ON RELAXATION OF ELECTRON AND ION TEMPERATURES OF FULLY IONIZED PLASMA IN A STRONG MAGNETIC FIELD

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Under conditions when the Larmor radii of the plasma particles do not exceed the screening radius of the Coulomb field, the effect of a magnetic field on particle-collision events becomes significant. Under these conditions, the relaxation time of electron and ion temperatures is dependent upon the magnetic field. The appropriate values of relaxation time are determined both in a broad interval of relationships between electron and ion temperatures, and in a broad interval of magnetic-field values.

1. Temperature relaxation in a completely ionized electron-ion plasma was investigated by many workers. It can be assumed here that, owing to the small mass ratio of the electron and the ion, the electrons and ions can be assumed to have Maxwellian momentum distributions with different temperatures over a wide range of the parameters characterizing the plasma. Denoting the electron and ion temperatures by T_e and T_i, we have after Landau [1]

$$\frac{dT_e}{dt} = -(T_e - T_i)\frac{2m}{M}\nu_{\text{eff}}, \tag{1}$$

where m is the electron mass, M the ion mass, and the effective collision frequency is given by the formula

$$\nu_{\text{eff}} = \frac{4}{3}\frac{\sqrt{2\pi}\, e^2 e_i^2 N_i}{\sqrt{m}\,(\varkappa T_e)^{3/2}} \ln\frac{r_D}{r_{min}}. \tag{2}$$

Here e and e_i are respectively the charges of the electron and the ion, N_i is the number of ions per unit volume, r_D is the Debye screening radius of the Coulomb field in the plasma, given by the relation

$$r_D^{-2} = \frac{4\pi e^2 N_e}{\varkappa T_e} + \frac{4\pi e_i^2 N_i}{\varkappa T_i}, \tag{3}$$

and finally r_{min} is the minimum impact parameter, which is determined either by the inapplicability of perturbation theory ($r_{min} \sim e^2/\kappa T$, where T is the larger of T_e and T_i), or by the inapplicability of the classical analysis ($r_{min} \sim \hbar/\sqrt{m\kappa T_e}$).

Formula (1) is obtained under the assumption that the thermal velocity of the ions is small compared with the thermal velocity of the electrons. This corresponds to the inequality

$$T_e/m \gg T_i/M. \tag{4}$$

In the derivation of formula (1) it was assumed, in addition, that the magnetic field does not influence the act of collision. This means that in order for formula (1) to be applicable, the inequality $r_B \gg r_D$ must hold, where r_B is the minimum Larmor radius of the plasma particles. We shall assume that the temperatures of the electrons and the ions satisfy the relation

$$T_e/M \ll T_i/m. \tag{5}$$

Then the Larmor radius of the electron is minimal. Therefore the corresponding inequality, for which the magnetic field B does not influence the collisions of the particles, can be written in the form

$$\frac{mc}{|e|B}\sqrt{\frac{\varkappa T_e}{m}} \gg r_D. \tag{6}$$

In the present communication we undertook to obtain results that extend Landau's result (1) to the case of strong fields, when it is necessary to take into account the influence of the magnetic field on the act of collision between the electron and the ion.

It must be noted that an attempt to study the temperature relaxation in the case of a completely ionized plasma situated in a strong magnetic field was made by Kihara [2,3]. However, the results obtained in this case are in our opinion inexact. The reason for it was primarily the neglect of the ratio of the electron mass to the ion mass (see below). Therefore, there still remains a gap in plasma relaxation theory, connected with the region of large values of magnetic field intensity. The contents of the present paper is devoted to filling this gap.

2. To investigate the temperature relaxation in a completely ionized electron-ion plasma situated in a strong magnetic field, we use the previously obtained [4] collision integral (see also [5]). Being interested in processes that change little over the characteristic collision time, we can write this collision integral in the following form [1)]

$$\frac{\partial}{\partial p_a^i}\sum_b N_b \int_{-\infty}^{0} d\tau \left\{D_{ij}^{(a,b)}(\mathbf{p}_a,\tau)\frac{\partial}{\partial P_a^j[\tau,\mathbf{p}_a]} - A_i^{(a,b)}(\mathbf{p}_a,\tau)\right\} \times f_a(\mathbf{P}_a[\tau,\mathbf{p}_a]), \tag{7}$$

where N_b is the number of particles of type b per unit volume; f_a is the distribution function of the particles of type a, normalized to unity; the function $\mathbf{P}_a[\tau,\mathbf{p}_a]$ is the momentum of the particle a, moving in a constant magnetic field, at the instant τ, if its momentum at zero time was $\mathbf{p}_a$; finally, the retarded diffusion and friction coefficients, D_{ij} and A_i respectively, have the form

$$D_{ij}^{(a,b)}(\mathbf{p}_a,\tau) = \int d\mathbf{p}_b \int \frac{d\mathbf{k}}{(2\pi)^3}\left(\frac{4\pi e_a e_b}{k^2}\right)^2 k_i k_j \mathscr{E}^{(ab)}(\tau,\mathbf{k},\mathbf{v}_a,\mathbf{v}_b) \times f_b(\mathbf{P}_b[\tau,\mathbf{p}_b]), \tag{8}$$

$$A_i^{(a,b)}(\mathbf{p}_a,\tau) = \int d\mathbf{p}_b \int \frac{d\mathbf{k}}{(2\pi)^3}\left(\frac{4\pi e_a e_b}{k^2}\right)^2 k_i k_j \mathscr{E}^{(ab)}(\tau,\mathbf{k},\mathbf{v}_a,\mathbf{v}_b) \times \frac{\partial f_b(\mathbf{P}_b[\tau,\mathbf{p}_b])}{\partial P_b^j[\tau,\mathbf{p}_b]}. \tag{9}$$

Here

$$\mathscr{E}^{(ab)}(\tau,\mathbf{k},\mathbf{v}_a,\mathbf{v}_b) = \exp\left\{i\mathbf{k},\ \mathbf{B}\frac{(\mathbf{B}\mathbf{v}_a)}{B^2}\tau - \frac{1-\cos\Omega_a\tau}{\Omega_a}\frac{[\mathbf{B}\mathbf{v}_a]}{B} - \frac{\sin\Omega_a\tau}{\Omega_a}\frac{[\mathbf{B}[\mathbf{B}\mathbf{v}_a]]}{B^2} - \mathbf{B}\frac{(\mathbf{B}\mathbf{v}_b)}{B^2}\tau + \frac{1-\cos\Omega_b\tau}{\Omega_b}\frac{[\mathbf{B}\mathbf{v}_b]}{B} + \frac{\sin\Omega_b\tau}{\Omega_b}\frac{[\mathbf{B}[\mathbf{B}\mathbf{v}_b]]}{B^2}\right\}, \tag{10}*$$

$\Omega_a = e_aB/m_ac$ is the Larmor frequency of the particle of type a.

Under conditions when f_b is Maxwellian, the diffusion coefficient (8) can be represented, after integration over the momenta of particle b, in the form

$$D_{ij}^{(a,b)0}(\mathbf{p}_a,\tau) = \int \frac{d\mathbf{k}}{(2\pi)^3}\left(\frac{4\pi e_a e_b}{k^2}\right)^2 k_i k_j \exp\left\{i\mathbf{k},\ \mathbf{B}\frac{(\mathbf{B}\mathbf{v}_a)}{B^2}\tau - \frac{1-\cos\Omega_a\tau}{\Omega_a}\frac{[\mathbf{B}\mathbf{v}_a]}{B} - \frac{\sin\Omega_a\tau}{\Omega_a}\frac{[\mathbf{B}[\mathbf{B}\mathbf{v}_a]]}{B^2}\right\} \times \exp\left\{-\frac{\varkappa T_b}{2m_b}\left(\frac{\mathbf{k}\mathbf{B}}{B}\right)^2 - 2\frac{\varkappa T_b}{m_b}\frac{[\mathbf{k}\mathbf{B}]^2}{B^2}\frac{\sin^2(\Omega_b\tau/2)}{\Omega_b^2}\right\}. \tag{11}$$

After an analogous transformation of the friction coefficient, we can write down the collision integral for particles of type a, colliding with particles of type b, the latter having a Maxwellian distribution, in the following form:

$$N_b\frac{\partial}{\partial p_a^i}\left\{\int_{-\infty}^{0} d\tau D_{ij}^{(a,b)0}(\mathbf{p}_a,\tau)\left[\frac{\partial}{\partial P_a^j[\tau,\mathbf{p}_a]} + \frac{P_a^j[\tau,\mathbf{p}_a]}{m_a\varkappa T_b}\right] \times f_a(\mathbf{P}_a[\tau,\mathbf{p}_a])\right\}. \tag{12}$$

In our problem both types of particles (electrons and ions) have Maxwellian momentum distributions. Therefore, multiplying the kinetic equation for the electrons by $p^2/3m$ and integrating over the momenta of the electrons, we obtain the equation

$$dT_e/dt = (T_i - T_e)/\tau_T, \tag{13}$$

where

$$\frac{1}{\tau_T} = \frac{2}{3}\frac{N_i}{m^2\varkappa^2 T_i T_e}\int_{-\infty}^{0} d\tau \int d\,\mathbf{p}_e f_e p_e^n D_{nl}^{(e,i)0} P_e^l[\tau,\mathbf{p}_e]. \tag{14}$$

Bearing in mind formula (11) and integrating in the right half of (14) over the momenta, we obtain

$$\frac{1}{\tau_T} = \frac{2}{3}\frac{N_i}{Mm}\int_0^{\infty} d\tau \int \frac{d\mathbf{k}}{(2\pi)^3}\left(\frac{4\pi e e_i}{k^2}\right)^2 \times \left\{\frac{(\mathbf{k}\mathbf{B})^2}{B^2}\tau + \frac{[\mathbf{k}\mathbf{B}]^2}{B^2}\frac{\sin\Omega_e\tau}{\Omega_e}\right\}\left\{\frac{(\mathbf{k}\mathbf{B})^2}{B^2}\tau + \frac{[\mathbf{k}\mathbf{B}]^2}{B^2}\frac{\sin\Omega_i\tau}{\Omega_i}\right\} \times \exp\left\{-\left[\frac{\varkappa T_e}{2m} + \frac{\varkappa T_i}{2M}\right]\frac{(\mathbf{k}\mathbf{B})^2}{B^2}\tau^2 - 2\frac{[\mathbf{k}\mathbf{B}]^2}{B^2}\left[\frac{\varkappa T_e}{m}\frac{\sin^2(\Omega_e\tau/2)}{\Omega_e^2} + \frac{\varkappa T_i}{M}\frac{\sin^2(\Omega_i\tau/2)}{\Omega_i^2}\right]\right\}. \tag{15}$$

In formula (15) the integration with respect to $\mathbf{k}$, as follows from the derivation of the collision integral [4], must be carried out over a region bounded both on the side of large values of k ($k_{max} = r_{min}^{-1}$) and of the small ones ($k_{min} = r_D^{-1}$). Taking this fact into account, we can transform (15) into

$$\frac{1}{\tau_T} = \frac{2m}{M}\frac{4}{3}\frac{\sqrt{2\pi}e^2e_i^2N_i}{\sqrt{m}(\varkappa T_e)^{3/2}}L, \tag{16}$$

where

[1)]This collision integral is suitable both for spatially-homogeneous distributions and for distributions which vary little over distances on the order of the effective interaction region of the particles.

*[Bv] = B × v.

$$L=\int_0^\infty \frac{dt}{t}\int_0^1 \frac{dx}{\varphi^{3/2}(t,x)}\left[x^2+(1-x^2)\frac{\sin 2\delta t}{2\delta t}\right]$$
$$\times\left[x^2+(1-x^2)\frac{\sin 2t}{2t}\right]\left\{\Phi\left(2t\frac{\omega_{max}}{|\Omega_e|}\varphi^{1/2}(t,x)\right)\right.$$
$$-\Phi\left(2t\frac{\omega_{min}}{|\Omega_e|}\varphi^{1/2}(t,x)\right)-\frac{4t}{\sqrt{\pi}}\frac{\varphi^{1/2}(t,x)}{|\Omega_e|}$$
$$\times\left(\omega_{max}\exp\left[-\frac{4t^2\omega_{max}^2}{\Omega_e^2}\varphi(t,x)\right]\right.$$
$$\left.\left.-\omega_{min}\exp\left[-\frac{4t^2\omega_{min}^2}{\Omega_e^2}\varphi(t,x)\right]\right)\right\}. \qquad (17)$$

Here

$$\delta=e_i m/|e|M,\quad \omega_{max}=\sqrt{\varkappa T_e/2m}\,k_{max},$$
$$\omega_{min}=\sqrt{\varkappa T_e/2m}\,k_{min},\qquad \Phi(z)=(2/\sqrt{\pi})\int_0^z e^{-t^2}dt; \qquad (18)$$
$$\varphi(t,x)=\left[1+\frac{mT_i}{MT_e}\right]x^2+[1-x^2]\psi(t), \qquad (19a)$$
$$\psi(t)=\frac{1}{t^2}\left\{\sin^2 t+\frac{mT_i}{MT_e}\frac{1}{\delta^2}\sin^2\delta t\right\}. \qquad (19b)$$

In the integrand of (17), x is the cosine of the angle between the direction of the magnetic field and the wave vector $\mathbf{k}$, while t is the time, which is measured in units of $|\Omega_e|^{-1}$. The instant $t = 0$ corresponds here to the shortest distance between particles, and the instant $t = \infty$ corresponds to their flying apart, provided, of course, this is possible.

For $t \ll 1$, which corresponds to a scattering time of the colliding particles which is much smaller than the Larmor revolution period of the electrons, the integrand in the right half of (19b) simplifies to

$$\frac{1}{t}\left\{\Phi\left(2t\frac{\omega_{max}}{|\Omega_e|}\right)-\Phi\left(2t\frac{\omega_{min}}{|\Omega_e|}\right)+\frac{4}{\sqrt{\pi}}\frac{t}{|\Omega_e|}\right.$$
$$\left.\times\left[\omega_{min}\exp\left(-4t^2\frac{\omega_{min}^2}{\Omega_e^2}\right)-\omega_{max}\exp\left(-4t^2\frac{\omega_{max}^2}{\omega_e^2}\right)\right]\right\}. \qquad (20)$$

Account was taken here of the inequality (4) and also of the fact that $\omega_{max} \gg |\Omega_e|$.

For weak magnetic fields ($|\Omega_e| \ll \omega_{min}$) expression (20) actually differs from zero only when $t < |\Omega_e|/2\omega_{min}$. Therefore for such weak fields we obtain

$$L_0=\ln\frac{\omega_{max}}{\omega_{min}}\approx\ln\frac{r_D}{r_{min}}. \qquad (21)$$

Here and below we confine ourselves to logarithmic accuracy. In other words, we neglect quantities on the order of unity compared with large logarithms. Formula (21), in conjunction with formula (16), leads to the value of the temperature relaxation time obtained in [1], where the influence of the magnetic field on the particle collision was completely neglected. This corresponds to formula (1).

3. In the case of interest to us, that of strong magnetic fields, when the Larmor frequency of the electrons is appreciably larger than ω_{min}, we also confine ourselves to logarithmic accuracy. Then the integral (17) can be represented in the form of the sum

$$L=L_\Omega+\delta L. \qquad (22)$$

The first term corresponds to integration over the region $t < 1$, when the integrand is approximated by (20). This region corresponds to a short distance between particles, at which the influence of the magnetic field on the collision act is insignificant. We then obtain for L_Ω

$$L_\Omega=\ln\frac{\omega_{max}}{|\Omega_e|}. \qquad (23)$$

The second term in (22) is determined principally by the values of the integrand of (20) corresponding to $t \gg 1$. Bearing this in mind, we can first assume the maximum frequency to be infinite. This corresponds to the fact that when $t > 1$ only long-range collisions occur. Then

$$\delta L=\int_1^\infty\frac{dt}{t}\int_{\sqrt{\psi(t)}}^1\frac{dx}{x^2}\frac{1}{\sqrt{x^2-\psi(t)}}$$
$$\times\left\{\frac{\sin 2t}{2t}+\left[1-\frac{\sin 2t}{2t}\right][x^2-\psi(t)]\right\}$$
$$\times\left\{\frac{\sin 2\delta t}{2\delta t}+\left[1-\frac{\sin 2\delta t}{2\delta t}\right][x^2-\psi(t)]\right\}\left\{1-\Phi\left(\frac{2\omega_{min}}{|\Omega_e|}tx\right)\right.$$
$$\left.+\frac{2}{\sqrt{\pi}}\frac{2\omega_{min}}{|\Omega_e|}tx\exp\left(-\left[\frac{2\omega_{min}}{\Omega_e}tx\right]^2\right)\right\}. \qquad (24)$$

Bearing in mind the inequality $|\Omega_e| \gg \omega_{min}$, we can assume, if we confine ourselves to logarithmic accuracy, that

$$1-\Phi\left(\frac{2\omega_{min}}{|\Omega_e|}tx\right)+\frac{2}{\sqrt{\pi}}\left(\frac{2\omega_{min}}{|\Omega_e|}tx\right)\exp\left[-\left(\frac{2\omega_{min}}{\Omega_e}tx\right)^2\right]$$
$$\approx\eta\left(\frac{2\omega_{min}}{|\Omega_e|}tx\right)=\begin{cases}1 & 2\omega_{min}tx<|\Omega_e|\\ 0 & 2\omega_{min}tx>|\Omega_e|\end{cases}. \qquad (25)$$

Using formula (25) we can integrate with respect to x in the right half of (24). Bearing in mind at the same time that in accordance with (19b) the function $\psi(t)$ is small compared with unity when t is large, and also neglecting all expressions which make contributions of order of unity to δL, we obtain

$$\delta L = \delta L_1 + \delta L_2, \tag{26}$$

$$\delta L_1 = \frac{1}{2}\ln\frac{|\Omega_e|}{2\omega_{min}} + \left(-\frac{1}{2} + \ln\frac{2}{e}\right)\int_1^{|\Omega_e|/2\omega_{min}} \frac{dt}{t}\,\frac{\sin 2\delta t}{2\delta t}$$
$$+ \ln\frac{2}{e}\int_{|\Omega_e|/2\omega_{min}}^{\infty} \frac{dt}{t}\,\frac{\sin 2\delta t}{2\delta t}\,\eta\left(\frac{2\omega_{min}}{|\Omega_e|}\,t\sqrt{\psi(t)}\right), \tag{27}$$

$$\delta L_2 = \frac{1}{2}\int_1^{\infty}\frac{dt}{t}\,\frac{\sin 2\delta t}{2\delta t}\,\eta\left(\frac{2\omega_{min}}{|\Omega_e|}\,t\sqrt{\psi(t)}\right)\ln\frac{1}{\psi(t)}$$
$$+ \int_{|\Omega_e|/2\omega_{min}}^{\infty}\frac{dt}{t}\,\frac{\sin 2\delta t}{2\delta t}\,\eta\left(\frac{2\omega_{min}}{|\Omega_e|}\,t\sqrt{\psi(t)}\right)\ln\frac{|\Omega_e|}{2\omega_{min}t}. \tag{28}$$

The first term of (26) yields logarithmic expressions in the first power, while the second term leads to doubly logarithmic additions. Indeed, from formula (27) we obtain

$$\delta L_1 = \frac{1}{2}\ln\frac{\Omega_i}{2\omega_{min}} + \ln\frac{2}{e}\ln\left|\frac{\Omega_e}{\Omega_i}\right| \quad \text{for } 2\omega_{min} \ll \Omega_i, \tag{29}$$

$$\delta L_1 = \ln\frac{2}{e}\ln\left|\frac{\Omega_e}{\Omega_i}\right| \quad \text{for } \Omega_i \ll 2\omega_{min} \ll \sqrt{\frac{MT_e}{mT_i}}\,\Omega_i, \tag{30}$$

$$\delta L_1 = \ln\frac{2}{e}\left\{\ln\frac{|\Omega_e|}{2\omega_{min}} + \ln\sqrt{\frac{MT_e}{mT_i}}\right\}$$
$$\text{for } 2\omega_{min} \gg \sqrt{\frac{MT_e}{mT_i}}\,\Omega_i. \tag{31}$$

In the limit of large fields, when $2\omega_{min} \ll \Omega_i$, the second term of formula (28) is immaterial. In addition, the function η is equal to unity everywhere. All this leads to an expression that does not depend on the magnetic field

$$\delta L_2' = F\left(\frac{mT_i}{MT_e};\,\delta^2\right)$$
$$\equiv \int_1^{\infty}\frac{dt}{t}\,\frac{\sin 2\delta t}{2\delta t}\ln\frac{t}{\sqrt{\sin^2 t + (mT_i/MT_e)(\sin^2\delta t)/\delta^2}}. \tag{32}$$

Assuming that $\ln(MT_e/mT_i)$ and $\ln(mT_i/MT_e\delta^2)$ greatly exceed unity, we can write with doubly-logarithmic accuracy the following approximate expression for the right half of (32):

$$\delta L_2 = \frac{1}{4}\ln\frac{MT_e}{mT_i}\ln\left(\sqrt{\frac{mT_i}{MT_e}}\,\frac{\Omega_e^2}{\Omega_i^2}\right) \quad \text{for } 2\omega_{min} \ll \Omega_i. \tag{33}$$

For a hydrogen plasma and for the case of approximately equal electron and ion temperatures, the right half of (33) is approximately equal to 21.

Numerical integration of the right of (32) yields 22 in this case.

At somewhat smaller magnetic fields, when the following inequality holds true

$$\Omega_i \ll 2\omega_{min} \ll \Omega_i\sqrt{MT_e/mT_i},$$

the second term of (28) also makes an appreciable contribution. Bearing in mind in this case that ω_{min} is itself determined only accurate to a factor of order of unity, we obtain

$$\delta L_2 = F\left(\frac{mT_i}{MT_e};\,\delta^2\right) - \frac{1}{2}\left(\ln\frac{2\omega_{min}}{\Omega_i}\right)^2. \tag{34}$$

Finally, in the last case, when the inequality

$$|\Omega_e| \gg 2\omega_{min} \gg \sqrt{MT_e/mT_i}\,\Omega_i$$

is satisfied, formula (28) yields

$$\delta L_2 = \frac{1}{2}\ln\frac{MT_e}{mT_i}\ln\frac{\Omega_e}{2\omega_{min}} + \frac{1}{4}O\left(\ln\frac{MT_e}{mT_i}\right). \tag{35}$$

Calculation with higher accuracy is meaningless, since only the order of magnitude of ω_{min} is determined. The same considerations allow us to state that we must not attach great significance to the second term of (31). The first term of (31) should be retained only if it is large compared with the second. Formulas (22), (23), and (29)—(35) in conjunction with (16) determine τ_T, the temperature relaxation time of an electron-ion plasma situated in a strong magnetic field. As follows from these formulas, such a relaxation time turns out to depend on the magnetic field intensity.

A few words should be said also on the numerical values of the quantities, due to the influence of the strong magnetic field. For this purpose we note first that when using the plasma parameters that are nowadays of interest, the usual Coulomb logarithm L_0 [see formula (21)] can assume values from 5 to 25. In our analysis the assumption $\omega_{min} \ll |\Omega_e|$ denotes that the logarithm of the ratio of the frequencies is large compared with unity. This means in turn that the use of our asymptotic formulas is possible for plasmas with $L_0 > 10$. The latter is connected with the fact that within the framework of the assumption of weak Coulomb interaction, which is the basis of the collision integral which we are using, and when we confine ourselves to logarithmic accuracy, the ratio ω_{max}/Ω_e must be large. This ratio is large if the inequality $B \ll 10T^{3/2}$ is satisfied, where T is the temperature in degrees and B is in Gausses. Bearing in mind this limitation on the field, we point out that in the case of very strong fields, as was already mentioned earlier, for an almost-isothermal hydrogen plasma the doubly logarithmic addition (33) reaches an order of 20. This value is attained at fields $B \geq 5\sqrt{N}$. In particular, this may be realized, for example, for $N \sim 5 \times 10^7\ cm^{-3}$ and $B \sim 3 \times 10^4$ G. Further, when $B \sim \sqrt{N}/6$, which corresponds to the maximum value given by (35), the doubly-logarithmic addition has a value ~ 14 also for a hydrogen plasma with nearly equal electron and ion temperatures. The figures presented show that the formulas obtained above disclose the appreciable influence of the magnetic field on the temperature relaxation in a plasma.

4. In conclusion we must dwell on a comparison of the results obtained above and the result found by Kihara [2,3] for the temperature relaxation time in a fully ionized plasma in the presence of a strong magnetic field. In this investigations, in calculating the relaxation time, a method different from ours was used. We therefore actually indicate those items in [2,3] which cause the discrepancy with our results. However, it is first necessary to point out the discrepancy itself, which consists in the fact that the doubly-logarithmic addition obtained in [2,3] has the form of the square of the logarithm of the ratio $|\Omega_e|/\omega_{min}$, whereas in our case the doubly logarithmic addition is given by expressions (32)—(35). By the same token, the influence of the magnetic field and the relaxation turns out to be quite different.

The reason for obtaining different results is due primarily to the fact that in [2,3] the ion mass was assumed to be infinite. We did not assume so, and our final formulas depend appreciably on the ratio of the ion and electron masses. Such a dependence is so essential, that an attempt to go to the limit $M \rightarrow \infty$ leads to logarithmically diverging expressions. It is therefore necessary to expect in [2,3] a diverging expression for the addition to the Coulomb logarithm. The absence of such divergences in those papers is due to the fact that although their calculations are within the framework of perturbation theory, the Coulomb interaction is used, which is not suitable in this analysis for either large or short distances.

We point out, finally that the doubly logarithmic addition (35) coincides, apart from a coefficient $^3/_2$, with the additions obtained by Belyaev [6] and by Gurevich and Firsov [7], who calculated the coefficient of diffusion in the plasma transversely to the magnetic field. It must be said that at large fields our doubly logarithmic expressions differ appreciably from those obtained by Belyaev [6]. This is not surprising, since our analysis pertains to an entirely different relaxation process, and it can be verified that, generally speaking, different relaxation times arise for different relaxation processes in a plasma situated in a strong magnetic field.

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