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DISPERSION RELATIONS FOR THE REFRACTIVE INDEX AND ABSORPTION COEFFICIENT IN MEDIA WITH EXCITON ABSORPTION

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Spatial dispersion in isotropic media with exciton absorption is studied. It is shown that the role of spatial dispersion is greater, the greater the width of the band of exciton states. In the range of frequencies where there is spatial dispersion, the formation of additional transverse waves with large values of the refractive index and absorption coefficient is possible. The Kramers-Kronig dispersion relations for the refractive index and absorption coefficient should only apply to the normal transverse waves.

1. INTRODUCTION

As is well known, the passage of electromagnetic waves through dielectric media is characterized by a complex permittivity $\epsilon = \epsilon' + i\epsilon''$. The real and imaginary parts of the permittivity as functions of the frequency ω satisfy the Kramers-Kronig dispersion relations

$$\varepsilon'(\omega) - 1 = \frac{2}{\pi} \mathrm{P} \int_0^\infty \frac{z\varepsilon''(z)\, dz}{z^2 - \omega^2}, \quad \varepsilon''(\omega) = \frac{2\omega}{\pi} \mathrm{P} \int_0^\infty \frac{\varepsilon'(z) - 1}{\omega^2 - z^2}\, dz, \tag{1}$$

where P signifies that the principal value of the integral is taken at the point $z = \omega$.

The derivation of the integral relations (1) is based on the absence of singular points of the analytic function $\epsilon(z)$ in the upper half-plane of the complex variable z. This property is in its turn a consequence of the principle of causality (see [1]), according to which the polarization of the medium at a given instant of time depends only on the values of the electric field at all preceding instants of time.

If the frequency of the electromagnetic wave falls in a region corresponding to collective excited states of the medium (for example, exciton states), the permittivity will also be a function of the wave vector **k**. Such a dependence is called spatial dispersion. The generalization of the Kramers-Kronig formulae to a medium with spatial dispersion was first made by Leontovich,[2] who considered the case when the permittivity is a function of the frequency ω and the real wave vector **k**. Such a case is realized when there are additional electrical charges in the medium. For example, the field of an electrical charge moving in the medium with velocity **v** corresponding to a wave vector **k** is characterized by the frequency $\omega = \mathbf{v} \cdot \mathbf{k}$. Consequently, for a given **k** the frequency can have any value.

When a transverse electromagnetic wave is propagating in an isotropic medium with no free charges, the frequency and the complex wave vector $\mathbf{k} = \mathbf{Q} + i\Lambda$ are related by the equation

$$\omega^2 c^{-2} \varepsilon(\omega, \mathbf{k}) = \mathbf{k}^2. \tag{2}$$

Thus, for a given frequency, not all values of **k** are possible in this case, but only certain selected and generally complex values $\mathbf{k}_i = \mathbf{k}_i(\omega)$. Therefore the propagation of a light wave in the medium is characterized by several values of the permittivity $\epsilon_i(\omega) \equiv \epsilon[\omega, \mathbf{k}_i(\omega)]$. If the refractive index μ and absorption coefficient η are introduced by the relation

$$\mathbf{k} = \omega(\mu + i\eta)\mathbf{s}/c, \tag{3}$$

where **s** is the unit vector defining the propagation direction of the wave, then, by separating the real and imaginary parts in $\epsilon(\omega, \mathbf{k})$, Eq. (2) can be transformed to

$$\mu^2 - \eta^2 = \varepsilon'(\omega, \mu, \eta), \quad 2\mu\eta = \varepsilon''(\omega, \mu, \eta). \tag{4}$$

The variation of ϵ' and ϵ'', and of μ and η with ω is given by quantum theory. If such variation is known, equation (4) allows the dispersion law, i.e., the variation of μ and η with frequency ω, to be found. In the range of frequencies where there is spatial dispersion, to each value of ω correspond several possible values of μ and η. In other words, for a given frequency, direction **s**, and polarization, several types of waves can propagate in the medium. When generalizing equations (1) such a possibility must be taken into account. When the integrations in (1) are performed, frequencies corresponding to regions of absorption are also involved. Therefore it is very important to study the spatial dispersion in the absorption region. As will be shown in Sec. 2, it is, in fact, in the region of absorption that spatial dispersion has its greatest magnitude.

The propagation of electromagnetic waves in media showing spatial dispersion has been studied in papers by Pekar[3] and Ginzburg.[4] However, in these papers only the region of transparency was considered. In Sec. 2 of this paper the dispersion law in an isotropic, non-gyrotropic medium is studied in the neighborhood of an exciton absorption band; also studied is the possibility of the existence of additional transverse electromagnetic waves in such a medium, and the variation with frequency of the refractive index and absorption coefficient for the normal and additional waves. In Sec. 3 are considered the dispersion relations for the refractive index and absorption coefficient of light in media with spatial dispersion.

2. THE DISPERSION LAW IN MEDIA WITH EXCITON ABSORPTION

As has already been shown in papers by the author and Rashba[5] and by the author and Lubchenko[6], the real and imaginary parts of the permittivity of a medium in the range of frequencies close to one of the bands of excited exciton states can be expressed by the formulae

$$\varepsilon' = \varepsilon_0(\omega_0) + \frac{A(\omega_Q - \omega)}{(\omega_Q - \omega)^2 + \gamma^2(\omega)}, \quad \varepsilon'' = \frac{A\gamma(\omega)}{(\omega_Q - \omega)^2 + \gamma^2(\omega)}, \tag{5}$$

where $A = 2\pi e^2 fN/m\omega_0$; e and m are the charge and mass of the electron; N is the number of molecules in unit volume; f is the oscillator strength of the corresponding quantum transition in the molecule; $\gamma(\omega)$ is a function basically different from zero in some range of frequencies. A possible form of this function is cited in [6].

The quantity ω_Q appearing in (5) is the frequency of the exciton excitation. It can be expressed by the approximate formula

$$\omega_Q = \omega_0 + \tfrac{1}{2}\tau\,[\cos(\omega_Q a\mu/c) - 1],$$

where ω_0 is the frequency corresponding to the edge of the exciton band; a is the distance between molecules; τ is a quantity characterizing the width of the exciton band of excited states. Taking into account that for frequencies in the visible and ultraviolet $\omega_Q a/c \sim 10^{-3}$, it is possible to write for values $\mu < 10^3$:

$$\omega_Q \approx \omega_0 - \tfrac{1}{4}\tau\,(\omega_Q a\mu/c)^2 \approx \omega_0 - \tfrac{1}{4}\tau\,(\omega_0 a/c)^2\mu^2. \tag{6}$$

If $\tau < 0$, it is said that the exciton has a positive effective mass; when $\tau > 0$, the effective mass of the exciton is negative.

Substituting (6) in (5), we find the approximate expressions:

$$\varepsilon'(\omega, \mu) = \frac{\varepsilon_0(\omega)}{1 - \mu^2 D(\omega)}, \quad \varepsilon''(\omega, \mu) = \frac{\Gamma(\omega)}{1 - \mu^2 D(\omega)}, \tag{7}$$

$$D(\omega) = \frac{(\omega_0 - \omega)(\omega_0 a/c)^2\tau}{2[(\omega_0 - \omega)^2 + \gamma^2(\omega)]}, \tag{8}$$

$$\varepsilon_0(\omega) = \varepsilon_0(\omega_0)[1 - \mu^2 D(\omega)] + A[\omega_0 - \omega - \tfrac{1}{4}\tau(\omega_0 a/c)^2\mu^2]/[(\omega_0 - \omega)^2 + \gamma^2(\omega)], \tag{9}$$

$$\Gamma(\omega) = \frac{A\gamma(\omega)}{(\omega_0 - \omega)^2 + \gamma^2(\omega)}. \tag{10}$$

Substituting (7) in (4), we find the equations determining the variation of the refractive index and absorption coefficient with the frequency ω:

$$\mu^2 - \eta^2 = \frac{\varepsilon_0(\omega)}{1 - \mu^2 D(\omega)} \equiv \varepsilon', \quad 2\mu\eta = \frac{\Gamma(\omega)}{1 - \mu^2 D(\omega)} \equiv \varepsilon''. \tag{11}$$

The derivation of (11) is based on the use of the approximate expression (6); therefore they are valid only for $\mu < 10^3$.

The absorption coefficients η and ϵ'' are always positive. Consequently, $\mu \geq 0$ also. The value of $\epsilon_0(\omega)$ can be negative. In the region of absorption this is possible when $\mu^2 < \eta^2$, and in the region of transparency when $\mu = 0$. Large values of η signify rapid attenuation of the wave in the medium. If μ is small this attenuation is mainly associated with reflection. The true absorption of the waves is given by the product $\mu\eta$.

The case $D(\omega) = 0$ corresponds to the absence of spatial dispersion. For each frequency ω satisfying this condition the transverse waves in the isotropic medium are characterized by one value of the refractive index and a single absorption coefficient, which are determined by the equations

$$\mu^2 \approx \tfrac{1}{2}\left[\sqrt{\varepsilon_0^2(\omega) + \Gamma^2(\omega)} + \varepsilon_0(\omega)\right],$$

$$\eta^2 \approx \tfrac{1}{2}\left[\sqrt{\varepsilon_0^2(\omega) + \Gamma^2(\omega)} - \varepsilon_0(\omega)\right].$$

In the particular case of frequencies corresponding to the region of transparency $[\gamma(\omega) \approx 0]$ we have

$$\mu^2 \approx \varepsilon_0(\omega), \quad \eta(\omega) \approx 0.$$

As will be shown below, spatial dispersion can only be observed in the range of frequencies for which $D(\omega) > 0$. According to (8) this condition is fulfilled when the effective mass of the exciton is positive ($\tau < 0$) for frequencies exceeding ω_0, and when the effective mass of the exciton is negative ($\tau > 0$) for frequencies smaller than ω_0. In other words, spatial dispersion can be observed for electromagnetic waves with frequencies falling in the range of the exciton excitation band. The quantity $D(\omega)$ is proportional to the width ($|\tau|$) of the band of excited exciton states, and to the square of the ratio of the lattice constant a to the wavelength ω_0/c of the radiation in vacuum. Spatial dispersion can be neglected for an exciton band of sufficiently narrow energy.

We consider the region of weak absorption, i.e., the case when $\Gamma(\omega) < \epsilon_0(\omega)$ (consequently, $\epsilon'' < \epsilon'$). In this case from (11) follow the approximate equalities

$$\mu^2 = \frac{\varepsilon_0(\omega)}{1 - \mu^2 D(\omega)}, \tag{12}$$

$$\eta^2 = \frac{\Gamma^2(\omega)}{4\varepsilon_0(\omega)\,[1 - \mu^2 D(\omega)]}. \tag{13}$$

If we neglect in the first approximation the weak variation of $\epsilon_0(\omega)$ with μ^2, then, solving (12), we find two roots:

$$\mu_{1,2}^2 = (1 \mp \sqrt{1 - 4\varepsilon_0(\omega) D(\omega)})/2D(\omega). \tag{14}$$

In the visible and ultraviolet regions of the spectrum the inequality $\epsilon_0(\omega) D(\omega) \ll 1$ is usually satisfied; therefore, from (14) follow the approximate equalities

$$\mu_1{}^2 \approx \varepsilon_0(\omega)\,[1 + \varepsilon_0(\omega) D(\omega)] \approx \varepsilon_0(\omega),$$

$$\mu_2^2 \approx [1 - \varepsilon_0(\omega) D(\omega)]/D(\omega). \tag{15}$$

By substituting the values obtained into (13), the corresponding absorption coefficients can be calculated.

Thus, in the range of frequencies corresponding to exciton excitations in the medium, the propagation of two types of transverse waves having identical polarization is possible:

a) The normal transverse waves which remain when $D(\omega) \to 0$. The refractive index and absorption coefficient of these transverse waves are, according to (15) and (13), given by the formulae

$$\mu_1^2 \approx \varepsilon_0(\omega),$$

$$\eta_1{}^2 = \Gamma^2(\omega)/4\varepsilon_0(\omega)\,[1 - \varepsilon_0(\omega) D(\omega)] \approx \Gamma^2(\omega)/4\varepsilon_0(\omega). \tag{16}$$

In other words, the refractive index and absorption coefficient of the normal transverse waves are, in the first approximation, determined by the same formulae as when spatial dispersion is absent.

b) Additional transverse waves, the refractive index and absorption coefficient of which are given by the formulae

$$\mu_2^2 \approx [1 - \varepsilon_0(\omega) D(\omega)]/D(\omega),$$

$$\eta_2^2 \approx \Gamma^2(\omega)/4\varepsilon_0^2(\omega) D(\omega). \tag{17}$$

It follows from (17) that the additional transverse waves cannot exist in the range of frequencies for which $D(\omega) < 0$ since such waves would correspond to imaginary values of the refractive index and absorption coefficient.

Comparing (16) and (17), we find

$$\eta_1^2/\eta_2^2 \approx \varepsilon_0(\omega) D(\omega) \ll 1. \tag{17a}$$

Consequently, the absorption coefficient of the additional transverse waves in the isotropic medium greatly exceeds the absorption coefficient of the normal transverse waves. Thus, the observation of the additional transverse waves in a propagating beam of light is difficult to carry out.

We now consider the region of strong absorption. In this case $\epsilon_0(\omega) < \Gamma(\omega)$ or $\epsilon' < \epsilon''$, and from (11) follow the approximate equations

$$\mu^2 \approx \frac{\varepsilon'' + \varepsilon'}{2} = \frac{\varepsilon_0(\omega) + \Gamma(\omega)}{2[1 - \mu^2 D(\omega)]}, \quad \eta^2 \approx \frac{\varepsilon'' - \varepsilon'}{2} = \frac{\Gamma(\omega) - \varepsilon_0(\omega)}{2[1 - \mu^2 D(\omega)]}. \tag{18}$$

Solving the first of these equations for μ^2 we find two roots:

$$\mu_{1,2}^2 = [1 \mp \sqrt{1 - 2D(\omega)\,[\varepsilon_0(\omega) + \Gamma(\omega)]}]/2D(\omega).$$

When $D(\omega)[\epsilon_0(\omega) + \Gamma(\omega)] \ll 1$ we can write

$$\mu_1^2 \approx [\varepsilon_0(\omega) + \Gamma(\omega)]/2,$$

$$\mu_2^2 \approx \left[1 - \tfrac{1}{2} D(\omega)\,[\Gamma(\omega) + \varepsilon_0(\omega)]\right] / D(\omega). \tag{19}$$

Substituting (19) in the second equation of (18) we find the corresponding absorption coefficients:

$$\eta_1^2 \approx \tfrac{1}{2}\,[\Gamma(\omega) - \varepsilon_0(\omega)],$$

$$\eta_2^2 \approx 2\,[\Gamma(\omega) - \varepsilon_0(\omega)]/D(\omega)\,[\Gamma(\omega) + \varepsilon_0(\omega)]. \tag{20}$$

It follows from (19) and (20) that for strong absorption the refractive index and absorption coefficient of the normal waves do not in the first approximation depend on the presence of spatial dispersion. The absorption coefficient of the additional transverse waves is in this case also much greater than the absorption coefficient of the normal waves, since

$$\eta_1^2 / \eta_2^2 \approx D(\omega)\,[\Gamma(\omega) + \varepsilon_0(\omega)] \ll 1. \tag{20a}$$

Thus, additional transverse waves are possible in a medium with exciton absorption only in the frequency range for which $D(\omega) > 0$. According to (8), the function $D(\omega)$ is zero at the edge of the exciton band of excited states ($\omega = \omega_0$) and increases with increasing frequency in the band. The value of the function $D(\omega)$, as well as depending on the quantity τ, is essentially dependent on the form of the function $\gamma(\omega)$ which defines the structure of the absorption band. When the frequency is further increased, the function $D(\omega)$ again diminishes to zero. Consequently, the additional transverse waves can appear only inside the absorption band and close to its high frequency side if the effective mass of the exciton is positive, and the low frequency side if the effective mass of the exciton is negative. The absorption coefficient of the additional waves always greatly exceeds the absorption coefficient of the normal waves 1).

To illustrate the relationships obtained above, values of the function $D(\xi)$ were calculated (see Fig. 1), where $\xi = (\omega - \omega_0)/\omega_0$, for a single isolated exciton absorption band with parameters corresponding to a molecular crystal of the naphthalene type: $\omega = 5 \times 10^{15}$ sec^{-1}, $\tau = -10^3$ cm^{-1}. The function $\gamma(\omega)$ was, for simplicity, taken as zero for $\omega < \omega_0$ and as 100 cm^{-1} for $\omega > \omega_0$. In real crystals the function $\gamma(\omega)$ has a more complex structure; in particular it contains some small contribution independent of ω (see [6]). For an absorption band with negative effective exciton mass ξ in Fig. 1 must be replaced by $-\xi$.

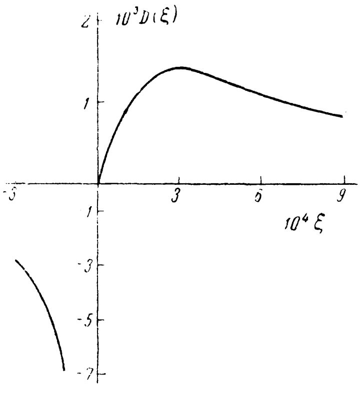

FIG. 1. The function describing spatial dispersion ($\xi = (\omega - \omega_0)/\omega_0$). Spatial dispersion exists when $D(\xi) > 0$.

Graphs of the functions $\epsilon_0(\xi)$ and $\Gamma(\xi)$, calculated from (9) and (10) with the parameters $f = 0.01$, $N = 4 \times 10^{21}$ cm^{-3} and $\epsilon_0(\omega) = 4$ (the remaining parameters are given above), are shown in Fig. 2 for positive effective mass, and in Fig. 3 for negative effective mass.

From the values obtained the squares of the refractive indices for the normal and additional

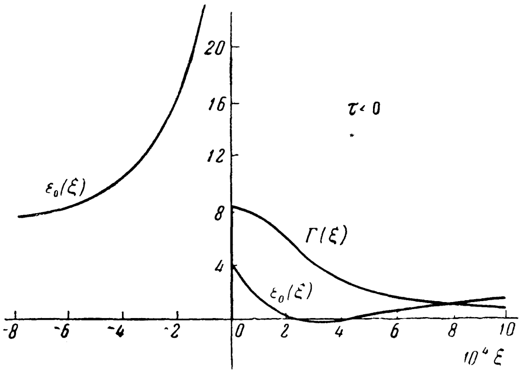

FIG. 2. The values of $\epsilon_0(\xi)$ and $\Gamma(\xi)$ in the neighborhood of a resonance frequency corresponding to an exciton band of excited states with positive exciton effective mass.

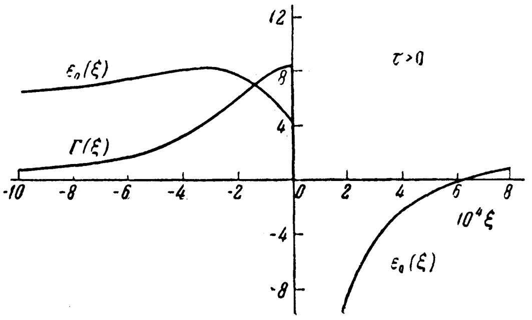

FIG. 3. The values of $\epsilon_0(\xi)$ and $\Gamma(\xi)$ in the neighborhood of a resonance frequency corresponding to a band with negative exciton effective mass.

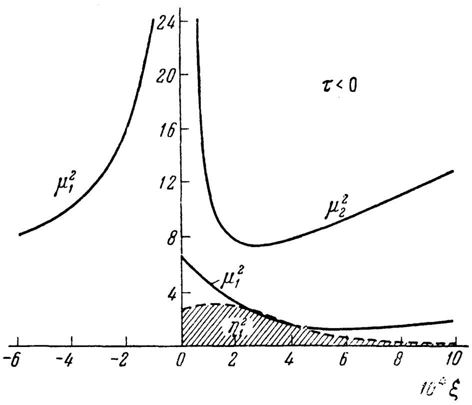

FIG. 4. The squares of the refractive indices of the normal (μ_1^2) and additional (μ_2^2) transverse waves in the frequency range corresponding to an exciton band with positive effective mass. The shaded region describes the absorption coefficient of the normal waves.

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1)The theory of spatial dispersion has usually been developed without taking absorption into account, and this essential fact was not always considered. For example, in the paper by Pekar[3] it is said that "a second wave freely proceeds through the crystal plate." Pekar[3] proposed on the basis of this assumption a number of methods for observing the additional waves in a crystal. In a paper by Brodin and Pekar[7] attempts were even made to interpret experimental data under the assumption that both types of waves were absorbed almost equally.

transverse waves were calculated (see Fig. 4). We see that the additional transverse waves only appear in a small frequency interval on the right-hand side of the resonance frequency. At the limits of this region the square of the refractive index of these waves has a very large value. Using the values of $D(\xi)$ shown in Fig. 1 and formulae (17a) and (20a), it can be seen that the absorption coefficient of the additional transverse waves should exceed the absorption coefficient of the normal waves by a factor of more than 100.

Ginzburg[4] studied spatial dispersion by expanding the inverse dispersion in powers of the refractive index

$$\varepsilon^{-1}(\omega,\mu)=\varepsilon^{-1}(\omega)+\beta\mu^2 \quad \text{or} \quad \varepsilon(\omega,\mu)=\varepsilon(\omega)/(1+\beta\mu^2\varepsilon(\omega)). \tag{21}$$

In the transparent region (21) agrees with (7) if we put $D(\omega) = -\beta\epsilon(\omega)$. However, instead of (8), Ginzburg took $\beta\epsilon(\omega) = \beta(\epsilon_0 - A/\xi)$ with β a constant. Such an expression is applicable only far from the resonance frequency. Thus, Figs. 1 and 2 given in the reference specified cannot reflect the real situation in the frequency range close to the resonance frequency. The work of Pekar has the same inadequacy (see [3]).

3. THE KRAMERS-KRONIG INTEGRAL RELATIONS

When spatial dispersion is present retarded interactions and the principle of causality lead to the following general expression for the dielectric polarizability:

$$\varepsilon(\omega,\mathbf{k}) = 1+\int_0^\infty d\tau \int\limits_{c\tau>|\boldsymbol{\rho}|} d\boldsymbol{\rho}\, S(\tau,\boldsymbol{\rho})\, e^{i(\omega\tau+\mathbf{k}\boldsymbol{\rho})}, \tag{22}$$

where $S(\tau,\boldsymbol{\rho})$ is a real function; the range of values of τ and $\boldsymbol{\rho}$ for which this function differs significantly from zero is determined by the time taken to establish polarization in the medium and by some small spatial region. When there is no spatial dispersion $S(\tau,\boldsymbol{\rho}) = S(\tau)\,\delta(\boldsymbol{\rho})$.

For an isotropic medium this function does not depend on the direction $\boldsymbol{\rho}$; therefore, integrating (22) over the angular variables, we obtain

$$\varepsilon(\omega,k) = 1+4\pi\int_0^\infty d\tau e^{i\omega\tau}\int_0^{c\tau} S(\tau,\rho)\frac{\sin(k\rho)}{k}\rho\, d\rho. \tag{22a}$$

In the general case ω and k in (22a) are independent. In the case of transverse electromagnetic waves ω and k are related by the dispersion equation (2). Using this relation and (3) we can transform (22a) to the form

$$\mu^2-\eta^2+2i\mu\eta=\varepsilon_{\mu\eta}(\omega)\equiv 1 + 4\pi\int_0^\infty d\tau e^{i\omega\tau}\int_0^{c\tau} S(\tau,\rho)\frac{\sin\{\omega(\mu+i\eta)/c\}}{\omega(\mu+i\eta)/c}\rho\, d\rho, \tag{23}$$

where the relation between ω, μ, and η is defined by (4).

The dielectric polarizability defined in this way characterizes the properties of the medium only relative to the propagation of electromagnetic waves satisfying Maxwell's equations with no free electric charges. This quantity should differ from the permittivity (22), where ω and k are assumed independent.

As was said in Sec. 2, if ω falls in the frequency range corresponding to exciton excitations, there can propagate in the medium two types of waves with differing values of $\mu(\omega)$ and $\eta(\omega)$. With each such wave can be associated its own value of $\epsilon_{\mu\eta}(\omega)$. We consider the value $\epsilon'_{\mu\eta}(\omega)$ for the normal wave. In this case μ_1 and η_1 always have finite values, and because of the presence of the function $S(\tau,\rho)$, which is different from zero for small values of τ and ρ, integration over the spatial variables in (23) should lead to finite values for $0 \le \tau \le \infty$ and for any real ω. Consequently, as ω tends to infinity, $\epsilon'_{\mu\eta}(\omega)$ tends to unity, since the integral in (23) tends to zero because of the presence under the integral sign of the oscillating multiplier $e^{i\omega t}$. Taking this fact into account, and using (4), we obtain

$$\lim_{|\omega|\to\infty}\mu_1(\omega)=1, \quad \lim_{|\omega|\to\infty}\eta_1(\omega)=0. \tag{24}$$

In order to obtain the Kramers-Kronig relations for $\epsilon'_{\mu\eta}(\omega)$, we need to consider the properties of the function (23) when it is continued analytically into the upper half-plane of complex values of ω, which we denote by z. We assume that the limiting properties (24) are preserved also for the functions $\mu_1(z)$ and $\eta_1(z)$, which are the analytical continuations of $\mu_1(\omega)$ and $\eta_1(\omega)$, when z tends to infinity along any path in the upper half-plane of the variable z. Then it can be seen that the function will tend to unity under the same limiting process. Thereby our assumption on the limiting properties of $\mu_1(z)$ and $\eta_1(z)$ is justified.

Thus, the function $\epsilon'_{\mu\eta}(z)$ is analytic in the upper half-plane of the variable z and tends to zero as $|z| \to \infty$ in the upper half-plane of z including the real axis. Consequently, just as in the absence of spatial dispersion (see [1]), we can write the integral relation

$$\varepsilon^1_{\mu\eta}(\boldsymbol{\omega})-1=\frac{}{i\pi}P\int_{-\infty}^{\infty}\frac{\varepsilon^1_{\mu\eta}(z)-1}{z-\omega}\,dz. \tag{25}$$

Separating the real and imaginary parts in this equation, using (4) and the equation $\epsilon(\omega, \mu, \eta) = \epsilon^*(-\omega, \mu, -\eta)$ which follows from (23) taking into account the fact that the function $S(\tau, \rho)$ is real, we obtain the dispersion relations

$$\mu_1^2(\omega) - \eta_1^2(\omega) - 1 = \frac{4}{\pi} P \int_0^\infty \frac{z\mu_1(z)\eta_1(z)\,dz}{z^2 - \omega^2},$$

$$\mu_1(\omega)\eta_1(\omega) = \frac{\omega}{\pi} P \int_0^\infty \frac{\mu_1^2(z) - \eta_1^2(z) - 1}{\omega^2 - z^2}\,dz. \qquad (26)$$

Thus, when there is spatial dispersion, the refractive index and absorption coefficient of the normal transverse waves satisfy the usual Kramers-Kronig dispersion relations.

For the additional transverse waves μ, η, and ϵ assume infinite values in some ranges of frequencies, i.e., they lose their physical significance as macroscopic characteristics of the medium. Consequently, the function $\epsilon^2_{\mu\eta}(\omega)$ is not analytic in the upper half-plane of the variable z including the real axis, and it is impossible to write down for it integral relations of type (24).

Experimental verification of Eqs. (26) is possible only when the values of $\mu_1(\omega)$ and $\eta_1(\omega)$ are measured directly for the normal transverse waves. A measurement of the refractive index by interferometric methods allows only the refractive index of the normal waves proceeding through the medium to be determined. The additional transverse waves decay rapidly and as a rule do not get out of the plate. When measuring the absorption coefficient by the method of the attenuation of a transmitted beam of light, the mean absorption coefficient of all transverse waves is measured. In order to determine the absorption coefficient of the normal transverse waves, the attenuation of the beam of light must be measured when the thickness of the layer of substance is increased starting with a thickness in which occurs complete absorption of all additional transverse waves.

The formulae obtained in this article refer to an isotropic medium. However, these formulae can be used also for certain selected directions of propagation of light in crystals. Such directions will be the directions of the principal axes of the tensor permittivity (if the principal axes of the tensors ϵ' and ϵ'' in the crystal coincide).

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