi \ll n_A.$$

The radiation intensity, the attenuation coefficient, and the equilibrium radiation intensity are determined in this case by the expressions

$$w_{s1} = w_0 \frac{\omega_{s1}^2}{\omega_{Hi}^2} \frac{2s^{2s}\beta_\perp^{2s-2} n_A^{2s-1}}{(s!)^2}$$
$$\times \left(\frac{\sqrt{u_i}}{\sqrt{u_i}-1}\right)^{s-1/2} \chi^{2s-2} \exp(-\alpha_{s1} R), \quad (35)$$

$$\alpha_{s1} = \frac{\omega_{Hi}^2}{c\omega_{s1}} \sqrt{\frac{\pi}{2}} \frac{2^{s-1}\beta^{2s-3} s^{2s} n_A^{2s-2}}{s!}$$
$$\times \left(\frac{\sqrt{u_i}}{\sqrt{u_i}-1}\right)^{s-2} \chi^{2s-2} \exp(-z_s^2), \quad (36)$$

$$I = I_{RJ} \frac{4n_A^2 \sqrt{u_i}}{\sqrt{u_i}-1}. \quad (37)$$

*ctg = cot

The second solution corresponding to the point 2 for $\chi \to 0$ is

$$n_{\|} = n_A \sqrt{\frac{u_i}{u_i-1}} \left[1 + \frac{n_A^2}{2(u_i-1) n_\perp^2}\right],$$

where

$$n_\perp = n_A \frac{u_i^{1/6}}{\sqrt{u_i-1}\, \chi^{1/3}} \gg n_A.$$

In this case the radiation intensity of the individual ion, the attenuation coefficient, and the equilibrium radiation intensity assume the form

$$w_{s1} = w_0 \frac{\omega_{s1}^2}{3\omega_{Hi}^2} \frac{s^{2s}\beta_\perp^{2s-2} n_A^{2s-1} u_i^{s/3-1/2}}{2^{2s}(s!)^2 (u_i-1)^{s-1/2} \chi^{2(s+3)/3}} \exp(-\alpha_{s1} R). \quad (38)$$

$$\alpha_{s1} = \frac{\omega_{Hi}^2}{c\omega_{s1}} \sqrt{\frac{\pi}{2}} \frac{s^{2s}\beta^{2s-3} n_A^{2s-2} (u_i-1)}{2^s s! u_i}$$
$$\times \left(\frac{u_i^{1/6}}{\sqrt{u_i-1}\, \chi^{1/3}}\right)^{2s-2} \exp(-z_s^2), \quad (39)$$

$$I = I_{RJ} \frac{n_A^2 u_i^{1/3}}{3(u_i-1)\chi^{8/3}}. \quad (40)$$

It is interesting to note that in this case the angle $\theta \to \pi/2$ when $\chi \to 0$. The conditions

$$\cos^2\theta \gg m/M\,(u_i-1), \qquad \beta_\perp n_\perp \ll 1, \qquad \beta n \ll 1$$

assume in this case the form

$$\chi^{2/3} \gg m/M\,(u_i-1), \qquad \chi^{1/3} \gg v_\perp / v_A \sqrt{u_i-1},$$
$$\chi^{1/3} \gg v_{Ti}/v_A \sqrt{u_i-1}.$$

It follows from (38) that the maximum radiation intensity occurs at angles χ close to zero ($\theta \approx \pi/2$). Let us estimate the order of magnitude of the summary radiation intensity at the first harmonic, for particles with

$$v_{Ti} \ll v_{\|} \sim v_\perp \ll v_A.$$

In this case

$$w(\chi) \sim w_0 n_A / \sqrt{u_i-1}\, \chi^{8/3}.$$

Integrating this expression with respect to χ from $\chi_{\min}$ to $\chi \sim 1$, we obtain

$$w_{\text{tot}} \sim w_0 n_A / \sqrt{u_i-1}\, \chi_{min}^{2/3}.$$

From the condition for the applicability of (38) it follows that

$$\chi_{min}^{2/3} \sim m/M\,(u_i-1) \text{ for } m/M > \beta_\perp^2 n_A^2,$$
$$\chi_{min}^{2/3} = \beta_\perp^2 n_A^2/(u_i-1) \text{ for } m/M < \beta_\perp^2 n_A^2.$$

From the radiation condition we obtain

$$n_{\|} \sim \sqrt[3]{n_A^2/\beta_{\|}},$$

so that

$$\sqrt{u_i - 1} \sim \sqrt[3]{n_A \beta_{\|}}.$$

Therefore

$$w_{\text{tot}} \sim w_0 n_A \sqrt[3]{n_A \beta_{\|}} (M/m) \quad \text{for} \quad m/M > \beta_{\perp}^2 n_A^2,$$

$$w_{\text{tot}} \sim w_0 n_A \sqrt[3]{n_A \beta_{\|}} / \beta_{\perp}^2 n_A^2 \quad \text{for} \quad m/M < \beta_{\perp}^2 n_A^2.$$

This estimate is applicable also when $v_{\|} \sim v_{\perp} \lesssim v_{Ti}$. It is only necessary to replace $\beta_{\|}$ and $\beta_{\perp}$ by $\beta = v_{Ti}/c$.

An analogous estimate can be readily carried out also for the total radiation intensity at the multiple harmonics.

3. CYCLOTRON RADIATION OF SLOW IONS AT THE FUNDAMENTAL FREQUENCY

The results obtained above for the intensity of cyclotron radiation at the fundamental frequency are valid only for fast ions, when $z_1 = v_{\|}/\sqrt{2}\, v_{Ti} \gg 1$. Let us consider now the radiation of the ions, without imposing any limitations on z_1. Here, however, we shall assume that $v_{\|} \ll v_A$ and $v_{\perp} \ll v_A$ (the case of large $v_{\|}$ and $v_{\perp}$ was investigated in [3]).

If the frequency of the radiated wave is close to ω_{Hi}, then the coefficients a, b, ... are equal to

$$a = i\sqrt{\frac{\pi}{8}} \frac{v_i w(z_1)}{\beta n \cos\theta} - \frac{v_i}{4} - \frac{v_i}{2} \operatorname{tg}^2\theta\, q(z_1),$$

$$b = \frac{v_i}{2\cos^2\theta} q(z_1), \qquad e = -i\frac{v_i}{\cos^2\theta}\left[q(z_e) + \frac{1}{2} q(z_1)\right],$$

$$d = -\sqrt{\frac{\pi}{8}} \frac{v_i w(z_1)}{\beta n \cos\theta} - \frac{3}{4} i v_i - i v_i \operatorname{tg}^2\theta \left[q(z_e) + \frac{1}{2} q(z_1)\right],$$

$$f = i\sqrt{\pi} \frac{m}{M} \frac{n_A^2 w(z_e)}{z_e \cos^2\theta} + \frac{v_i}{2\cos^2\theta} q(z_1),$$

$$c = 2\frac{M}{m} v_i z_e^2 q(z_e) - \frac{v_i \cos 2\theta}{2\cos^2\theta} q(z_1) - i\sqrt{\frac{\pi}{8}} \frac{v_i w(z_1)}{\beta n \cos\theta} - \frac{v_i}{4}. \tag{41}$$

In the case under consideration the solution of the dispersion equation corresponding to a weakly damped wave has the form (2), where $n_{\|}$ is determined by formula (7), n_e' by formula (8), and n_1' is equal to

$$n_1' = \sqrt{\frac{\pi}{8}} \frac{\beta n_{\perp}^4}{16 n_A^2 |w(z_1)|^2} \exp(-z_1^2). \tag{42}$$

In the particular case $z_1 \gg 1$ formula (42) goes over into formula (9).

The field of the fundamental harmonic in the wave zone has the form

$$E_{\varphi} = -2P_1 \cos\varphi, \qquad E_{\chi} = -2P_1 \cos\chi \sin\varphi,$$

$$H_{\varphi} = -\frac{n_A}{\sqrt{2}} P_1 \frac{\cos\chi}{\sqrt{1+\sin^2\chi}} \sin\varphi,$$

$$H_{\chi} = \frac{n_A}{\sqrt{2}} P_1 (1 + \sin^2\chi)\cos\varphi; \tag{43}$$

$$P_1 = \frac{e\beta\beta_{\perp}\omega n_A}{cR\,|w(z_1)|} \frac{\cos\chi \sin^2\chi}{(1+\sin^2\chi)^2}. \tag{43'}$$

The phase φ is determined by expression (16) with s = 1.

The radiation intensity of an individual ion is in this case

$$w = w_0 \frac{\sqrt{2}\beta_{\perp}^2 n_A^3}{\pi |w(z_1)|^2} \frac{\sin^4\chi \cos^2\chi}{(1+\sin^2\chi)^{5/2}} e^{-2\varkappa R}. \tag{44}$$

In order of magnitude we have when $v_{\|} \lesssim v_{Ti}$

$$w \sim w_0 n_A (v_{Ti}/v_A)^2.$$

In the particular case when $v_{\|} \gg v_{Ti}$ formula (44) goes over into (20).

The contribution of the radiation of the ions at the fundamental frequency to the radiating ability of the plasma is, according to (44) and (22), equal to

$$\eta = \frac{M\omega^3\beta^3 n_A^4}{2\sqrt{2\pi}\pi^3 c\,|w(z_1)|^2} \frac{\sin^4\chi\cos\chi}{(1+\sin^2\chi)^4} \exp(-z_1^2). \tag{45}$$

In order of magnitude we have

$$\eta \sim e^2 n_0 \omega\beta v_{Ti}^2 / c v_A^2$$

when $z_1 \lesssim 1$. If $z_1 \gg 1$, formula (45) goes over into (23) [in this case it is necessary to put $\omega = \omega_{Hi}$ in (23)]. The radiating ability then decreases:

$$\eta \sim \frac{e^2 n_0 \omega\beta}{c}\left(1 - \frac{\omega_{Hi}}{\omega}\right)^2 \exp(-z_1^2).$$

The equilibrium radiation intensity for the wave under consideration is

$$I = I_{RJ} \frac{2n_A^2}{(1+\sin^2\chi)^2}.$$

4. CERENKOV RADIATION OF THE ELECTRONS

Let us consider the Cerenkov radiation from an electron moving along a helix in a plasma, in the low frequency region ($\omega \lesssim \omega_{Hi}$). Using formulas (2)–(6) for the refractive indices $n_{\|}(n_{\perp})$ and the damping coefficients, and the general expressions for the fields (2.10) from [5], we obtain for the electric and magnetic fields produced by the electron in the plasma the following expressions:

$$E_{\varphi}^{(j)} = -A\left\{\frac{n_{\|}^2 - \varepsilon_1}{\sqrt{u_i}}\left(-v_{\perp}\beta_{\perp}n_{\perp} + \frac{v_{\|}\operatorname{tg}\theta}{z_e^2}\right)\sin\Psi_{0j} + \frac{v_{\|}n_{\|}n_{\perp}\varepsilon_2(a\sin\Psi_{0j} - b\cos\Psi_{0j})}{v_i z_e^2(a^2+b^2)}\right\},$$

$$E_{\chi}^{(j)} = A\cos\chi\left\{\frac{\varepsilon_2}{\sqrt{u_i}}\left(v_{\perp}\beta_{\perp}n_{\perp} - \frac{v_{\|}\operatorname{tg}\theta}{z_e^2}\right)\cos\Psi_{0j} - \frac{v_{\|}n_{\|}n_{\perp}(n^2-\varepsilon_1)(a\cos\Psi_{0j} + b\sin\Psi_{0j})}{v_i z_e^2(a^2+b^2)}\right\},$$

$$H_\varphi^{(j)} = An\left\{\frac{\varepsilon_2\cos\theta}{\sqrt{u_i}}\left(v_\perp\beta_\perp n_\perp - \frac{v_{||}\operatorname{tg}\theta}{z_e^2}\right)\cos\Psi_{0j} - \frac{v_{||}(\varepsilon_1 n^2 - \varepsilon_1^2 + \varepsilon_2^2)\sin\theta\,(a\cos\Psi_{0j} + b\sin\Psi_{0j})}{v_i z_e^2(a^2+b^2)}\right\},$$

$$H_\chi^{(j)} = An\cos(\chi-\theta)\left\{\frac{n_{||}^2-\varepsilon_1}{\sqrt{u_i}}\left(-v_\perp\beta_\perp n_\perp + \frac{v_{||}\operatorname{tg}\theta}{z_e^2}\right)\sin\Psi_{0j} + \frac{v_{||}n_{||}n_\perp\varepsilon_2(a\sin\Psi_{0j} - b\cos\Psi_{0j})}{v_i z_e^2(a^2+b^2)}\right\}, \quad (46)$$

where

$$A = \frac{(-1)^{j-1}\,em\sqrt{n_\perp}}{c^2Rn_{||j}(n_{||1}^2 - n_{||2}^2)M\beta_{||}\,|\,dn_{||}/d\omega\,|\sqrt{\sin\chi\,|\cos\chi d^2n_{||}/dn_\perp^2\,|}},$$

$$a = 1 - 2z_e\int_0^{z_e}\exp(t^2 - z_e^2)\,dt, \qquad b = \sqrt{\pi}z_e\exp(-z_e^2).$$

The radiated frequency is obtained from the condition $\omega_j = k_{||j}v_{||}$.

From this we obtain the intensity of the Cerenkov radiation per unit solid-angle interval

$$w_j(\chi) = \frac{m^2e^2\omega n_j n_{\perp j}S(\chi)\exp(-2\varkappa_j R)}{8\pi c^3M^2n_{||j}^2(n_{||1}^2 - n_{||2}^2)^2\sin\chi\beta_{||}\,|\cos\chi(d^2n_{||j}/dn_\perp^2)(dn_{||j}/d\omega)\,|}, \quad (47)$$

where

$$S(\chi) = \cos\chi\cos\theta\left[\frac{\varepsilon_2^2}{u_i}\left(\frac{v_{||}\operatorname{tg}\theta}{z_e^2} - v_\perp\beta_\perp n_\perp\right) + \frac{v_{||}^2n_\perp^2(n^2-\varepsilon_1)(\varepsilon_1n^2 - \varepsilon_1^2 + \varepsilon_2^2)}{v_i^2z_e^4(a^2+b^2)} + \frac{2av_{||}\varepsilon_2\operatorname{tg}\theta(v_{||}\operatorname{tg}\theta - v_\perp\beta_\perp z_e^2n_\perp)(\varepsilon_1n^2 - \varepsilon_1^2 + \varepsilon_2^2)}{\sqrt{u_i}\,v_iz_e^4(a^2+b^2)}\right]$$

$$+ \cos(\chi-\theta)\left[\frac{(n_{||}^2-\varepsilon_1)^2}{u_i}\left(\frac{v_{||}\operatorname{tg}\theta}{z_e^2} - v_\perp\beta_\perp n_\perp\right)^2 + \frac{v_{||}^2n_{||}^2n_\perp^2\varepsilon_2^2}{v_i^2z_e^4(a^2+b^2)} + \frac{2av_{||}n_{||}n_\perp\varepsilon_2(n_{||}^2-\varepsilon_1)(v_{||}\operatorname{tg}\theta - v_\perp\beta_\perp z_e^2n_\perp)}{\sqrt{u_i}\,v_iz_e^4(a^2+b^2)}\right].$$

The damping coefficient κ_j is given by formula (16). In order of magnitude we have when $v_A \sim v_{||} \sim v_\perp \sim v_{Te}$ the estimate $w_j \sim w_0n_A(n/M)^2$, i.e., at the first harmonics the electron radiates $(M/m)^2$ times less than an ion having the same velocity.

However, the Cerenkov radiation can make the main contribution to the thermal radiation at frequencies $\omega \sim \omega_{Hi}$. Indeed, the contribution of the Cerenkov radiation of the electrons to the radiating ability of the plasma is, according to (22) and (47),

$$\eta_e = \frac{m^2e^2n_0\omega v_{Te}nn_\perp\operatorname{tg}^2\theta P\cdot\exp(-z_e^2)}{2(2\pi)^{3/2}M^2c^2n_{||}^3(n_{||1}^2 - n_{||2}^2)^2z_e^2\sin\chi\,|\cos\chi d^2n_{||}/dn_\perp^2\,|}, \quad (48)$$

where

$$P = \cos\chi\cos\theta\left[\frac{\varepsilon_2^2}{u_i} + \frac{n_{||}^2(n^2-\varepsilon_1)(\varepsilon_1n^2 - \varepsilon_1^2 + \varepsilon_2^2)}{v_i^2(a^2+b^2)}\right] + \cos(\chi-\theta)\left[\frac{(n_{||}^2-\varepsilon_1)^2}{u_i} + \frac{n_{||}^4\varepsilon_2^2}{v_i^2(a^2+b^2)}\right].$$

In order of magnitude for $v_A \sim v_{Te}$ and $\omega \sim \omega_{Hi}$ we have

$$\eta_e \sim m^2e^2n_0\omega v_{Te}/M^2c^2.$$

At the same time, the contribution of the cyclotron radiation of the ions is exponentially small if ω is not close to $s\omega_{Hi}$, $s = 1, 2, 3, \ldots$ When $\omega = \omega_{Hi}$ (for a weakly damped wave) and $\omega = 2\omega_{Hi}$ (for both waves), the cyclotron radiating ability is $\sqrt{M/m}$ times larger than (48). However even when $\omega = 3\omega_{Hi}$ we get $\eta_{3j} \sim \sqrt{m/M}\,\eta_e$.

In the case of a magnetic-sound wave ($\omega \ll \omega_{Hi}$) with $\sin^2\chi \gg \omega/\omega_{Hi}$ we have $\theta = \chi$ and

$$w(\chi) = \frac{m^2e^2\omega^2n_A}{8\pi M^2c^3\,|1 - v_{||}^2/v_A^2\,|}\frac{\omega^2}{\omega_{Hi}^2}\left(\frac{v_{||}\operatorname{tg}\chi}{z_e^2} - v_\perp\beta_\perp n_A\sin\chi\right)^2e^{-2\varkappa R},$$

$$\varkappa = \varkappa_e = \sqrt{\frac{\pi}{8}}\frac{m\omega\beta_en_A^2\sin^2\chi}{Mc\cos\chi}\exp(-z_e^2),$$

$$\eta_e = \frac{m^2\omega^3n_0e^2\beta_e^3n_A^2\sin^2\chi}{(2\pi)^{3/2}M^2c\omega_{Hi}^2\cos\chi}\exp(-z_e^2), \qquad I = I_{RJ}n_A^2. \quad (49)$$

In order of magnitude we have for $v_{||} \sim v_\perp \sim v_{Te} \sim v_A$

$$w(\chi) \sim m^2e^2\omega^2v_{Te}^2/M^2c^3, \qquad \varkappa \sim m/M\lambda\!\!\!^-,$$

where $\lambda\!\!\!^- = c/\omega n_A$. The radiated frequency ω is

$$\omega = \sqrt{2/3}\,\omega_H\sin\chi\sqrt{1 - v_A/v_{||}\cos\chi}.$$

It is obvious that the condition $\omega \ll \omega_{Hi}$ is satisfied if $|1 - v_A/v_{||}\cos\chi| \ll 1$. Thus, the Cerenkov radiation of the magnetic-sound wave occurs only when $v_A < v_{||}$ near the angle $\chi = \chi_0$, where $\cos^2\chi_0 = v_A/v_{||}$.

If $v_{||}$ is close to v_A, so that the condition $\chi_0^2 \gg \omega/\omega_{Hi}$ is not satisfied, then it is necessary to use the general formula (47). On the other hand, if $\chi_0^2 \ll \omega/\omega_{Hi}$, we have for the Alfven and magnetic-sound wave

$$n_{||} = n_A(1 \pm \omega/2\omega_{Hi} - \chi^2), \qquad n_\perp = 2n_A\chi,$$

$$w(\chi) = \frac{2m^2e^2\omega^3n_A\chi^2}{\pi M^2c^3\omega_{Hi}}\left(\frac{v_{||}}{z_e^2} - \beta_\perp v_\perp n_A\right)^2e^{-2\varkappa_eR},$$

$$\varkappa_e = \sqrt{2\pi}m\omega\beta_en_A\chi^2\exp(-z_e^2)/Mc, \qquad z_e = v_{||}/\sqrt{2}v_{Te}. \quad (50)$$

The radiated frequency is in this case

$$\omega = \pm 2\omega_{Hi} (v_A / v_{\|} - 1).$$

Inasmuch as $\omega > 0$, only the Alfven wave is radiated when $v_A > v_{\|}$, and only the magnetic-sound wave when $v_A < v_{\|}$.

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