CERTAIN THRESHOLD SINGULARITIES IN TOTAL INTERACTION CROSS SECTIONS

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Threshold singularities in total interaction cross sections are studied near two-particle production thresholds. The singularity in the total π^+p cross section is investigated near the threshold for the reaction $\pi^+p \to K^+\Sigma^+$.

THRESHOLD singularities in total particle interaction cross sections are found easiest with the help of the optical theorem. Utilizing the form of the elements of the scattering matrix near a two-particle production threshold obtained by Baz'[1] we get

$$\sigma_t(E) = \sigma_t(E_{\text{th}}) - \frac{2\pi |k| Ra_0}{k_1^2} \begin{cases} \sin 2\delta, & E < E_{\text{th}} \\ -\cos 2\delta, & E > E_{\text{th}} \end{cases}. \quad (1)$$

If there are no inelastic processes below the threshold in question then by subtracting from Eq. (1) the cross section $\sigma_{\text{inel}} = 2\pi |k| Ra_0/k_1^2$ for $E > E_{\text{th}}$ we obtain

$$\sigma_{\text{el}}(E) = \sigma_{\text{el}}(E_{\text{th}}) - \frac{2\pi |k| Ra_0}{k_1^2} \begin{cases} \sin 2\delta, & E < E_{\text{th}} \\ 2\sin^2\delta, & E > E_{\text{th}} \end{cases} \quad (2)$$

In Eqs. (1) and (2) δ stands for the phase shift in the $l = 0$ state in the case of spinless particles. For particles with spin 0 and $1/2$ δ stands for the phase shift in the $s_{1/2}$ state if the product of intrinsic parities of the particles remains unchanged in the reaction, and for the phase shift in the $p_{1/2}$ state in the opposite case. For higher spin cases Eqs. (1) and (2) involve all elements of the elastic scattering matrix referring to the same total angular momentum and parity. The remaining notation is the same as the notation of [1]. For the case of spinless particles Eq. (2) has also been given by Prokoshkin.[2]

Equation (1) shows that in total interaction cross sections the same four types of threshold singularities are possible as were indicated in [1] for fixed angle scattering cross sections. At the same time the total elastic scattering cross section can have only two types of singularities, since for $E > E_{\text{th}}$ we have $\partial\sigma_{\text{el}}/\partial |k| < 0$ always.

The above mentioned general considerations were applied to the analysis of threshold singularities in the total interaction cross section of π^+ mesons with protons near the threshold for the reaction $\pi^+p \to K^+\Sigma^+$ (E_{th} = 891.2 MeV). The straight line $\sigma_t^{(0)}(E) = 4\pi(-0.086 \pm 0.218 \quad 10^{-2} E)$ mb (where E is the laboratory energy in MeV) was drawn through the 44 points in the interval from 664 MeV to 1300 MeV obtained by Klepikov, Meshcheryakov, and Sokolov[3] and supplemented by the data of Delvin et al[4]. The interpolating curve found in [3] was not used, since in just this energy region it did not describe the experimental data sufficiently well. Then an additional parameter was introduced in the threshold region:

$$\sigma_t(E) = \sigma_t^{(0)}(E) + \sigma_t^{(1)}(E), \quad (3)$$

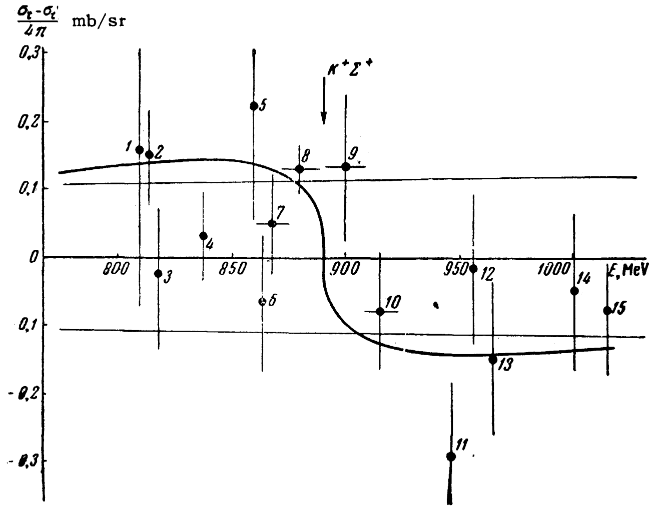
where

$$\sigma_t^{(1)}(E) = 4\pi \frac{0.040\sqrt{|E - E_{\text{th}}|}}{1 + 0.02|E - E_{\text{th}}|} \begin{cases} 1, & E < E_{\text{th}}, \\ -1, & E > E_{\text{th}} \end{cases} \quad (4)$$

with only the parameter in the numerator being varied. For interpolation $\sigma_t^{(0)}(E)$ we have $v^2 = \chi^2/f = 3.2$, where χ^2 refers to 15 points near threshold, and where f = 15. For the total interpolation (3) in the same region $v^2 = 1.41$ and f = 14. It is obvious that the second interpolation is substantially better than the first.

The experimental points for $(\sigma_t - \sigma_t^{(0)})/4\pi$ and the curve $\sigma_t^{(1)}/4\pi$ are shown in the figure. The points 1, 5, 8, 9, 12 were taken from [4], the point 15 was obtained by combining the data from [4] with the data indicated in [3]; the origin of the remaining points was given in [3]. The thin lines, nearly parallel to the E axis, indicate the range of error for interpolation without the threshold parameter. The range of error of the curve $\sigma_t^{(1)}/4\pi$ has its largest value, equal to 0.05, near energies of 840 and 940 MeV.

The form of the threshold term is consistent with the value $\delta = -67.5°$. The accuracy of the experimental data does not allow to make this result more precise by introducing different coefficients to the right and to the left of the threshold. If



one extrapolates the phase shifts $\delta(s_{1/2})$ and $\delta(p_{1/2})$ from the values they are known to have at lower energies, one finds it most likely that the singularity is in the s phase shift. In that case the parity of the $K^+\Sigma^+$ system is the same as that of the π^+p system.

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