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SCATTERING OF ELECTRONS ON PROTONS WITH ACCOUNT OF DIPOLE MOMENTS

A. A. BOGUSH and I. S. SATSUNKEVICH

Physics Institute, Academy of Sciences, Belorussian S.S.R.

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The differential cross section for scattering of electrons on protons that possess a dipole structure is derived by a direct calculation of the matrix elements, initial and final longitudinal polarizations of the particles being taken into account. The cross-section correction, calculated under the assumption that the proton has dipole moments (electric and magnetic), can be invoked to explain the discrepancy between experiment and the generally accepted phenomenological model of the process.

THE latest experimental data obtained by the Hofstadter and Wilson groups [1-3] apparently lead to the conclusion that at large momentum transfers q ($q^2 > 30 f^{-2}$) the Rosenbluth formula [4] for the electron-proton scattering cross section is inadequate for the interpretation of the experimental results. The experimental cross sections clearly exceed the theoretical ones, calculated by extrapolation of the values of the form factors to the region of large q. This excess may be due to the large contribution of the two-photon exchange [2,3]. On the other hand, assumption that the proton has an electric dipole moment also leads to a correction which, in principle, can explain the difference in the cross sections.

We calculate below in the Born approximation the differential cross sections for the scattering of an electron on a proton that has a dipole structure along with the ordinary structure, for cases when the initial and final polarizations of both particles are fixed, when only the initial polarizations are taken into account, and for unpolarized particles.

When polarizations are taken into account, the calculation of the scattering cross section, determined by the formula (see [5])

$$d\sigma/d\Omega = \alpha^2 p q^{-4} |M|^2,$$

$$q = p_1 - p_2, \quad p^2 = -\varkappa^2, \quad \varkappa = M_0\hbar/c, \tag{1}$$

is based on the use of the covariant procedure of direct calculation of the matrix elements M [6], and not the squares of their moduli $|M|^2$. For the process under consideration, the matrix elements M are written according to [6] in the form

$$M = \mathrm{Sp}\,(\Gamma_\mu \Lambda^{r_2}(-p_2)\, S^{-1})\; \mathrm{Sp}\,(\gamma_\mu \Lambda^{r_1'}(p_1')\, S'^{-1}), \tag{2}$$

where (compare with [7])

$$\Gamma_\mu = \gamma_\mu (f_1 - i f_2 \hat{q} - f_3 \hat{q}\gamma_5)$$

is the vertex operator for a proton with non-point-like charge, and distributed magnetic and electric dipole moments, where

$$f_2 = f_2(q^2) = l F_2(q^2)/2\varkappa, \quad f_3 = \lambda F_3(q^2)/2\varkappa, \quad \hat{q} = q_\mu \gamma_\mu, \tag{3}$$

while

$$\Lambda^{r_2}(-p_2) = \varphi^{r_2}(-p_2)\cdot\overline{\varphi}^{r_2}(-p_2),$$

$$\Lambda^{r_1'}(p_1') = \psi^{r_1'}(p_1)\cdot\overline{\psi}^{r_1'}(p_1')$$

are the projective dyad matrices [8], which determine the initial states of the proton and the electron; S and S′ are the transformation matrices of the Lorentz group representation in bispinor space, realizing the transitions from the initial state of the particle to the finite state of the particle with account of the variation of the four-momentum ($-p_2 \to -p_1$, $p_1' \to p_2'$) and of the spin variable of the particles ($r_2 \to r_1$, $r_1' \to r_2'$) [6].

We confine ourselves to the case of longitudinal polarization. In this case we obtain for the proton (in the c.m.s.)

$$\Lambda^{\varepsilon_2}(-p_2)\, S^{-1} = \frac{\sqrt{d}}{4\varkappa\sqrt{2}}(\varepsilon_2|\mathbf{p}| + p_0\gamma_5 + \varkappa\gamma_5\gamma_4) \times \left(\gamma_5\gamma_4 - i\frac{\hat{\mathbf{z}}}{d}\right)(\delta_{\varepsilon_2\varepsilon_1} + i\gamma_4\delta_{\varepsilon_2,-\varepsilon_1}) \tag{4}$$

and for the electron

$$\Lambda^{\varepsilon_1'}(p_1')S'^{-1} = \frac{\sqrt{d'}}{4\varkappa\sqrt{2}}(\varepsilon_1'|\mathbf{p}| + p_0'\gamma_5 - \varkappa'\gamma_5\gamma_4)\left(\gamma_5\gamma_4 + i\frac{\hat{\mathbf{z}}'}{d'}\right) \times (\delta_{\varepsilon_1'\varepsilon_2'} + i\gamma_4\delta_{\varepsilon_1',-\varepsilon_1'}), \tag{5}$$

where

$$d = d(\varepsilon_1, \varepsilon_2) = 1 + \varepsilon_1\varepsilon_2 s_1 s_2, \quad d' = d(\varepsilon_1', \varepsilon_2'),$$

$$z = \varepsilon_1 s_1 + \varepsilon_2 s_2 + i\varepsilon_1\varepsilon_2 [s_1 s_2], \ z' = z(\varepsilon_1', \varepsilon_2'), \quad \hat{z} = \gamma_a z_a,$$

and where $\epsilon_1, \epsilon_1', \epsilon_2, \epsilon_2' = \pm 1$ determine the signs of the spin projection on the directions $\mathbf{s}_1 = \mathbf{p}_1/|\mathbf{p}|$ and $\mathbf{s}_2 = \mathbf{p}_2/|\mathbf{p}|$.

After simple calculations with account of (1)—(5) we obtain the following expressions for the 16 differential cross sections corresponding to all possible combinations of initial and final electron and proton polarization:

$$d\sigma_{\mathrm{cms}\varepsilon_2, \varepsilon_1'; \varepsilon_1, \varepsilon_2'} / d\Omega = \alpha^2 [2\mathbf{p}^2\nu(p_0 + p_0')]^{-2} \{|M_{\varepsilon_2, \varepsilon_1'; \varepsilon_1, \varepsilon_2'}|^2\}; \quad (6)$$

$$|M_{\varepsilon, \varepsilon; \varepsilon, \varepsilon}|^2 = \mu^2 f_1^2 (\mathbf{p}^2 + p_0 p_0')^2,$$

$$|M_{\varepsilon, -\varepsilon; \varepsilon, -\varepsilon}|^2 = \nu^2 \{f_1 [\eta^{-1} (\mathbf{p}^2 + p_0 p_0') + 2\mathbf{p}^2] + 4 f_2 \varkappa \mathbf{p}^2\}^2,$$

$$|M_{\varepsilon_1, \varepsilon_1'; \varepsilon_1, -\varepsilon_1'}|^2 = \mu\nu f_1^2 \varkappa'^2 p_0^2,$$

$$|M_{\varepsilon_1, \varepsilon_1'; -\varepsilon_1, \varepsilon_1'}|^2 = \mu\nu \{[f_1 \varkappa p_0' - 2 f_2 \mathbf{p}^2 (p_0 + p_0')]^2 + 4 f_3^2 \mathbf{p}^2 (\mathbf{p}^2 + p_0 p_0')^2\},$$

$$|M_{\varepsilon_1, \varepsilon_1'; -\varepsilon_1, -\varepsilon_1'}|^2 = \nu^2 \{\varkappa'^2 [(f_1 \varkappa - 2 f_2 \mathbf{p}^2)^2 + 4 f_3^2 \mathbf{p}^2 p_0^2]\};$$

$$\mu = 2\cos^2(\vartheta/2), \nu = 2\sin^2(\vartheta/2), \eta = \nu/\mu = \mathrm{tg}^2(\vartheta/2), \quad (7)^*$$

where ϑ is the scattering angle in the c.m.s.

It follows from (7) that the influence of the anomalous magnetic and electric dipole moments of the proton on the scattering cross section manifest itself differently for different initial and final particle polarizations.

In summing over the final spin states of the proton and the electron, we find

$$d\sigma_{\mathrm{cms}\,\pm}/d\Omega = \alpha^2 [2\mathbf{p}^2\nu (p_0 + p_0')]^{-2} \{|\mathfrak{M}|^2_{\mathrm{cms}} \mp |\mathfrak{R}|^2_{\mathrm{cms}}\}, \quad (8)$$

where

$$|\mathfrak{M}|^2_{\mathrm{cms}} = 2\nu\{ f_1^2 [\eta^{-1} (\mathbf{p}^2 + p_0 p_0')^2 + (\mathbf{p}^2)^2 \nu + \varkappa^2 \varkappa'^2] + 2 f_2^2 (\mathbf{p}^2)^2 [\mu (p_0 + p_0')^2 + \nu (2\varkappa + \varkappa'^2)] - 4\varkappa f_1 f_2 \mathbf{p}^2 (\varkappa'^2 - \nu \mathbf{p}^2)\ 2 f_3^2 \mathbf{p}^2 [\mu (\mathbf{p}^2 + p_0 p_0') + \nu \varkappa'^2 p_0^2]\},$$

$$|\mathfrak{R}|^2_{\mathrm{cms}} = 2\nu \mathbf{p}^2 \{f_1^2 (2\mathbf{p}^2 + \mu p_0 p_0') + 4 f_2^2 \nu \varkappa^2 \mathbf{p}^2 + 2 f_1 f_2 \varkappa [(2+\nu)\mathbf{p}^2 + \mu p_0 p_0']\}. \quad (9)$$

It can be readily verified that when $f_3 = 0$, the scattering cross section obtained in this way agrees with the known formulas [5,9].

*tg = tan.

We note that the presence of a dipole structure in the proton makes no contribution to the polarization correction $\sim |\mathfrak{R}|^2$ [see (9)] if only the initial spin states of the particles are taken into account. This can be used to observe the characteristic behavior of the dipole moment in the experiment. By making two measurements of the cross-section difference $d\sigma_- - d\sigma_+ \sim |\mathfrak{R}|^2$, we can determine the values of f_1 and f_2. Substituting them into the formula for the cross section of the unpolarized particles ($\sim |\mathfrak{M}|^2$) and using the corresponding experimental cross section, we obtain f_3.

In the laboratory frame we can write for the scattering cross section of unpolarized particles as $\kappa' \to 0$, with account of (3),

$$\frac{d\sigma_{\mathrm{lab}}}{d\Omega} = \frac{\alpha^2 p_{\mathrm{lab}}}{q^4_{\mathrm{lab}}} \{|\mathfrak{M}|^2_{\mathrm{lab}}\} = \left(\frac{\alpha}{2p_{10}'}\right)^2 \frac{\cos^2(\vartheta/2)}{\sin^4(\vartheta/2)} \left(1 + \frac{2p_{10}'}{\varkappa}\sin^2\frac{\vartheta}{2}\right)^{-1} \times \left\{F_1^2 + \frac{q^2_{\mathrm{lab}}}{4\varkappa^2}\left[2(F_1 + lF_2)^2 \mathrm{tg}^2\frac{\vartheta}{2} + (lF_2)^2 + (\lambda F_3)^2\right]\right\}.$$

It is easy to see that when $\lambda F_3 = 0$, i.e., in the absence of dipole moments in the proton, formula (10) coincides with the formula of Rosenbluth [4]. It follows from (10) that at large values of q the form factor λF_3 makes a definite contribution to the scattering cross section.

At small values of q satisfactory agreement between the experimental data of Hofstadter et al. and the Rosenbluth formula indicates that λF_3 is small. Assuming that the foregoing divergence of the cross sections at large values of q is due only to the dipole structure of the protons, we can estimate λF_3. A preliminary estimate on the basis of the employed data of [3,10] shows that when $q^2 = 30 f^{-2}$ the value of λF_3 is on the order of 10^{-2}. Taking into account the experimental error, one must approach this value of λF_3 with some caution. In addition, it is necessary to bear in mind that corrections of a different physical origin to the scattering cross section are also possible.

The appearance of a form factor connected with the dipole structure of the proton in the formula for the cross section necessitates a corresponding modification in the procedure used to determine form factors. Rewriting (10) in the notation of Hofstadter [10] we have

$$d\sigma/(d\Omega\sigma_{NS}) = a_{11}F_1^2 + a_{12}F_1F_2 + a_{22}F_2^2 + a_{33}F_3^2, \quad (11)$$

where $a_{33} = \hbar^2 q^2 \lambda^2 / 4M_0^2 c^2$.

It follows therefore that in order to find the three unknowns F_1, F_2, and F_3 in the cross section (11) we need at least three experimental cross sections (for the same value of q^2). Further, in place of the intersection of two ellipses it is necessary to consider the intersections of three ellipsoids.

It is important to emphasize that unlike a_{11}, a_{12}, and a_{22}, the coefficient a_{33} does not depend on the scattering angle ϑ for a fixed value of q^2, which may be useful in the determination of the specific influence of the dipole structure.

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327