NON-EQUILIBRIUM ELECTRON AND PHONON SYSTEMS IN AN EXTERNAL MAGNETIC FIELD

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The quantum as well as classical kinetic equations for electrons interacting with a quantized Bose field in the presence of an external homogeneous magnetic field are presented. The equations describe the interaction of electrons with various oscillation branches of an electron-ion plasma as well as of a solid body. Closed formulas are derived for some of the kinetic coefficients. The Fokker-Planck coefficients are calculated for the case of a strong magnetic field.

1. The classical and quantum theories of a non-equilibrium electron system interacting with a wave field were considered by Klimontovich [1] and by Pines and Schrieffer [2] in the absence of external fields. The presence of the latter introduces essential changes in the formulas for the kinetic coefficients, particularly in the limiting quantum cases. It becomes necessary to deal with quantum systems in the study of kinetic coefficients of solids, both metals and semiconductors. A typical classical (non-quantum) system is a high-temperature rarefied electron-ion plasma. In this connection, it is of interest to study the non-equilibrium quantum and classical systems.

2. The state of the electrons will be described by the diagonal elements of the density matrix $\rho_{\nu\nu'} = f_\nu \delta_{\nu\nu'}$, in the representation of the single-particle Hamiltonian H_0, with

$$\hat{H}_0 | \nu \rangle = E_\nu | \nu \rangle. \qquad (1)$$

Thus, for example, for a system situated in mutually perpendicular static magnetic and electric fields $\mathbf{B} \equiv \{0, 0, B\}$ and $\mathbf{E} \equiv \{0, E, 0\}$, in the Landau representation, we have

$$| \nu \} \equiv | k_x, k_z, n \rangle = (2\pi \sqrt{\alpha})^{-1} \exp\{ik_x x + ik_z z\} \Phi_n [(y - y_0)/\alpha], \qquad (2)$$

$$E_\nu = \hbar\Omega (n + 1/2) + \frac{\hbar^2}{2\mu} [k_x^2 + k_z^2] - \left(\frac{\hbar B}{\mu c} k_x + E\right)^2 \frac{\mu c^2}{2B^2};$$
$$\Omega = |e| B/\mu c, \quad \alpha^2 = \hbar/\mu\Omega, \quad y_0 = -(c\hbar k_x + \mu c^2 E / B)/eB, \qquad (3)$$

$\Phi_n(y)$ is the wave function of the harmonic oscillator.

In describing spatially-homogeneous distributions, only diagonal elements $\rho_{\nu\nu}$ will play any role, and moreover the $\rho_{\nu\nu}$ will be functions of only two quantum numbers n and k_z. The elements $\rho_{\nu\nu}$ which depend on k_x will describe particle distributions in space which are already inhomogeneous in y. Therefore with the aid of the diagonal matrix $\rho_{\nu\nu}$ elements in the $\hat{H}_0$ representation we can describe also the spatially inhomogeneous distributions. We note that in describing the spatially inhomogeneous distributions that have, say, cylindrical symmetry, we can also confine ourselves to diagonal elements of the density matrix, but it is necessary to take in place of the Landau representation the eigenfunctions of the single-particle problem in cylindrical coordinates.

The state of the phonon system will be described by a distribution function N_q. Let, further, C_q be the constant of interaction between the electrons and the phonons. We then obtain in first order of perturbation theory in the interaction of the electrons with the Bose field

$$\frac{\partial f(\nu)}{\partial t} = \sum_{\nu', q} \frac{2\pi}{\hbar} |C_q|^2 \{|\langle \nu' | e^{i\mathbf{qr}} | \nu \rangle|^2 \delta(E_{\nu'} - E_\nu - \hbar\omega_q)$$
$$\times [f(\nu')(1 - f(\nu))(N_q + 1) - f(\nu)(1 - f(\nu')) N_q]$$
$$+ |\langle \nu' | e^{-i\mathbf{qr}} | \nu \rangle|^2 \delta(E_{\nu'} - E_\nu + \hbar\omega_q)$$
$$\times [f(\nu')(1 - f(\nu)) N_q - f(\nu)(1 - f(\nu'))(N_q + 1)]\}, \qquad (4)$$

$$\frac{\partial N_q}{\partial t} = \sum_{\nu\nu'} \frac{4\pi}{\hbar} |C_q|^2 \{|\langle \nu' | e^{i\mathbf{qr}} | \nu \rangle|^2 \delta(E_{\nu'} - E_\nu - h\omega_q)$$
$$\times [(f(\nu') - f(\nu)) N_q + f(\nu')(1 - f(\nu))]\}. \qquad (5)$$

In the case of the eigenfunctions (2), the matrix element $\langle \nu' | e^{i\mathbf{q}\cdot\mathbf{r}} | \nu \rangle$ has been already calculated earlier [3]. We note that the equations written out are suitable for the analysis of spatially-inhomogeneous distributions, if the characteristic scales of these inhomogeneities are large compared with the mean free path and with the wavelengths under consideration.

For an electron-ion plasma, the functions C_q can be readily obtained by the following elementary method. For large values of N_q ($N_q \gg 1$) Eq. (5) assumes the form

$$\dot{N}_q = -\gamma_q N_q. \tag{6}$$

Inasmuch as N_q is proportional to the square of the wave amplitude in the plasma, γ_q is double the damping decrement of the latter. Therefore, by finding the damping decrement of the wave in the plasma in terms of its dielectric constant and comparing the expression thus obtained with γ_q, we obtain C_q. Indeed, according to [4], we have

$$\varepsilon(\omega q) = 1 - \lim_{\Delta \to 0} \frac{4\pi e^2}{q^2 V} \sum_{\nu\nu'} |\langle \nu | e^{i\mathbf{qr}} | \nu' \rangle|^2 \frac{f(\nu') - f(\nu)}{E_{\nu'} - E_\nu - \hbar\omega_q + i|\Delta|}, \tag{7}$$

then $\gamma_q = 2\omega_q \operatorname{Im} \epsilon(\omega q)$, and

$$|C_q|^2 = \frac{2\pi e^2}{q^2 V} \hbar\omega_q^{(p)} \tag{8}$$

for the plasma oscillations; for the acoustic oscillations we can obtain in the same manner

$$|C_q|^2 = \frac{2\pi e^2}{\operatorname{Re}\varepsilon(0, q)\, q^2 V} \hbar\omega_q^{(a)}. \tag{9}$$

Such an expression was first obtained in [5] for the interaction constant of the electrons with acoustic phonons, except in a different notation, and somewhat later independently by Bardeen and Pines [6].

The frequencies $\omega_q^{(p)}$ and $\omega_q^{(a)}$ are roots of the dispersion equation

$$\operatorname{Re}\varepsilon(\omega, q) = 0, \tag{10}$$

where $\epsilon(\omega, q)$ is a trivial generalization of (7), namely

$$\varepsilon(\omega, q) = 1 - \lim_{\Delta \to 0} \frac{4\pi}{q^2 V} \sum_{\nu_i, \nu_i', i} e_i^2 |\langle \nu_i' | e^{i\mathbf{qr}} | \nu_i \rangle|^2 + \frac{f_i(\nu_i') - f_i(\nu_i)}{E_{\nu_i'} - E_{\nu_i} - \hbar\omega_q + i|\Delta|}; \tag{11}$$

$i = 1$ pertains to the electrons and $i = 2$ to the ions.

In other cases, for example in the study of the interaction between the electrons and optical and acoustic oscillations in a solid, C_q was determined by a different method.

When taking into account collisions between electrons and between electrons and ions (if they are important), Eq. (4) must be supplemented by the corresponding collision integral, obtained in [3].

To describe the nonequilibrium states of a classical plasma one frequently employs the collision integral written in the form proposed by Landau and corresponding to the Fokker-Planck equation [7]. Equations of this type are readily obtained from (4) and (5), by going to the limit as $\hbar = 0$. In the case of a constant magnetic field directed along the z axis, the classical limit of the matrix element $\langle \nu' | e^{i\mathbf{q}\cdot\mathbf{r}} | \nu \rangle$ is given in [3].

As a result of such a limiting transition we obtain

$$\frac{\partial f(p_z, p_\perp, y_0, t)}{\partial t} = \frac{V}{(2\pi)^3} \sum_{m=-\infty}^{\infty} \int d\mathbf{q} \frac{2\pi}{h} |C_q|^2 \times \hat{D} \{J_m^2(p_\perp q_\perp/\mu\Omega)\, \delta(p_z q_z/\mu + m\Omega - \omega_q)[(\hat{D}f)\, \Theta_q/\omega_q + f]\}, \tag{12}$$

$$\frac{\partial \Theta_q}{\partial t} = -\frac{4\pi}{\hbar} |C_q|^2 \times \sum_{m=-\infty}^{\infty} \int dp_z\, 2\pi dp_\perp\, p_\perp\, dy_0 \delta(p_z q_z/\mu + m\Omega - \omega_q) \times J_m^2(p_z q_z/\mu\Omega)\{(\hat{D}f)\, \Theta_q - \omega_q f\}. \tag{13}$$

Here

$$\hat{D} \equiv \left\{ q_z \frac{\partial}{\partial p_z} + \frac{m\Omega\mu}{p_\perp} \frac{\partial}{\partial p_\perp} - \frac{c q_x}{eB} \frac{\partial}{\partial y_0} \right\}, \tag{14}$$

$J_m(x)$ is a Bessel function and $\Theta_q = N_q \hbar\omega_q$. In the case of weak fields and spatially-homogeneous distributions, (12) and (13) go over into the equations obtained first by Klimontovich [1] and then by Pines and Schrieffer [2].

3. With the aid of (4), (5) and (12), (13) it is possible to obtain formulas for the kinetic coefficients in closed form. Let us consider first the kinetic coefficients connected with the equalization of the gradients of the concentrations, electrons, and temperature in the direction transverse to the magnetic field. To obtain the quantum coefficients it is convenient to determine with the aid of (4) the macroscopic equations of diffusion and heat conduction. Let us assume that the deviations from thermodynamic equilibrium are small, and then the distribution functions for determining the diffusion coefficient and the heat conductivity can be represented in the form

$$f(n, k_z, y_0, t) = f_0[(\mu_0 - E_{n,k_z})/T_0] + (\partial f_0/\partial\mu_0)(\partial\mu_0/\partial N)\,\delta N(y_0, t) \quad (15)$$

$$f(n, k_z, y_0, t) = f_0[(\mu_0 - E_{n,k_z})/T_0] + \{(\mu_0 - E_{n,k_z})/T_0 - \partial\mu_0/\partial T\} \times (\partial f_0/\partial\mu_0)\,\delta T(y_0, t). \quad (16)$$

Substituting (15) and (16) in (4), linearizing with respect to δN and δT, and then expanding the collision integral in powers of q_x, we obtain for Maxwellian statistics

$$\frac{\partial}{\partial t}\delta N(y_0, t) = D_\perp \frac{\partial^2}{\partial y_0^2}\delta N(y_0, t), \quad (17)$$

$$\frac{\partial}{\partial t}\delta T(y_0, t) = \varkappa_\perp \frac{\partial^2}{\partial y_0^2}\delta T(y_0, t), \quad (18)$$

$$D_\perp = \frac{\alpha^4}{2}\frac{V}{(2\pi)^3}\int dq \frac{2\pi}{\hbar^2}|C_q q_x|^2\{(N_q+1)\Phi_1(q_z, q_\perp, \omega_q) + N_q\Phi_1(-q_z, q_\perp, -\omega_q)\}, \quad (19)$$

$$\varkappa_\perp = \frac{\alpha^4}{2}\frac{V}{(2\pi)^3}\int dq \frac{2\pi}{\hbar^2}|C_q q_x|^2\{(N_q+1)\Phi_2(q_z, q_\perp, \omega_q) + N_q\Phi_2(-q_z, q_\perp, -\omega_q)\}; \quad (20)$$

$$\Phi_1 = b_1^{-1}\sum_{n, n', k_z}\hbar\delta(E_{n',k_z+q_z} - E_{n,k_z} - \hbar\omega)|F_{nn'}(\alpha^2 q_\perp^2/2)|^2(\partial f_0(E_{n',k_z+q_z})/\partial\mu_0), \quad (21)$$

$$\Phi_2 = b_2^{-1}\sum_{n, n', k_z}\hbar\delta(E_{n',k_z+q_z} - E_{n,k_z} - \hbar\omega)|F_{nn'}(\alpha^2 q_\perp^2/2)|^2 \times\{(\mu_0 - E_{n',k_z+q_z})/T_0 - \partial\mu_0/\partial T\} \times (E_{nk_z} - \mu_0)\{\partial f_0(E_{n',k_z+q_z})/\partial\mu_0\};$$

$$b_1 = \sum_n \int dp_z \frac{\partial f_0}{\partial\mu_0},$$

$$b_2 = \sum_n \int dp_z\left(\frac{\mu_0 - E_{n,k_z}}{T_0} - \frac{\partial\mu_0}{\partial T}\right)\left(E_{nk_z} - \mu_0\right)\frac{\partial f_0}{\partial\mu_0},$$

$$F_{nn'}(x) = x^{|n'-n|}e^{-x}L_n^{(n'-n)}(x), \quad (22)$$

$L_n^{(n'-n)}(x)$ is the generalized Laguerre polynomial.

From (19) we can obtain an expression for the electric conductivity transversely to the magnetic field. Using the Einstein relation, we obtain with the aid of (19) a formula for the electric conductivity, which coincides with formula (8) of the paper by Gurevich and Firsov [8], obtained in a different way. Inasmuch as in formula (19) the matrix element $\langle\nu|e^{i\mathbf{q}\cdot\mathbf{r}}|\nu'\rangle$ has already been calculated, the analysis of the different limiting cases entails no difficulty.

An account of the collisions between the electrons and ions leads to an additional contribution to the coefficients (19) and (20), which in many cases is more significant than the contribution from the scattering of the electrons by the phonons. From the collision integral for Maxwellian distributions

$$\sum_{i, \nu_i, \nu_i'}\int d\eta\frac{2\pi}{\hbar}\left|\int\frac{d\mathbf{q}}{(2\pi)^3}\langle\nu_1|e^{i\mathbf{q}\mathbf{r}}|\nu_1'\rangle\langle\nu_i'|e^{-i\mathbf{q}\mathbf{r}}|\nu_i\rangle\frac{4\pi e_1 e_i}{\varepsilon(\eta/\hbar, q)q^2}\right| \times\delta(E_1(\nu_1') - E_1(\nu_1) - \eta)\,\delta(E_i(\nu_i') - E_i(\nu_i) + \eta) \times\{f_1(\nu_1')f_i(\nu_i') - f_1(\nu_1)f_i(\nu_i)\}, \quad (23)$$

which was obtained earlier [3], we can derive equations analogous to (17) and (18). Putting $f_2 = f_2(nk_z)$ and using f_1 in the form of (15) and (16), we obtain equations of the same type as (17) and (18), while for the coefficients of diffusion and heat conduction transversely to the field we obtain

$$D_\perp^{\text{coll}} = b_1^{-1}\frac{1}{2}\sum\left(\frac{c\hbar}{e_1 B}\right)^2\frac{2\pi}{\hbar}\int\frac{d\mathbf{q}V}{(2\pi)^3}d\eta \times\left|\frac{4\pi e_1 e_i}{\varepsilon(\eta/\hbar, q)q^2}F_{n_1n_1'}(\alpha_1^2 q_\perp^2/2)F_{n_in_i'}(\alpha_i^2 q_\perp^2/2)\right|^2 \times\delta[E_1(n_1', k_z^1 + q_z) - E_1(n_1, k_z^1) - \eta]\,\delta[E_i(n_i', k_z^i + q_z) - E_i(n_i, k_z^i) + \eta]\,f_i^0(n_i', k_z^i + q_z)[\partial f_1(n_1', k_z' + q_z)]/\partial\mu_0, \quad (24)$$

where the summation is over i, n_i, n_i', k_z^i, k_x^i, n_1, n_1', and k_z^1.

An analogous formula is obtained for $\kappa_\perp^{\text{coll}}$, if we replace b_1^{-1} in front of the sum in (24) by b_2^{-1} and multiply the integrand by

$$\{(\mu_0 - E_{n_1, k_z^1+q_z})/T_0 - \partial\mu_0/\partial T\}(E_{n_1,k_z^1}\mu_0).$$

In the quantum formulas (19), (20), and (24) we can go to the classical limit by letting ħ go to 0. Then

$$D_\perp + D_\perp^{\text{coll}} = \left(\frac{c}{eB}\right)^2 4\pi\sum_{m_1=-\infty}^{\infty}\int\frac{V}{(2\pi)^3}d\mathbf{q}N_q|C_q q_x|^2\int dp_z^1 2\pi\int dp_\perp^1 \times p_\perp^1 J_m^2\left(\frac{p_\perp q_\perp}{\mu_1\Omega}\right)\delta(p_z^1 q_z/\mu_1 + m_1\Omega_1 - \omega_q)f_1(p_\perp^1, p_z^1)$$
$$+ \left(\frac{ce_1}{B}\right)^2\sum_{m_1, m_i=-\infty}^{\infty}2\pi\int_0^\infty dp_\perp^1 p_\perp^1\int dp_z^1 2\pi\int dp_\perp^i p_\perp^i\int dp_z^i \times\int\frac{dq}{(2\pi)^3}d\eta\,\pi\delta(q_z p_z^1/\mu_1 - m_1\Omega_1 - \eta) \times\delta(q_z p_z^i/\mu_i + m_i\Omega_i + \eta)\left|\frac{4\pi}{q^2\varepsilon(\eta/\hbar, q)}\right|^2 J_{m_1}^2\left(\frac{p_\perp^1 q_\perp}{\mu_1\Omega_1}\right)J_{m_i}^2 \times\left(\frac{p_\perp^i q_\perp}{\mu_i\Omega_i}\right)q_x^2 f_i^0(p_\perp, q_\perp)f_1^0(p_\perp, p_z), \quad (25)$$

where f_i^0 is the Maxwellian function.

An analogous formula can be written for $\kappa_\perp + \kappa_\perp^{coll}$. We note that, as it should be, the first term in (25) coincides with the coefficient preceding $\partial^2/\partial y_0^2$ in (12), averaged over $f_1(p_z^1, p_\perp^1)$, while the second term corresponds to the analogous coefficient if (12) is supplemented by the classical limit (23). From (25) we can immediately obtain an expression for the electric conductivity, using the formula $\sigma_\perp Tk = e^2 n_1(D_\perp + D_\perp^{coll})$ (n_1 is the electron density).

4. If the electrons have ordered motion due to different causes (for example drift in perpendicular electric and magnetic fields, electric current, etc.), then the electron-ion plasma becomes kinetically unstable if the drift velocity v_d exceeds v_{ph}, the phase velocity of the wave. Indeed, direct calculations show that for acoustic oscillations [$|C_q|^2$ is determined from (9)] γ_q has the form (for $\mathbf{q} \parallel \mathbf{B}$)

$$\gamma_q = \sum_{n_1} \frac{2\pi e^2}{q^2} \frac{\omega_q}{\varepsilon(0, q)} (v_{ph} - v_d) \frac{\mu_1^2 \Omega_1}{\hbar} \times \partial f_1^0 \{[(\mu_0 - \hbar\Omega_1 (n_1 + 1/2) - \mu_1 (v_{ph} - v_d)^2]/kT\}/\partial\mu_0. \tag{26}$$

When $v_{ph} < v_d$ we have $\gamma < 0$, that is, the oscillations grow. In the case of Fermi statistics, (26) describes giant oscillations of the growth increment of the sound waves for $v_{ph} < v_d$. Instabilities of this type can be used in principle to amplify and generate ultra and hyper sound. When $v_d = 0$, (26) goes over into the result obtained by Gurevich, Skobov, and Firsov [9].

5. In conclusion we note that Eqs. (12) and (13), if (12) is supplemented with the particle collision integral as determined in [3], may turn out to be useful in the investigation of electron heating and runaway. In place of the system (12) and (13) we can consider (12) alone, from which Θ_q is eliminated with the aid of (13), since the equation for Θ_q can be integrated in general form. The equation will have the simplest form in the case of weak fields, when the influence of the magnetic field on the collisions can be neglected ($r_L/r_D \gg 1$).

APPENDIX 1

Let us calculate the coefficients in (12), which can be written in the form

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_\alpha}\left(D_{\alpha\beta} \frac{\partial}{\partial p_\beta} f\right) + \frac{\partial}{\partial p_\alpha}(A_\alpha f), \tag{A.1}$$

$$\frac{\partial}{\partial \mathbf{p}} \equiv \left\{\frac{\partial}{\partial p_z}, \frac{\partial}{\partial p_\perp}, -\frac{c}{eB}\frac{\partial}{\partial y_0}\right\};$$

$$D_{\alpha\beta} = \sum_{m=-\infty}^{\infty} \frac{e^2}{2\pi} kT \int d\mathbf{q} \frac{Q_\alpha Q_\beta}{q^2} J_m^2 \times \left(\frac{p_\perp q_\perp}{\mu\Omega}\right) \delta(p_z q_z/\mu + m\Omega - \omega_q),$$

$$A_\alpha = \sum_{m=-\infty}^{\infty} \frac{e^2}{2\pi} \int d\mathbf{q} \frac{Q_\alpha}{q^2} \omega_q J_m^2 \left(\frac{p_\perp q_\perp}{\mu\Omega}\right) \delta(p_z q_z/\mu + m\Omega - \omega_q);$$

$$Q \equiv \{q_z, m\Omega\mu/p_\perp, q_x\}. \tag{A.2}$$

In the general case the integrals in (A.2) are not expressed in terms of investigated functions, but in the particular cases that are of interest, the quantities $D_{\alpha\beta}$ and A_α can be approximated by elementary formulas.

Let us consider, for example, the case of strong magnetic fields, when the Larmor radius is $r_L \ll 1/q_{max}$ ($q_{max} \sim 1/r_D$ and r_D is the Debye radius). We cannot neglect here the influence of the magnetic field on the collision process. Naturally, the collision integral which does not take into account the dependence of the cross sections on the magnetic field, is unsuitable for the analysis of this case. If the inequality $r_L q_{max} \ll 1$ is satisfied, the Bessel functions contained in $D_{\alpha\beta}$ and A_α can be replaced by their asymptotic values for small arguments, and then only terms with $m = 0$ are significant. Under these assumptions we have

$$D_{\alpha\beta} = \frac{e^2 kT\mu}{2\pi |p_z|} \int \frac{d\mathbf{q}}{q^2} \begin{pmatrix} q_z \\ 0 \\ q_x \end{pmatrix} \begin{pmatrix} q_z \\ 0 \\ q_x \end{pmatrix} \delta\left(q_z - \omega\frac{\mu}{p_z}\right),$$

$$A_\alpha = \frac{e^2}{2\pi}\left|\frac{\mu}{p_z}\right| \int \frac{d\mathbf{q}}{q^2} \omega_q \begin{pmatrix} q_z \\ 0 \\ q_x \end{pmatrix} \delta\left(q_z - \omega_q \frac{\mu}{p_z}\right). \tag{A.3}$$

The inequality $r_L q_{max} \ll 1$ enables us to approximate $\omega_q \approx \Omega$; then

$$D_\parallel = \frac{e^2 kT}{2}\left|\frac{\mu}{p_z}\right| \Omega^2 \ln\left[1 + \left(\frac{q_{max} p_z}{\mu\Omega}\right)^2\right],$$

$$D_{33} = \frac{e^2 T}{2}\left|\frac{\mu}{p_z}\right| \left\{q_{max}^2 - (\Omega\mu/p_z)^2 \ln\left[1 + \left(\frac{q_{max} p_z}{\mu\Omega}\right)^2\right]\right\},$$

$$D_{12} = D_{13} = D_{21} = D_{31} = D_{22} = 0,$$

$$A_1 = \frac{e^2}{2}\left(\frac{\mu}{p_z}\right)^2 \Omega^2 \ln\left[1 + \left(\frac{q_{max} p_z}{\mu\Omega}\right)^2\right],$$

$$A_2 = A_3 = 0. \tag{A.4}$$

If we approximate the spectrum by the expression $\omega_q^2 \approx \Omega^2 + \omega_0^2$ (ω_0 is the Langmuir frequency), we obtain, when $\Omega^2 \ll \omega_0^2$, simple interpolation formulas which go over into those obtained by Klimontovich[1], and when $\Omega^2 \gg \omega_0^2$ we again obtain (A.4).

APPENDIX 2

Let us calculate the coefficient of electron diffusion transversely to the magnetic field for electrons scattered by plasma oscillations. According to (25) we obtain for a Maxwellian distribution function

$$D_\perp = \sqrt{\frac{\pi}{2}}\left(\frac{c}{B}\right)^2 \mu\bar{v}\int \frac{dq_\perp q_\perp^3}{q^2|q_z|} dq_z \sum I_{|m|}(r_L^2 q_\perp^2)$$
$$\times \exp\{-r_L^2 q_\perp^2 - (\omega_q - m\Omega)^2/(2q_z^2\bar{v}^2)\}, \qquad (A.5)$$

where $\overline{v^2} = kT/\mu$, $r_L^2 = \overline{v^2}/\Omega^2$, and $I_m(x)$ is a Bessel function of imaginary argument.

If $(\omega_q - m\Omega)^2/(2\overline{v^2}) \gtrsim q_{max}^2$, then $D_\perp$ is an exponentially small quantity. The largest contribution to $D_\perp$ is made by the regions $(\omega_q - m\Omega)^2 \approx 0$, or more accurately $(\omega_q - m\Omega)^2/(2\overline{v^2}) \ll q_{max}^2$. If we approximate $\omega_q^2 \approx \Omega^2 + \omega_0^2$, we obtain for the oscillating part of $D_\perp$

$$D_\perp^{osc} \approx \frac{1}{4}\sqrt{\frac{\pi}{2}}\left(\frac{c}{B}\right)^2 \frac{\mu\bar{v}}{r_L^2}$$
$$\times \ln\left[\frac{2q_{max}^2\bar{v}^2}{(\omega_q - m_0\Omega)^2}\int_0^{(q_{max}r_L)^2} d\zeta e^{-\zeta} I_{|m_0|}(\zeta)\right].$$

Thus, whenever the magnetic field intensity is such that $\omega_q \approx m_0\Omega$, $D_\perp$ has a maximum; in other words, the dependence of $D_\perp$ on B has an oscillating character.

Note added in proof (November 15, 1962). Generalization of initial equations and further development of the ideas employed here for calculating the kinetic coefficients have made it possible to obtain the total particle density and energy fluxes with account of the phonon entrainment effect.

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