pear only on the right cut since $\Delta\Phi_l(t)$ is an analytic function of l. In fact, because of the unitarity condition they can appear for $t > 4\mu^2$ only if their residues vanish at the same time.

Let now $l \to -1$. Then $\Delta\Phi_l \to \infty$ and if the sum over the poles contains a finite number of terms then $\Phi_l(t) \to \infty$ for any t. However, as a consequence of the unitarity condition $\Phi_l(t)$ has its modulus bounded for $t > 4\mu^2$. Therefore the number of poles must be infinite in order for them to compensate the contribution from the left cut. Moreover, the poles must fill the whole of the real axis for $t < t'$ (see the figure), so that the distance between the poles must tend to zero. Otherwise the contributions from the poles and from the left cut would have different analytic properties and would not be able to compensate each other. This means that for given $t < t'$ as $l \to -1$ we should encounter in the l plane arbitrarily many poles in a small neighborhood of $l = -1$. It therefore follows that $l = -1$ is an essential singular point of $\Phi_l(t)$ for $t < t'$. It is obvious that this essential singularity exists for any t since its position is independent of t for $t < t'$.

Let us see now whether the situation is changed if there exist in the l plane in the interval $-1 < l < l_0$ of the real axis singularities other than Regge poles. If such a singularity is a branch point, whose position is independent of the energy t, then the restrictions on the asymptotic behavior of $A(s,t)$ become even stronger. The analytic properties of $\Phi_l(t)$ as a function of t are in that case unchanged. Only the unitarity condition for l to the left of the branch point is changed. The unitarity condition written in the form [2]

$$\frac{1}{2i}\left[\Phi_l - \Phi^*_{l^*}\right] = \frac{k}{\omega}\,\Phi_l(t)\Phi^*_{l^*}(t-4\mu^2)^l, \qquad (5)$$

is valid for any l, but to the left of the branch point it does not mean that Φ_l has a restricted modulus, because Φ_{l^*} is not equal to Φ_l in view of the presence of a cut in the l plane. From the unitarity condition, Eq. (5), it follows, however, that Φ_l cannot be unbounded on both sides of the cut. If $\Phi_l \to \infty$ on one side of the cut then it must equal $\pm(\omega/2ik)(t-4\mu^2)^{-l}$ on the other side. If we consider l on that side of the cut where Φ_l is finite then all of the above considerations remain unchanged including the conclusion about the existence of an essential singularity near $l = -1$. If in the indicated interval we meet a branch point whose position depends on t then, at least for $t < t'$, the restriction on the asymptotic behavior of $A(s,t)$ becomes even stronger. This problem will not be discussed here in detail because we do not understand how there can appear in the l plane moving cuts. [6]

In conclusion we wish to thank V. B. Berestetskiĭ, M. Gell-Mann, S. Mandelstam, L. B. Okun', M. Froissart, and K. A. Ter-Martirosyan for useful discussions.

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COMBINATION RADIATION IN UNIFORM MOTION OF A CHARGE IN A HOMOGENEOUS MEDIUM

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1. In the case when the velocity v of a uniformly moving charge and the dielectric constant $\epsilon(\omega)$ of a homogeneous medium satisfy the inequality $v^2\epsilon(\omega) < 1$ ($\hbar = c = 1$), no Cerenkov radiation is possible. It is shown below that under such conditions, in real media, a different type of radiation of a uniformly moving charge is possible, one which can naturally be called combination radiation. As is well known, the possibility of Cerenkov radiation in a medium is connected with the coherent scattering of a photon by atomic electrons, something accounted for by introducing the dielectric constant $\epsilon(\omega)$. Yet in addition to coherent photon scattering there exists also Raman scattering, wherein the quantum frequency changes upon scattering as a result of the transition of the atom

to a different state. The additional energy ΔE, acquired by the quantum in such a scattering, leads to a satisfaction of the energy and momentum conservation laws during the radiation. Indeed, the energy conservation law for the entire radiation process states that

$$E_1 + \Delta E = E_2 + \omega, \tag{1}$$

where E_1 and E_2 are the initial and final energy of the moving charge. In a homogeneous medium there is no preferred direction, therefore (if we neglect the influence of random inhomogeneities) the medium cannot acquire a momentum, and the momentum conservation law has the usual form $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{k}$, from which it follows that combination (Raman) scattering of the photon during radiation has a somewhat unusual character: in such a scattering the photon energy changes, but not the momentum, so that a virtual photon goes over into a real one, but such a process is impossible for a real photon in the initial and final states. The conservation laws determine the angle of emission of the quantum (ϵ pertains to a medium without excited atoms)

$$\cos\vartheta = \frac{1}{v\sqrt{\varepsilon}}\left(1 - \frac{\Delta E}{\omega}\right) + \frac{\omega(1-\varepsilon)}{2p\sqrt{3}} + \frac{\Delta E}{p\sqrt{\varepsilon}}\left(1 - \frac{\Delta E}{\omega}\right), \tag{2}$$

where the fundamental term is the first term in the right half, while the remaining terms take into account the small correction connected with the recoil of the particle upon radiation. It follows from (2) that radiation is possible only when $\Delta E > 0$, that is, when the atom (molecule) goes into a state with lower energy. From the requirement $\cos^2\vartheta \leq 1$ follows the condition for the existence of the radiation:

$$v^2\varepsilon(\omega) \geqslant [1 - (\Delta E/\omega)]^2. \tag{3}$$

In particular, when the radiated frequency is larger than the atomic frequencies, there follows from (3) the possibility of radiation, at any energy of the particle, if $\Delta E > \omega_L = [ne^2Z/m]^{1/2}$ and the existence of an energy threshold for the radiation if $\Delta E < \omega_L$. The radiated frequencies are bounded from above by a maximum value

$$\omega_+ = E(\Delta E/M)\left\{1 + v\sqrt{(E/M)^2 - (\omega_L/\Delta E)^2}\right\},$$

so that at superhigh energies $E > (M^2/\Delta E)$ radiation at frequencies $\omega \sim E$ is possible.

2. Combination radiation of a uniformly moving charge can be used to detect high-energy particles by the angle of emission of a photon of given frequency. At not very low temperatures in real media there always exists a noticeable concentration of excited molecules or atoms, with excitation energy $\Delta E \lesssim T$, owing to the thermal excitation. In this case, however, the near lying levels are simultaneously excited, and this leads to radiation of one and the same frequency at different angles. In addition, when $\omega \sim T$ it is difficult to separate the radiation of the moving charge against the background of the spontaneous radiation of the excited atoms, and it becomes meaningful to consider the radiation of frequencies $\omega \gg T$. However, as follows from (2), when $\omega \gg E$ the quantum emission angle ϑ is small, making the measurement of the energy more complicated.

It is more convenient to create excited states by artificial means, using an external exciting electromagnetic field. An advantage of such a method is the possibility of resonant excitation of one energy state with fixed value of ΔE, so that the combination radiation of a specified frequency occurs at a strictly defined angle. If the spontaneous emission line of the excited atom is sufficiently narrow, then it is possible to detect in this case too the fast particles by means of the combination radiation of frequencies $\omega \sim \Delta E$, provided only $\Delta E \gg T$. In this case ϑ can have a value ~ 1 even at ultrarelativistic particle energies. In this lies the advantage of detection by combination radiation, over detection by Cerenkov counters; a shortcoming of the method is the need for ensuring homogeneity of the exciting field in space.

3. The probability of combination radiation is determined by the properties of the excited state. We shall consider below the particular case of combination radiation, when ΔE coincides with one of the atomic frequencies, $T \ll \Delta E$, and $\Delta E \ll \omega \ll m_e$. The radiation matrix element contains the radius vector $\mathbf{R}_a$ of the scattering atom in the form of the combination $\exp[i(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{k})\cdot\mathbf{R}_a]$. Summation over the atoms and the assumption that the medium is homogeneous lead to the appearance in the matrix element of a factor

$$\sum_a \exp[i(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{k})\mathbf{R}_a] = n_0(2\pi)^3\delta(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{k}),$$

where n_0 is the number of excited atoms per unit volume. For the frequencies under consideration the principal role is played in the combination scattering of the photon by the transition of the scalar (Coulomb) virtual quantum into a real transverse photon via the intermediate states of an electron with negative energy.

The probability of combination radiation is independent of the mass of the particle M and contains only the mass of the electron m_e. The ratio of the probabilities of the combination radiation and bremsstrahlung in the frequency interval $d\omega$ has an order of magnitude

$$(dW_c/dW_b) \sim (n_0/n)\,(n_0/\omega m^2 e^4)\,(M/mZ)^2,$$

where n is the total number of atoms per unit volume. It follows therefore that when $n_0 \sim n$, for low frequencies, heavy particles, and light elements, combination radiation is more intense than bremsstrahlung.

For hydrogen, the radial integral in the matrix element can be calculated exactly. For example, for the 2p—1s transition the probability of radiation turns out to be

$$dW = 0.29\cdot\pi\,(e^2 n_0^2/m^4)\,\omega^{-2}\,d\omega.$$

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PRESTELLAR STATE OF MATTER

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In accordance with the presently observed expansion of the nebulas, it is deemed probable that in the earlier stages of the evolution of the universe there existed a homogeneous isotropic Friedmann nonstationary solution with the density of matter decreasing from an infinite value at the initial instant.

Let us consider the state of matter in the initial stage of evolution. We assume that the matter consists of a mixture of protons, electrons, and neutrinos in equal amounts, and that the entropy is low. Then at high density (on the order of nuclear density) and at zero temperature the neutrinos and the electrons form a degenerate relativistic Fermi gas, and owing to the helicity (two-component nature) of the neutrino their Fermi energy E_{F_ν} exceeds the Fermi energy of the electrons; at a density 2.5×10^{38} particles/cm^3 we have $E_{F_\nu} = 500$ MeV and $E_{F_e} = 400$ MeV. The process $e^- + p = n + \nu$, which leads to the formation of neutrons at high density of matter in the stars [1], turns out to be forbidden here, since the neutrino states that are energetically obtainable in this process are occupied. In the uniform model (closed or open) the neutrinos do not depart anywhere. Upon expansion such a substance turns into pure cold hydrogen.

According to modern views, it is deemed most probable that it is precisely pure hydrogen which is the initial substance from which the stars were originally formed, and that the heavier elements were the result of nuclear processes in stars.

Earlier, Gamow, Alpher, and Herman [2,3] suggested that in the initial state matter consists of neutrons or of approximately equal amount of neutrons and protons (see also [4]) and is at such a high temperature, that the radiation density is gigantically in excess of the nucleon density. This point of view, on the basis of which the authors attempted even to reconstitute the presently observed abundance of the elements, leads to unsurmountable contradictions. In the prestellar stage they obtain a large amount of helium (about 10—20%) and deuterium (about 0.5%). The radiation density (energy/c^2) remains approximately equal to the nucleon density after the matter has expanded to the modern average nucleon density 10^{-30} g/cm^3. These deductions are incompatible with the observations; the notion of matter consisting of protons, electrons, and neutrinos is the only one possible 1). At a density many times larger than nuclear, when the Fermi energy of the protons becomes comparable with their rest mass, various processes of neutron and hyperon productions become possible in principle (see [5]) ($p \to \Sigma^+$, $p \to n + \pi^+$, $p + e^- = n + \nu$). However, the expansion is slow (see [4]) compared with the relaxation time of these processes 2), and therefore only p, e^-, and ν remain by the time nuclear density is reached.

For the theory of the decay of homogeneous matter into individual clusters corresponding to the galaxies, an important factor may be the inhomogeneities of the density, arising during the course of expansion in the density interval 0.5—2 g/cm^3, where metallic hydrogen is transformed into molecular hydrogen [6], and where a phase transition from the solid body to the gas at density < 0.07 g/cm^3 is also possible. This question needs further investigation.

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1)This deduction is motivated in detail in an article by the author, submitted to the collection "Voprosy kosmogonii" (Problems of Cosmogony).

2)We note that the process inverse to neutron formation proceeds not as the decay of a free neutron, but under the influence of a Fermi gas of neutrinos, and therefore is practically terminated within the expansion time.