INITIAL AND FINAL STATE INTERACTIONS IN DIRECT NUCLEAR REACTIONS

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Within the framework of the dispersion method, a rigorous treatment is given of final state interactions in direct nuclear reactions. An exact solution is obtained for the singular integral equation which arises in this problem; numerical computations using the exact solution reduce to a single quadrature. A comparison of the exact solution with the first iteration is used to give a qualitative explanation of the success of the distorted wave method in describing direct processes.

1. The dispersion theory of direct reactions, formulated in a paper of Shapiro,[1] allows quite rigorous inclusion of the interaction of a particle with the nucleus in the initial and final states. In the present paper we show that when such interactions are included, the amplitude for the direct process satisfies a certain integral equation which relates the amplitude for the reaction to the amplitudes for elastic scattering of the incident and emergent particles. This equation is based on unitarity and analyticity. It is convenient to represent the amplitude corresponding to inclusion of final state interaction by means of diagrams (Fig. 1). Since there are a large number of vertices corresponding to elastic scattering of the particle by the nucleus (strong interaction), one cannot stop at the few terms shown in Fig. 1. The solution of the integral equation represents a summation of an infinite number of terms.

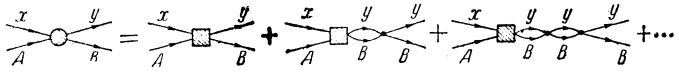

FIG. 1. Diagrams corresponding to final state interaction.

2. The integral equation for the partial amplitude of the direct reaction A + x → B + y, including final state interaction,[1)] can be written in the form

$$M(E) = M_0(E) + \frac{1}{\pi}\int_{E_0}^{\infty} \frac{M(E')\,h^*(E')}{E' - E - i\eta}\,dE', \qquad (1)$$

where E is the kinetic energy of the emerging particle in the cms, $M_0(E)$ is the partial amplitude for the direct interaction omitting final state interaction (the first term in Fig. 1), $h(E) = e^{i\delta(E)} \sin\delta(E)$, and δ is the partial phase for scattering of the emergent particle by the final nucleus.

We consider the wide class of reactions in which there is no anomalous threshold in the energy variable, so that $E_0 = 0$. The integral equation (1) was obtained from the dispersion relation without subtractions; these have to be made if the amplitude does not fall off sufficiently rapidly with energy. Including the subtractions does not complicate the computations. The integral equation in the form (1) is rigorously correct if we neglect effects associated with the spin of the final nucleus, since the elastic scattering amplitude is then diagonal in the orbital angular momentum l.

Singular integral equations of the type (1) and their solutions have been studied in detail in the monograph of Muskhelishvili,[2] and the equation in just the form of (1) has also been treated by Omnés. The solution of (1) has the form

$$M(E) = M_0(E) + \frac{\rho^+(E)}{\pi}\int_0^{\infty} \frac{M_0(E')\,h^*(E')}{\rho^-(E')(E' - E - i\eta)}\,dE',$$

$$\rho^{\pm}(E) = \rho(E)\exp\{\pm i\delta^*(E)\},$$

$$\rho(E) = \exp\left\{\frac{E - E_0}{\pi}\,P\int_0^{\infty} \frac{\delta^*(E')}{(E' - E_0)(E' - E)}\,dE'\right\}. \qquad (2)$$

The solution (2) is a particular solution of the inhomogeneous equation (1), to which one can add the general solution of the homogeneous equation. But it is easy to see that the solution (2) is the only one which vanishes for $M_0 \to 0$. This solu-

1)Simultaneous inclusion of both initial and final state interactions does not cause any difficulties in principle, but to be specific we have limited ourselves to final state interactions.

tion coincides with the iteration series and goes over into $M_0(E)$ for $h \to 0$.

Direct computations using formulas (2) are quite complicated. But the solution can be simplified considerably if we approximate the phase as a function of energy by the expression

$$\tan \delta^*(E) = \sqrt{E}Q(E)/P(E), \qquad (3)$$

where Q(E) and P(E) are arbitrary polynomials. This expression is a generalization of the effective range approximation. For nucleon-nucleon S-scattering, Q = const, $P(E) = -1/a + r_0E$. When the scattering is determined by one resonance level, we have $\tan \delta(E) = -\Gamma/2(E - E_r)$, where Γ is the level width, proportional to $\sqrt{E}$. This expression is valid when there are no inelastic channels; including inelastic channels somewhat changes the appearance of the formula, but it again corresponds to formula (3). Apparently formula (3) can be used in practice for any case of scattering of a particle by a nucleus. We note that formula (3) does not include the term in the phase which corresponds to potential scattering and which does not change $\rho(E)$.

From formula (3) we get $\rho(E)$ in the form (similar computations were done by Galanin and Grashin [4])

$$\rho(E) = \text{const} \cdot \prod_{k=1}^{n^-} \sqrt{E - E_k^-} \Big/ \prod_{k=1}^{n^+} \sqrt{E - E_k^+}, \qquad (4)$$

where E_k^- are the roots of the equation $P(E) - i\sqrt{E}\,Q(E) = 0$, E_k^+ are the roots of $P(E) + i\sqrt{E}\,Q(E) = 0$, on the sheet where $\operatorname{Im}\sqrt{E} > 0$. Going to the limit $\eta \to 0$ in (2), we get

$$M(E) = e^{i\delta^*(E)} \left[M_0(E)\cos\delta^*(E) + \frac{1}{\pi} P \int_0^\infty \frac{\rho(E)}{\rho(E')} \frac{M_0(E')\sin\delta^*(E')}{E' - E}\, dE' \right], \qquad (5)$$

where $\rho(E)$ for the case of the model (3) is given by (4). Thus the problem is reduced to one quadrature.

3. The distorted wave method is widely used at present to describe direct nuclear reactions. Since the distorted wave method and the dispersion method of this paper solve essentially the same problem, it is desirable to understand the connection between the two methods. In studying the expressions for the reaction amplitude, one notes a relation between the direct reaction amplitude in the distorted wave method and the approximate solution of (1) corresponding to the first iteration. Unfortunately it is difficult to trace this connection all the way. It is nevertheless worthwhile to understand what conclusions follow if these expressions are assumed to be equivalent. For this purpose we compare the exact solution of Eq. (1) with the first iteration:

$$M_1(E) = M_0(E)e^{-i\delta^*(E)}[\cos\delta^*(E) + 2i\sin\delta^*(E)] + \frac{1}{\pi} P \int_0^\infty \frac{M_0(E')e^{-i\delta^*(E')}\sin\delta^*(E')}{E' - E}\, dE'. \qquad (6)$$

It is natural to compare the solutions of (5) and (6) for the two limiting cases, using the model (3): 1) $P(E) \to 0$ ($\delta \to \pi/2$) and 2) $Q(E) \to 0$ ($\delta \to 0$).

In the first case the exact solution and the first iteration are very different. For example, for scattering through a single resonance level, neglecting inelastic channels, formula (5) gives the physically obvious result $M(E_r) \approx 0$, where $M_1(E_r) \approx 2M_0(E_r)$ (where E_r is the resonance energy).

In the second case, where $\tan\delta^* \sim \sin\delta^* \ll 1$, the solutions of (5) and (6) coincide to first order in $\sin\delta^*$:

$$M(E) \approx M_1(E) \approx M_0(E)[1 + i\sin\delta^*(E)] + \frac{1}{\pi} P \int_0^\infty \frac{M_0(E')\sin\delta^*(E')}{E' - E}\, dE'. \qquad (7)$$

This is easily seen by comparing (5) and (6) directly for small δ^*. Thus a sufficient condition for the validity of (6) is the inequality

$$\sin\delta^* \ll 1, \text{ i.e., } h^*(E) \ll 1. \qquad (8)$$

4. The treatment given above permits one to understand qualitatively the success of the method of distorted waves in describing direct processes. In fact, direct nuclear reactions are preferentially peripheral (cf., for example, [5]). The region of the nuclear surface with radius R corresponds to values $l \sim kR$, for which it is known, for example from studies of the optical model for neutrons, that $|\tan\delta_l| < 1$ (Fig. 2). If in addition, values of E'

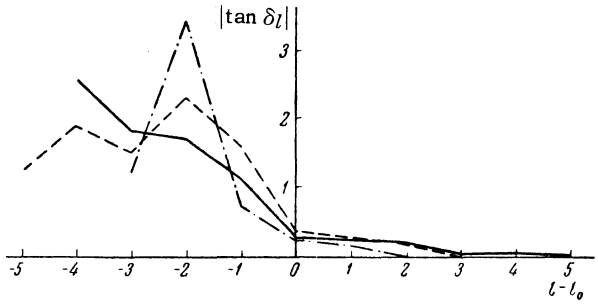

FIG. 2. Schematic dependence of $|\tan\delta_l|$ on l, determined from the optical model description of scattering of neutrons by nuclei.[6] Solid curve for Cu and 14 MeV neutrons, the dashed curve for In and 14 MeV neutrons, the dot-dash curve for Sn and 3.2 MeV neutrons. The farthest left value of l in each case corresponds to $l = 0$. The value of l_0 is $\approx kR$.

which are much greater than E do not give a large contribution to the integral in formula (5), this inequality can also be used in the integrand.

As we see from Fig. 2, condition (8) is satisfied for $l \geq l_0$ ($l_0 \approx kR$). But it may break down for $l < l_0$ and even for $l = l_0 - 1$. For example, in the case of Sn (E = 3.2 MeV, $l_0 = 3$), for $l_1 = 1$, $|\tan \delta| = 3.44$. Thus one can understand why the distorted wave method, which in general gives a good description of direct processes, in some cases gives poor agreement with experiment (cf., for example, [7]), especially at large angles, where the results are most sensitive to changes in the partial reaction amplitudes. In those cases where the agreement is poor, they change the parameters of the optical model, but still use a potential which describes the elastic scattering poorly. This procedure is hard to justify.

The dispersion method treated here does not have the difficulties of the distorted wave method, and should be applied to treat specific nuclear reactions.

In conclusion we should like to thank Prof. I. S. Shapiro for continued interest and valuable discussions.

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