EQUILIBRIUM SHAPES AND FISSION OF A ROTATING NUCLEUS

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The shape of the total-energy surface of a rotating nucleus is studied in the zeroth ellipsoidal approximation on the basis of the liquid drop model. An essential change occurs in the surface for the fission parameter value $x_0 = 0.81$; for $x < x_0$ a new stable equilibrium state is possible without axial symmetry. The range of x is determined for the previously studied[1] fission behavior of a rotating nucleus. It is shown that the approximation used for the equilibrium shape is valid for $x = 0$ and $x \approx 1$.

THE experimental investigation of compound nuclei formed in reactions with multicharged ions, and therefore possessing high excitation energy and large angular momentum, has made it necessary to investigate theoretically the effect of large angular momentum on the equilibrium shape of a rotating nucleus and to determine the stability of the equilibrium shape. These properties of a rotating nucleus must be known as a basis for investigating the disintegration processes of the compound nuclei.

The total excitation energy of these nuclei can be divided into the thermal excitation energy and the potential energy, which depends on the nuclear shape (including the rotational energy). When a nucleus possesses high excitation energy the effects of nucleon pair correlation and shell structure can be neglected; under these conditions the nuclear moment of inertia can be regarded as that of a rigid body. The equilibrium shapes of a rotating nucleus correspond to maximum thermal excitation energy. Therefore stable equilibrium corresponds to a minimum of potential energy, while unstable equilibrium is represented by a saddle point on the potential energy surface.

The influence of angular momentum on the equilibrium shape of a nucleus is determined on the basis of the liquid drop model, which appears to be entirely adequate in cases of high excitation energy. Despite its simplicity the liquid-drop model enables us to consider the nuclear properties that are most essential in determining the shape of a rotating nucleus and the character of its stability. In this model the potential energy, which depends on the nuclear shape, is the sum of the surface, Coulomb, and rotational energies. This model has already been used to study the influence of angular momentum on the equilibrium shapes and stability of heavy nuclei.[1] Relatively light nuclei ($A \sim 50$) have recently been studied similarly;[2] in these cases the shapes of rotating nuclei were approximated by families of prolate and oblate axisymmetric ellipsoids. For each such family and at all values of the angular momentum a minimum potential energy exists which is lowest for an oblate ellipsoid rotating about its axis of symmetry in the case of small angular momentum. For large angular momentum the minimum is lower for a prolate ellipsoid rotating about an axis perpendicular to its axis of symmetry. The approximation of nuclear shape used in [2] cannot be used to determine the character of the transition from one equilibrium shape to another. It is thus impossible to determine whether this transition is continuous or whether an energy barrier exists between the equilibrium shapes. In [2] the equilibrium shapes were studied for a single value of A; the shapes can be determined accurately for a neutral nucleus.[3] However, the stability of these shapes must be redetermined, since in [3] the stability of the equilibrium shape was investigated at constant angular velocity, whereas angular momentum is conserved in a rotating nucleus.

In the present work the equilibrium shapes of a rotating nucleus and the character of their stability are studied for the case in which the shape can be approximated by a triaxial ellipsoid. In this approximation the stability can be studied over the entire range of the fission parameter. Although an ellipsoid is not a true equilibrium shape of a rotating nucleus, we can expect that this approximation will be sufficiently accurate for the angular momentum range under consideration.

EQUILIBRIUM SHAPES IN THE ELLIPSOIDAL APPROXIMATION

Let the nuclear surface be represented by

$$X^2/a^2 + Y^2/b^2 + Z^2/c^2 = R_0^2, \tag{1}$$

where R_0 is the radius of a sphere of equal volume and the semiaxes of the ellipoid satisfy the relation $abc = 1$ (conservation of volume). We shall consider the case in which a and b are nearly equal. It is convenient to introduce a parameter δ characterizing the degree of asymmetry of the ellipsoid as follows:

$$a^2 = (\sqrt{1+\delta^2} + \delta)/c, \qquad b^2 = (\sqrt{1+\delta^2} - \delta)/c.$$

For $\delta \ll 1$ the sum of the surface and Coulomb energies (expressed in units of $4\pi R_0^2 O$, where O is the coefficient of surface tension) is given by

$$U_S + U_C = \sum_{k=0}^{\infty} \varphi_k(c)\,\delta^{2k},$$

where

$$\varphi_0(c) = \frac{1}{2c} + \frac{c^2}{2}\int_0^1 \frac{du}{\Psi(u)} + \frac{2x}{c}\int_0^1 \frac{du}{\Phi(u)},$$

$$\varphi_1(c) = \frac{c^2}{4}\int_0^1 \frac{(1-u^2)^2\,du}{\Psi^3(u)} - \frac{x}{c^4}\int_0^1 \frac{u^2(1-u^2)\,du}{\Phi^3(u)},$$

$$\varphi_2(c) = -\frac{c^2}{16}\left\{\int_0^1 \frac{(1-u^2)^2\,du}{\Psi^3(u)} + 3c^3\int_0^1 \frac{u^2(1-u^2)^3\,du}{\Psi^5(u)}\right\} + \frac{x}{4c^4}\left\{\int_0^1 \frac{u^2(1-u^2)\,du}{\Phi^3(u)} + \frac{3}{c^3}\int_0^1 \frac{u^4(1-u^2)^2\,du}{\Phi^5(u)}\right\};$$

$$\Psi(u) = 1 + (c^3 - 1)u^2, \quad \Phi(u) = 1 + \left(\frac{1}{c^3} - 1\right)u^2.$$

Here $x = (3Z^2e^2/10R_0)/(4\pi R_0^2 O)$ is the fission parameter; Ze is the nuclear charge. The rotational energy of an ellipsoid rotating about the z axis is represented by

$$U_{rot} = yc\{1 - \tfrac{1}{2}\delta^2 + \tfrac{3}{8}\delta^4 + \ldots\},$$

where $y = (M^2/2I_0)/(4\pi R_0^2 O)$, M is the angular momentum, and I_0 is the moment of inertia of a spherical nucleus.

In the first approximation with respect to δ^2 the potential energy surface $U = U_S + U_C + U_{rot}$ of a rotating nucleus possesses three extremal points which are determined from the following equations:

$$\delta = 0;\ \varphi_0'(c_1) + y = 0; \tag{2}$$

$$\delta \neq 0;\ \varphi_0'(c_2) + y + \delta^2[\varphi_1'(c_2) - \tfrac{1}{2}y] = 0,$$
$$\varphi_1(c_2) - \tfrac{1}{2}c_2 y + 2\delta^2[\varphi_2(c_2) + \tfrac{3}{8}c_2 y] = 0. \tag{3}$$

Here $\varphi'(c) \equiv d\varphi(c)/dc$.

For small y Eq. (2) gives the stable equilibrium point at which the nucleus is an axisymmetric oblate ellipsoid ($\delta = 0$) with its angular momentum vector along the axis of symmetry. For all values of the fission parameter x there exists a critical rotational energy y_{cr} at which the oblate ellipsoidal shape begins to become unstable. The values of y_{cr} and of the smallest semiaxis c_0 of an oblate ellipsoid for y_{cr} are determined from

$$\varphi_0'(c_0) + y_{cr} = 0, \qquad \varphi_1(c_0) - \tfrac{1}{2}c_0 y_{cr} = 0.$$

These values are given in Tables I and II. Figure 1 shows the approximate dependence of $(1-c_1)/(1-c_0)$ on y/y_{cr}, which is determined from (2) and which holds for all x to within a few percent.

The behavior and properties of the extremal points given by (2) and (3) are investigated most simply for $y \sim y_{cr}$. Then for $\Delta y = y - y_{cr} \ll y_{cr}$ we obtain from (2) in first approximation

$$\delta = 0, \qquad c_1 - c_0 = -\Delta y/\varphi_0''(c_0), \tag{4}$$

and for the points of (3), where the nucleus is a triaxial ellipsoid, we have

$$\delta^2 = \frac{N(x)}{F(x)}\Delta y, \qquad c_2 - c_0 = -\frac{P(x)}{F(x)}\Delta y,$$

$$P(x) = \tfrac{1}{2}c_0[\varphi_1'(c_0) - \tfrac{1}{2}y_{cr}] + 2[\varphi_2(c_0) + \tfrac{3}{8}c_0 y_{cr}],$$

$$N(x) = \tfrac{1}{2}c_0\varphi_0''(c_0) + \varphi_1'(c_1) - \tfrac{1}{2}y_{cr},$$

$$F(x) = 2\varphi_0''(c_0)[\varphi_2(c_0) + \tfrac{3}{8}c_0 y_{cr}] - [\varphi_1'(c_0) - \tfrac{1}{2}y_{cr}]^2. \tag{5}$$

Table I

x	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0.8
y_{cr}	0.2831	0,248	0,214	0.180	0,147	0.115	0.0835	0.0559	0.0302
c_0	0,7779	0.783	0,790	0.798	0,807	0.819	0.834	0.853	0.880
$10P(x)$	0.852	0.824	0.796	0.771	0.751	0.728	0,711	0.701	0.702
$N(x)$	0.729	0,673	0.620	0.563	0,504	0.447	0.387	0,323	0.261
$F(x)$	0.186	0.152	0.120	0.0923	0,0678	0.0461	0.0278	0.0129	0.0010
$\varphi''_0(c_0)$	1.94	1,75	1.55	1.36	1,16	0.960	0.765	0.564	0.372
$A(x)$	—0.743	—0.860	—1.03	—1.27	—1,62	—2.26	—3,62	—7,18	—92
$10K_0(x)$	0.372	0,315	0.257	0,209	0.162	0,120	0.0805	0.0471	0.0207
$K_1(x)$	0.222	0,218	0,210	0.202	0,193	0,181	0,166	0,147	0,120
$K_2(x)$	0,258	0.287	0.322	0,368	0.432	0.521	0.654	0.886	1.34

Table II

x	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
$10\,y_{cr}$	0.257	0.213	0.171	0.133	0.0975	0.0665	0.0397	0.0195	0.0045
c_0	0.887	0.894	0.902	0.911	0.922	0.933	0.946	0.961	0.980
$10P(x)$	0.703	0.705	0.708	0.711	0.716	0.720	0.727	0.735	0.745
$N(x)$	0.246	0.232	0.218	0.203	0.189	0.177	0.158	0.146	0.130
$100F(x)$	−0.095	−0.286	−0.463	−0.623	−0.764	−0.887	−1.03	−1.14	−1.22
$\varphi''_0(c_0)$	0.332	0.292	0.253	0.215	0.180	0.146	0.098	0.069	0.032
$A(x)$	96	32.2	20.3	15.4	13.0	12.1	12.3	13.6	21.3

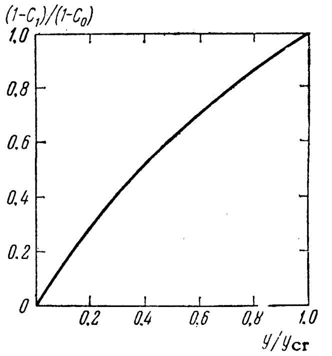

FIG. 1

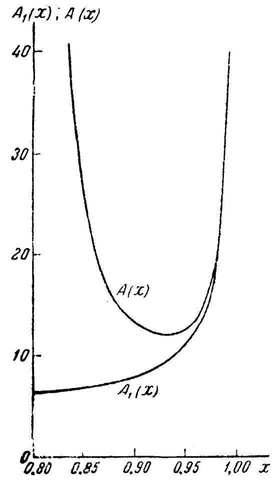

FIG. 2

The values of these functions are given in Tables I and II.

The character of the instability of the oblate ellipsoid for $y > y_{cr}$ changes at $x_0 = 0.81$, which is determined from $F(x_0) = 0$. For nuclei with $x > x_0$ the instability of (4) for $y > y_{cr}$ results from the fission process discussed in detail in [1]. It was here possible to determine the interval of variation of the fission parameter, $0.81 < x \le 1$, for which the description of the fission of a rotating nucleus at $y \sim y_{cr}$ considered in [1] is valid. In the ellipsoidal approximation we determine the fission barrier, which is equal to the potential-energy difference between the saddle point (5) and the point (4):

$$E_f(y) = A(x)\,(\Delta y)^2 + \ldots$$
$$= -(\Delta y)^2 N^2(x) \,/\, 2\varphi''_0(c_0)\, F(x) + \ldots, \qquad (6)$$

while from [1] at $y \sim y_{cr}$ we obtain

$$E_f(y) = A_1(x)\,(\Delta y)^2 + \ldots$$
$$= \left[\frac{5}{16}\frac{1}{1-x} + \frac{15\,685}{3264} + \ldots\right](\Delta y)^2 + \ldots \qquad (7)$$

The functions $A(x)$ and $A_1(x)$ are represented in Fig. 2, which shows that the expression for the fission barrier in [1], holding true for $y \sim y_{cr}$, can be used only for a very narrow range of x. The unlimited growth of $A(x)$ and of the coefficients of Δy in (5) for $x \to x_0$ is associated with the fact that at y_{cr} for $x < x_0$ the fission barrier is of finite height and its top is located at a finite distance from stable equilibrium. Thus the instability of the oblate ellipsoid for $y > y_{cr}$ and $x < x_0$ is not caused by fission, but is associated with a new stable equilibrium state of (5) where the nucleus is a triaxial ellipsoid. We note that for nuclei with $x < x_0$ and $y < y_{cr}$ no extremum of (5) exists, but appears only when the shape of the oblate ellipsoid becomes unstable, i.e., when $y > y_{cr}$. Since we are here investigating only shapes very similar to an oblate ellipsoid, the instability of the equilibrium shape (5) due to fission is not being considered for nuclei with $x < x_0$, since the shape of these nuclei at the top of the fission barrier differs greatly from an ellipsoid.

The difference between the potential energy at stable equilibrium and that of a spherical nucleus, for $y \le y_{cr}$, is

$$\Delta U_T = K_0(x) + K_1(x)\,\Delta y + K_2(x)\,(\Delta y)^2,$$
$$K_0(x) = \varphi_0(1) + y_{cr} - \varphi_0(c_0) - c_0 y_{cr},$$
$$K_1(x) = 1 - c_0, \quad K_2(x) = 1/2\varphi''_0(c_0).$$

The thermal excitation energy for $y \le y_{cr}$ is the sum of the thermal excitation energy of a spherical nucleus and ΔU_T; for $y > y_{cr}$ and $x < x_0$ the quantity $E_f(y)$ must be added.

VALIDITY OF THE ELLIPSOIDAL APPROXIMATION

An ellipsoid is not a true equilibrium shape. For $y \gg y_{cr}$ a triaxial ellipsoid is an entirely unsuitable approximation to the equilibrium shape of a rotating nucleus. For $y \le y_{cr}$, however, the ellipsoidal approximation can be sufficiently accurate. The axisymmetric equilibrium shape in ellipsoidal coordinates,

$$X^2 + Y^2 = d^2(\xi^2 + 1)(1 - \mu^2), \qquad Z^2 = d^2\xi^2\mu^2,$$

where 2d is the distance between the foci of the ellipsoid, can be represented by

$$\xi = \xi_0\Big(1 + \sum_{l=1}^{\infty} \alpha_l P_l(\mu)\Big).$$

Here $P_l(\mu)$ represents Legendre polynomials and the coefficients α_l determine the deformation of the ellipsoid. For nuclei with $x \approx 1$ and $x = 0$ when the true shape and exact potential energy at equilibrium are known, the validity of the zeroth ($\alpha_l = 0$) ellipsoidal approximation can be determined.

For $x \approx 1$ at equilibrium the principal deformation of a spherical shape is of the form $\beta_{20}P_2(\cos\theta)$,[1] which represents an ellipsoid and is proportional to $1-x$. The deformation α_l of the ellipsoid is proportional in this case to a higher power of the small parameter $1-x$ than β_{20}. The heights of the fission barriers determined from (6) and (7) are very close, and the expressions for y_{cr} in the case of an ellipsoid

$$y_{cr} = \tfrac{7}{5}(1-x)^2[1 - \tfrac{9}{2}(1-x)]$$

and for the exact shape[1]

$$y_{cr} = \tfrac{7}{5}(1-x)^2[1 - \tfrac{504}{85}(1-x)]$$

differ only with regard to the numerical coefficient in the correction term.

In the case of a neutral nucleus ($x = 0$) we can compare the shape and energy for an ellipsoid and for an exact equilibrium shape[3] at the instant when an oblate ellipsoidal shape becomes unstable ($y_{cr} = 0.2831$). Table III shows that all values are close. The exact equilibrium shape also differs very little from an ellipsoid [$\xi_0^2 = c_0^3/(1-c_0^3)$; $\xi_0 = 0.9430$]:

$$\alpha_2 = -0.010, \quad \alpha_4 = -0.024, \quad \alpha_6 = +0.004.$$

We shall now determine the rotational energy for which instability of an exact axisymmetric equilibrium shape of a neutral nucleus arises[3] while angular momentum is conserved. Let the shape of the nucleus be

$$r(\zeta, \varphi) = aR_0\{r_0(\zeta) + \gamma(\zeta, \varphi)\},$$

where $aR_0r_0(\zeta)$ is the radius of the equilibrium shape, aR_0 is the equatorial radius, $\zeta = \cos\theta$ (θ and φ are the spherical angles), and $\gamma(\zeta, \varphi)$ is the deformation of the equilibrium shape ($\gamma \ll r_0$). The instability of the equilibrium shape with respect to the considered deformation arises when the change of potential energy associated with this deformation,

$$\Delta U = \Big[O\cdot\Delta S - \frac{M^2}{2I^2}\Delta I\Big] \Big/ 4\pi R_0^2 O,$$

becomes negative (ΔS is the change of the surface and ΔI is the change of the nuclear moment of inertia).

We shall consider $\gamma(\zeta, \varphi)$ in the form

$$\gamma(\zeta, \varphi) = \varepsilon_{00} + \varepsilon_{m\nu}(1-\zeta^2)^m \cos\nu\varphi.$$

Then for $\epsilon_{m\nu} \ll 1$ we obtain in first approximation

$$\Delta U_{m\nu} = [A_{m\nu} - hC_{m\nu}]\,\varepsilon_{m\nu}^2, \tag{8}$$

where $h = 15a^3y/8(I')^2$, $I' = I/I_0$, $\nu \ge 2$ and the expressions for $A_{m\nu}$ and $C_{m\nu}$ are given in the Appendix. It is found that $A_{m\nu}$ and $C_{m\nu}$ depend only very slightly on h, so that it is easy to determine the value h_{cr} at which the equilibrium shape becomes unstable with respect to the given deformation.

Table IV gives values of h_{cr} for three types of deformations with $\nu = 2$, and also the nuclear energy at h_{cr}. Instability arises first for these deformations, since ΔS increases with ν, while ΔI

Table III

	a	c	c/a	U_S	U_{rot}	I'	U
Ellipsoid $y_{cr} - 0.2831$	1.134	0.778	0.686	1.027	0.220	1.286	1.247
Exact shape $y_{cr} - 0.2831$ $h_{cr} = 0.4565$	1.129	0.760	0.672	1.028	0.219	1.294	1.247

Table IV

	ν = 2				ν = 2		
	m = 1	m = 2	m = 3		m = 1	m = 2	m = 3
h_{cr}	0.475	0.450	0.493	U_S	1.031	1.028	1.033
y_{cr}	0.2964	0.2784	0.3092	U_{rot}	0.227	0.216	0.235
a	1.134	1.128	1.138	I'	1.306	1.289	1.318
c	0.751	0.762	0.744	U	1.258	1.244	1.268
c/a	0.663	0.676	0.653				

is independent of ν. For $m > 3$, h_{cr} increases with m because the term $m^2\zeta^2$ in ΔS grows rapidly. Because of angular momentum conservation the instability of the equilibrium shape arises for a considerably smaller value of h than in the case of constant angular velocity when $h_{cr} = 1$.[3] 1) We note also that the rotational energy of a spherical nucleus $y_{cr} = 0.2784$ at which the exact equilibrium shape becomes unstable is very close to the value $y_{cr} = 0.2831$ for the instability of an oblate ellipsoid.

Thus at the limits of the range of the fission parameter x the exact equilibrium shape of a rotating nucleus is very well approximated by an ellipsoid for $y \lesssim y_{cr}$. Since all quantities (c_0, y_{cr}, etc.) vary smoothly with x, it can be expected that for intermediate values of x with $y \lesssim y_{cr}$ the zeroth ellipsoidal approximation will be quite accurate, especially since for all x with $y \lesssim y_{cr}$ only a small deformation of the equilibrium shape occurs.

In conclusion let us consider the consequences of the described behavior of equilibrium shapes in the case of a rotating nucleus. Figure 3 shows the values of y_{cr} and of the critical rotational energy $E_{rot}^{cr} = B^2A/10V$ (dashed curve)[5] above which a rotating nucleus can emit neutrons at zero thermal excitation energy ($B = 8$ MeV is the neutron binding energy, $V = 40$ MeV is the depth of the potential well for nucleons, $4\pi R_0^2 O = 17.8$ MeV, $r_0 = 1.21$ F). It follows that for nuclei with $x < 0.7$ after the emission of all neutrons γ quanta come from a nucleus having the shape of an oblate ellipsoid. Gamma-ray emission from a nucleus having large angular momentum must therefore fluctuate. The magnitude of E_{rot}^{cr} also determines the order of the rotational energy at which neutron emission from a nucleus having zero thermal excitation energy is no longer forbidden, and the neutron width becomes equal in order of magnitude to the width when the entire excitation energy is of thermal character. On the other hand, for nuclei with $x < 0.81$ at y_{cr} the fission barrier is of finite height; consequently the fission probability of a compound nucleus where $E_{rot}^{cr} < 4\pi R_0^2 O y_{cr}$ must be smaller than unity. This has been observed experimentally.[6] It was found that at high excitation energies the fission probability differs from unity and varies slowly with the incident-ion energy; the fission probability decreases as x diminishes. This behavior is observed for nuclei with $x < 0.65$.[6]

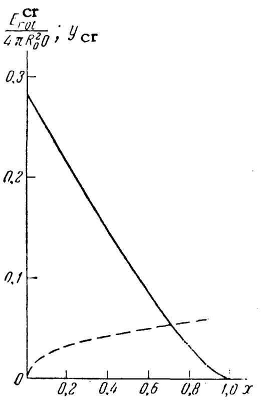

FIG. 3

The author wishes to than B. T. Geĭlikman and D. P. Grechukhin for discussions.

APPENDIX

The change of the equilibrium surface shape is

$$\Delta S = 4\pi R_0^2 a^2 \int_0^1 d\zeta \, [\varepsilon_{00} B_{00}(\zeta) + \varepsilon_{m\nu}^2 B_{m\nu}(\zeta)],$$

$$B_{00}(\zeta) = r_0(\zeta)\, \Phi^{1/2}(\zeta) \left[\frac{1}{r_0(\zeta)} + \frac{r_0(\zeta)}{\Phi(\zeta)}\right],$$

$$B_{m\nu}(\zeta) = r_0(\zeta)\, \Phi^{1/2}(\zeta) \frac{(1-\zeta^2)^{2m-1}}{\Phi(\zeta)} \left\{\frac{\nu^2}{4} + m^2\zeta^2 + \frac{1-\zeta^2}{4}\right.$$

$$+ \frac{(1-\zeta^2)[r_0(\zeta) - 2m\zeta r_0'(\zeta)]}{2r_0(\zeta)}$$

$$\left. - \frac{(1-\zeta^2)[r_0(\zeta) - 2m\zeta r_0'(\zeta)]^2}{4\Phi(\zeta)}\right\},$$

1) Sperber[4] recently considered a similar problem for $x = 0$, obtaining $h_{cr} = 2.414$. This large value is incorrect, since for all $h > 1$ the equilibrium shape must be unstable (for $h > 1$ the hydrostatic pressure on the axis of rotation is negative).[5]

where

$$r_0'(\zeta) = dr_0(\zeta)/d\zeta, \qquad \Phi(\zeta) = r_0^2(\zeta) + (1-\zeta^2)[r_0'(\zeta)]^2.$$

For the change of the moment of inertia of the equilibrium shape we have

$$\Delta I = 4\pi R_0^5 a^5 \rho_m \int_0^1 d\zeta\, [\varepsilon_{00} D_{00}(\zeta) + \varepsilon_{m\nu}^2 D_{m\nu}(\zeta)],$$

$$D_{00}(\zeta) = r_0^4(\zeta)(1-\zeta^2), \qquad D_{m\nu}(\zeta) = r_0^3(\zeta)(1-\zeta^2)^{2m+1}.$$

The conservation of volume enables us to express ϵ_{00} in terms of $\epsilon_{m\nu}$:

$$\varepsilon_{00} = -H_{m\nu}\varepsilon_{m\nu}^2,$$

$$H_{m\nu} = \left[\frac{1}{2}\int_0^1 d\zeta\, r_0(\zeta)(1-\zeta^2)^{2m}\right] \Big/ \int_0^1 d\zeta\, r_0^2(\zeta)$$

and from preservation of the center-of-mass position we obtain $\epsilon_{m1} = 0$, so that $\nu \geq 2$. Thus the quantities $A_{m\nu}$ and $C_{m\nu}$ in (8) are given by

$$A_{m\nu} = a^2\left[\int_0^1 d\zeta B_{m\nu}(\zeta) - H_{m\nu}\int_0^1 h\zeta B_{00}(\zeta)\right],$$

$$C_{m\nu} = a^2\left[\int_0^1 d\zeta D_{m\nu}(\zeta) - H_{m\nu}\int_0^1 d\zeta D_{00}(\zeta)\right].$$

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