PHASE-SHIFT ANALYSIS OF NUCLEON-NUCLEON SCATTERING AT 40, 95, 147, *AND* 310 MeV

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Simultaneous phase-shift analysis of np and pp scattering is carried out. The phase-shift sets thus obtained are given. The pion-nucleon interaction constant derived agrees with the value 0.08. An attempt is made to determine the pion mass from data on nucleon-nucleon scattering. The mass obtained does not contradict the known value.

THE satisfactory results of the joint phase-shift analysis obtained for 210 MeV [1] have encouraged the authors to continue the research and carry out a phase-shift analysis of nucleon-nucleon scattering at 40, 95, 147, and 310 MeV. A pp-scattering phase-shift analysis for these energies was made earlier [2-4]. Only the 95 MeV np-scattering data [5] were processed. In this case, however, MacGregor [5] did not take into account the one-meson exchange and did not determine the phase-shift errors.

The phase-shift analysis program was exactly the same as used by the authors previously [1] and is therefore not described here. The processed experimental data are listed in Table I. Table II lists the conditions under which the search for the solutions was started from the random points, and the results of the search.

RESULTS

It is seen from Table II that the initial search of solutions at T = 95 MeV [1] was made at $l_{max} = 2$. It was established, however, that the obtained solutions described poorly the dependence of the polarization on the scattering angle. This had made it necessary to increase l_{max} to 3 and to refine the obtained sets of phase shifts. One more solution was then obtained in the additional searches with $l_{max} = 3$ (see set 1 in Table IV below). At the level $\chi^2 < 1.5\overline{\chi^2}$ (the probability of the occurrence of $\chi^2 > 1.5\overline{\chi^2}$ is less than 0.05 in all cases), the number of solutions reaches five [1)], whereas at 310 and 147 MeV it does not exceed two. This circumstance is most likely due to lack of information.

It turned out, however, that one can now present arguments in favor of discarding the third and fourth solutions. Lapidus [6] has shown that the cross section of the exchange nd scattering has the same dependence on the scattering-matrix elements as the parameters of triple scattering, and by virtue of this it is possible to use for the phase-shift analysis triple-scattering data in lieu of the missing data. Sets 1^a, 2, 3, and 4 give for σ_{nd}/σ_{np} at $\vartheta = 180°$ values 0.548 ± 0.018, 0.45 ± 0.026, 0.18 ± 0.02, and 0.036 ± 0.020, respectively. The experimental value close to 180° is approximately 0.4. [22] If, in addition, we take into account the fact that the pion-nucleon scattering constant is $f^2 = 0.08$, then χ^2 increases for sets 3 and 4 to a value noticeably exceeding $1.5\overline{\chi^2}$. It must also be noted that these sets have no analogs at other energies.

Of the two solutions of the first type (sets 1 and 1^a) at T = 95 MeV for the phase shifts $^1S_0$, $^1F_3$, $^3F_2$, $^3F_3$, and $^3F_4$, set 1 is in best agreement with the solutions of the first type at large energies. Set 1^a gives apparently somewhat exaggerated values for the phase shifts of the F wave.

At 310 MeV, the addition of data on np-scattering did not alter the situation with respect to the ambiguity in the t = 1 phase shifts, first observed by several authors [2] in pp-scattering phase-shift analysis. In our case sets 1 and 2 likewise differ only in the phase shifts of the t = 1 waves. This may be the result of the fact that the experiments on pp scattering have been made only in the forward hemisphere, even in cases when there was no symmetry about the 90° angle (D_{pp}, A_{pp}, R_{pp}).

At 147 MeV, a single set was obtained with $\chi^2 < 1.5\overline{\chi^2}$. According to the phase shifts of the t = 1 waves, this set corresponds to sets 1 at 95 and 310 MeV. Attempts to find an analog of set 2

[1)]The authors are somewhat surprised that only one solution was found in [5] in a search for solutions with $l_{max} = 5$.

Table I

Average energy T, MeV	Measured quantity	No. of points	Energy at which measurements were made, MeV	Literature	Remarks
40	σ_{pp}	27	39.4	[4]	
	P_{pp}	1	39.4	[4]	
	σ_{np}	33	42	[7](H3),[21]	Data averaged
	P_{np}	6	40	[17]	
95	σ_{pp}	14	95	[4]	
	P_{pp}	14	95	[4]	
	D_{pp}	5	98	[4]	
	σ_{np}	36	90—93	[7](H3, S6, W2, S7, F2, C6)	Data renormalized in accordance with [8] and averaged
	P_{np}	15	95	[9]	
147	σ_{pp}	21	147	[15]	
	P_{pp}	14	147	[15]	
	D_{pp}	9	147.143	[16,17]	Data averaged
	R_{pp}	14	140.142	[18,19]	
	σ_{np}	24	156	[7](R2)	
	P_{np}	8	143	[20]	
310	σ_{pp}	14	340	[2]	
	P_{pp}	7	310	[10]	
	D_{pp}	6	310	[10]	
	R_{pp}	6	310	[10]	
	A_{pp}	3	316	[11]	
	C_{nn}^{pp}	1	320.310	[12,13]	Data averaged
	σ_{np}	17	300	[7](D8)	
	P_{np}	16	310	[10]	
	D_{np}	3	310	[14]	

Symbols: σ — differential scattering cross section, P — polarization, D — depolarization, R and A — triple-scattering parameters, C_{nn} — correlation of normal components of polarization.

Table II

Average energy T, MeV	l_{max}*	$\overline{\chi^2}$	Number of searches from random points	Number of solution	χ^2	Average energy T, MeV	l_{max}*	$\overline{\chi^2}$	Number of searches from random points	Number of solution	χ^2
				1	61.0		3			1	82.8
310	3	56	95	2	75.7					1a	120
				3	101.3					2	198
				1	73.5	95	2	72	71	3	235
147	3	73	78	2	146.6					4	178
				3	112.7	40	2	52		1	53.0
				4	112.1**		1			1	125.7

*At orbital momenta $l > l_{max}$ the amplitude of the nucleon-nucleon scattering was taken in the one-meson approximation in the search for solutions.
**According to the phase shifts, t = 1 corresponds to solution no. 2.

at this energy were unsuccessful. The set obtained from the initial phase-shift values obtained by extrapolation of set 2 from 210 [1] and 310 MeV to 147 MeV yields $\chi^2 = 146$ and is thus discarded.

It must be noted that in the energy interval 40—310 MeV the phase shifts of sets 1 as well as sets 2 depend monotonically on the energy and can be readily extrapolated from one energy to another. Extrapolation, for example, yielded set 1 for 40 MeV, whereas a search for the solutions in accord with the general program for $l_{max} = 2$ is quite difficult. In sixty attempts, eight solutions were found with $\chi^2 \sim 50$, of which half had unlikely high values of δ_{3D_1}. Set 1 can be readily observed in a search with $l_{max} = 1$.

Table III

$T = 210$ MeV					$T = 147$ MeV			$T = 310$ MeV		
μ, MeV	f^2	χ^2	f^2	χ^2	μ, MeV	f^2	χ^2	μ, MeV	f^2	χ^2
135	0,071	99.8						80	0.02	56,6
140	0,080	98,2	0.079	131.9	140	0.065	73.4	100	0,04	57.4
150	0.098	95.7						120	0,05	57,8
160			0.124	125.9	160	0.099	73.3	135	0.08	58,7
170	0.146	92,8						140	0,09	59
180	0.176	92.2			180	0.148	73.6			
190	0.210	92.0	0.227	121,1						
200	0,250	92.3	0.274	120.0						
210	0,297	92.8	0,380	120.2						
240			0.540	121						
	Set 1*		Set 2**			Set 1			Set 1	

*Set b_1 from [1].
**Set c from [1].

We also investigated at 147 MeV the extent to which the solutions obtained are shifted if the data on the depolarization in pp scattering are taken either from the paper of Hwang et al [16], or from the paper of Rose [17], without averaging the results of these two groups.[2)] It is observed here that the average values of the phase shifts practically coincide in these two cases, but the theoretical curve for set 1 fits closer the data of Hwang et al [16].

The phase-shift analysis of the nucleon-nucleon scattering was carried out by the authors under the assumption that the mass of the virtual pion exchanged by the nucleons in the collision is 140 MeV. In this connection it was interesting to see how strongly a change in the pion mass influences the results. For this purpose we found, for different values of μ, the minimum of the functional M with respect to all the phase shifts and with respect to the constant of the pion-nucleon interaction, of solutions 1 and 2 for $l_{max} = 3$. It was thus found that all phase shifts are quite stable against changes in μ and remain practically constant in the mass interval indicated in Table III. To the contrary, the pion-nucleon interaction constant depends quite strongly on the choice of μ, and as μ varies from 135 to 210 MeV the average value of f^2, for example, more than triples. In this case χ^2 ($\mu = 140$, $f^2 = 0.08$) exceeds χ^2_{min}, but by not more than several units. Thus, at the minimum of χ^2 μ and f^2 differ from the known values by not more than one or two errors.

[2)]The known discrepancy between the results of the D_{pp} measurements in Harvard and in Harwell was eliminated to a considerable degree, but the average values of D_{pp} obtained by these groups still differ systematically.

The phase-shift sets obtained, together with their errors, are listed in Tables IV—VII. The method used to determine the errors gives the correct results only if rapid convergence is observed during the course of finding the solution [1]. The latter usually occurs if the experiment is sufficiently complete and is carried out with good accuracy. On the other hand, if the experimental data are insufficiently complete or have large errors, then the average obtained phase-shift errors are too high. This occurred to a considerable degree at 95 MeV for set 1^a at $l_{max} = 3$. In this case the error was determined from the change in the phase shifts under random displacements of the experimental points, either relative to their average values or relative to the probable values of the experimental quantities.

Altogether, the points were shifted twelve times. In one case, to be sure, the deviations of the obtained parameters exceeded the triple error indicated in Table IV. The use of a similar procedure for sets 1 and 2, obtained at 210 MeV for $l_{max} = 3$, has shown that when the convergence is good both methods of determining the errors give the same result. It was also observed here that in this case the difference $\Delta\chi^2 = \chi_2^2 - \chi_1^2$ under random displacements of the experimental points changes relatively weakly, remaining approximately equal to the initial value $\Delta\chi^2 = 33$ (see Table VI). This indicates that both solutions are noticeably correlated and, consequently, the choice between them can hardly be based on the χ^2 criterion [1]. Estimates show that the probability of χ_2^2 being less than or equal to χ_1^2 upon further remeasuring of the experimental data is less than 1 per cent.

The phase shifts of the t = 1 waves are in good agreement with the known data of pp-scattering

Table IV. Phase shifts in degrees

	$T = 95$ MeV			$T = 40$ MeV
	Set 1[a]	Set 2	Set 1	Set 1
χ^2	81.0	86.4	82.8	53.0
$^1S_0$	16.43±2.50	−4.32±2.28	29.20±1.61	44.50±1.85
$^3S_1$	44.40±1.2	35.42±1.14	44.83±3.64	60.73±3.32
f^2	0.071±0.0083	0.074±0.012	0.075±0.011	0.08 fixed
$^3P_0$	12.25±1.79	−21.12±1.13	5.79±3.25	−0.13±7.85
$^1P_1$	−21.24±4.5	15.57±2.31	−24.77±5.93	−3.42±1.14
$^3P_1$	−11.72±0.47	8.40±1.55	−12.60±1.48	−6.06±2.81
$^3P_2$	12.98±0.5	13.06±0.52	10.54±0.46	5.87±0.40
ε_1	−5.86±1.40	24.15±1.13	−9.97±5.49	−21.16±5.94
$^3D_1$	−13.49±1.06	−7.36±1.53	−7.13±6.42	3.95±3.32
$^1D_2$	2.14±0.70	1.43±0.75	1.12±1.88	1.48±0.18
$^3D_2$	8.91±2.05	−9.10±1.37	1.00±3.43	−1.90±2.60
$^3D_3$	−0.45±0.63	−5.20±0.53	3.18±3.73	0.33±1.51
ε_2	−2.55±0.38	−1.14±0.72	0.57±1.79	
$^3F_2$	4.24±0.55	7.03±0.24	1.30±0.35	
$^1F_3$	−3.82±0.70	−6.53±0.55	−3.01±0.90	
$^3F_3$	−2.87±0.43	−3.79±0.17	1.92±0.75	
$^3F_4$	1.56±0.23	1.45±0.17	1.50±0.21	

Table V. Phase shifts in degrees for $T = 147$ MeV and $l_{max} = 3$

	Set 1 (f^2 fixed)	Set 1	Set 2
χ^2	79.19	73.45	146.57
$^1S_0$	16.81±0.63	17.13±0.64	−17.93±0.90
$^3S_1$	27.34±1.67	27.58±1.65	20.81±1.24
f^2	0.08 fixed	0.0645±0.0067	0.053±0.010
$^3P_0$	6.57±0.63	6.91±0.63	−23.60±0.22
$^1P_1$	−17.06±3.6	−18.05±3.26	17.28±2.82
$^3P_1$	−18.31±0.25	−18.28±0.25	7.14±0.33
$^3P_2$	14.31±0.17	14.29±0.17	15.12±0.23
ε_1	−1.93±2.29	−2.45±2.18	27.80±1.28
$^3D_1$	−13.49±1.59	−13.28±1.58	−4.03±1.24
$^1D_2$	6.03±0.21	6.05±0.22	7.07±0.23
$^3D_2$	23.72±2.05	23.65±2.05	−0.75±2.72
$^3D_3$	−0.73±1.65	−0.66±1.66	−4.36±0.71
ε_2	−2.39±0.22	−2.40±0.22	−2.84±0.24
$^3F_2$	−0.88±0.48	−1.02±0.5	−2.33±0.26
$^1F_3$	−1.64±1.46	−1.14±1.28	−5.61±1.10
$^3F_3$	−0.75±0.45	−0.71±0.48	−0.95±0.25
$^3F_4$	0.13±0.25	0.073±0.26	−1.03±0.15

analysis[2-4]. It is interesting to note that phase-shift sets 1 and 2 give an entirely different picture of the singlet np scattering. At 147, 210, and 310 MeV, and as was previously noted by MacGregor[5] at 95 MeV, scattering at angles close to 180° occurs essentially only in singlet states of the np system, if the np scattering cross section is calculated from the phase shifts of the first set. This is not observed, on the other hand, if set 2 is used in the calculations.

It is apparently possible to distinguish experimentally between sets 1 and 2 by measuring A_{pp}, R_{pp}, and D_{pp} at angles $\vartheta > 90°$ or C_{cr} at 40° for 210 and 310 MeV.

CONCLUSION

The results can be summarized as follows:

1. The known experimental data on nucleon-

Table VI. Phase shifts in degrees for $l_{max} = 3$

	$T = 310$ MeV		$T = 210$ MeV	
	Set 1	Set 2	Set 1	Set 2
χ^2	59.0	69.9	98	132
$^1S_0$	−6.05±1.56	−26.60±1.90	4.52±0.50	−16.15±0.67
$^3S_1$	−6.24±2.56	−2.65±4.03	12.86±1.66	1.72±3.49
f^2	0.090±0.007	0.100±0.009	0.0797±0.0045	0.0785±0.0058
$^3P_0$	−11.29±1.48	−27.95±3.98	−2.20±0.91	−27.07±0.89
$^1P_1$	−23.67±3.54	−23.88±3.37	−30.12±1.61	−20.75±2.24
$^3P_1$	−28.62±0.76	−9.55±1.06	−21.82±0.17	−2.24±0.44
$^3P_2$	16.38±0.55	21.22±1.30	15.98±0.13	18.08±0.33
ε_1	21.71±2.57	22.89±3.84	8.11±1.63	33.02±1.38
$^3D_1$	−22.96±1.60	−18.25±1.60	−18.54±1.39	−8.28±2.19
$^1D_2$	11.48±0.48	4.46±0.54	7.14±0.19	3.94±0.24
$^3D_2$	18.77±1.93	18.43±2.59	23.80±1.90	12.19±1.38
$^3D_3$	1.01±1.24	1.88±1.50	1.77±1.41	−0.80±1.12
ε_2	−2.08±0.38	−8.13±0.45	−2.56±0.14	−6.24±0.14
$^3F_2$	1.12±0.60	−0.33±1.27	1.82±0.34	1.60±0.52
$^1F_3$	−5.00±1.32	−0.59±1.62	−5.10±0.49	−3.29±0.73
$^3F_3$	−2.95±0.6	−0.22±0.43	−2.51±0.19	−2.29±0.14
$^3F_4$	3.15±0.32	3.15±0.72	2.09±0.18	1.60±0.33

Table VII. Phase shift for $l_{max} > 3$, in degrees

	310 MeV			
	Set 1	Set 1	Set 2	Set 2
χ^2	52,3	37.0	57,9	36,5
$^1S_0$	—6,23±1.62	—7,67±1.77	—23,28±2.85	—19,21±2.95
$^3S_1$	—6.82±3.21	—9,02±3.37	14,53±8.60	12,13±8,89
f^2	0.080±0.013	0,083±0.024	0,109±0.015	0,072±0.027
$^3P_0$	—11.24±1.53	—12,45±1.98	—31,61±6.62	—36.58±6.78
$^1P_1$	—25,32±3.93	—23,78±4,11	—25,09±5.98	—25,03±5.28
$^3P_1$	—28.17±0.88	—28.16±1.10	—10,04±1,04	—11.47±1.03
$^3P_2$	16,79±0.61	16,64±0.63	20,33±1.77	18,91±1,47
ε_1	21,24±3,47	19.30±4.27	17.01±3.52	18,32±4.18
$^3D_1$	—21,39±1.62	—22.45±1,81	—25.72±3,89	—28.31±4.82
$^1D_2$	11.36±0.49	10.70±0,85	4.22±0.54	4.13±0.60
$^3D_2$	20.82±2.81	22,02±3.00	14.73±5,02	12,60±5.76
$^3D_3$	1.50±1.43	1.95±1.44	—3,39±3.43	—1,85±3.71
ε_2	—2.00±0.40	—2.27±0.54	—8.43±0.40	—8,90±0.52
$^3F_2$	1.55±0.67	1.13±0.74	0.98±1,66	0.25+1.59
$^1F_3$	—5.30±1.49	—5.10±2.11	—0,65±2,81	—3.45±3.12
$^3F_3$	—3.19±0.66	—3.77±0.76	—0,53±0.54	—0.21±0.56
$^3F_4$	3,31±0.38	2.95±0.44	3.44±0,78	2.66±0,83
ε_3	3.66±2.16	3.03±2.40	11.04±1.30	12,06±1.31
$^3G_3$	—6.52±2.33	—7,41±2,87	4.09±1.77	—3,46±1.87
$^1G_4$	1.30±0.33	1.67±0.36	1.65±0.45	1.51±0.56
$^3G_4$	8.60±1.40	9,12±1.55	5.39±1.85	3.60±3.08
$^3G_5$	—1.04±1.17	—1.11±1.21	—2,03±1.54	—1.51±1.66
ε_4		—1,01±0.39		—1,66±0,33
$^3H_4$		1.72±0.53		2.16±0,56
$^1H_5$		—2.26±1.45		1.67±1,73
$^3H_5$		—0.80±0.65		—1.57±0.39
$^3H_6$		1.42±0.35		1,29±0,50

nucleon scattering in the energy region 95—310 MeV can be satisfactorily described under the assumption that, starting with orbital momenta $l > 3$, the scattering amplitude is given sufficiently accurately by the one-meson approximation (see Fig. 1). The average value of the pion-nucleon interaction constant is equal in this case to 0.078 ± 0.003 and is in good agreement with the value obtained from the πp-scattering experiments, $f^2 = 0.080 \pm 0.002$ [23].

2. It is difficult to choose the most reliable of the obtained solutions in each individual case. The energy dependence of the phase shifts suggests, however, that phase shift number 1 is the most probable. It should be noted that the energy dependences of the phase shifts of set 1 are quite simple. The phase shifts $\delta(^1S_0)$ and $\delta(^3S_1)$ are positive at 40 MeV, decrease with increasing energy, and pass through zero at 240—260 MeV. $\delta(^3P_0)$ is positive, has a maximum at 90—100 MeV, decreases with increasing energy, and passes through zero at approximately 180—190 MeV. The absolute values of the remaining phase shifts increase monotonically with energy in the investigated energy interval (Figs. 2 and 3).

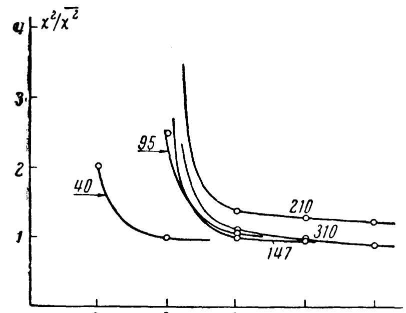

FIG. 1. Dependence of $\chi^2/\overline{\chi^2}$ on l_{max}.

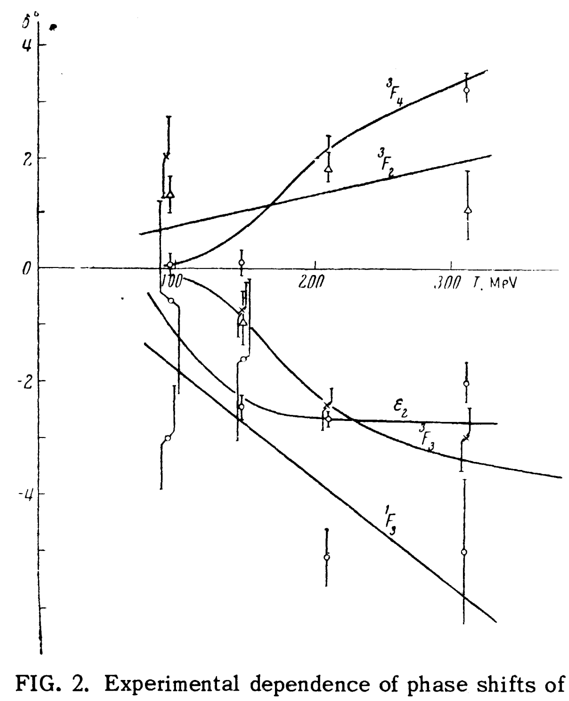

FIG. 2. Experimental dependence of phase shifts of the $^1F_3$, $^3F_2$, $^3F_3$, and $^3F_4$ waves.

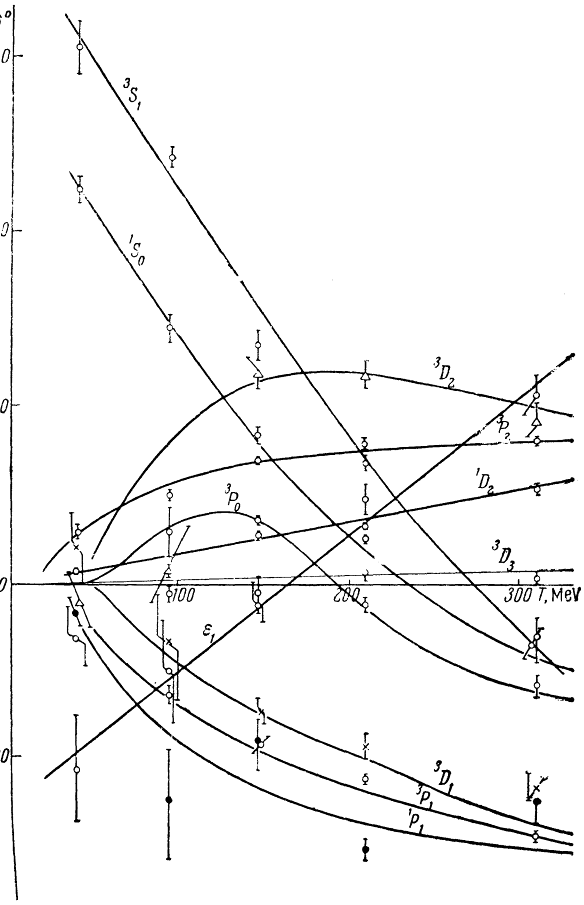

FIG. 3. Energy dependence of phase shifts of the $^1S_0$, $^3S_1$, $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$, $^1D_2$, $^3D_1$, $^3D_2$, and $^3D_3$ waves.

3. The phase shifts of the t = 0 and t = 1 waves have on the average approximately the same magnitude. Thus, nucleons in different isotopic-spin states interact in the investigated energy interval with equal intensity.

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Note added in proof (September 16, 1962). At 40 MeV, to reduce the number of solutions with $l_{\max} = 2$, one-meson phase shifts were taken for $^1D_2$, $^3D_2$, and $^3D_3$, while ϵ_1, which was not determined from the one-meson approximation, was determined from experiment, and $^3D_1$ was found from the condition that $\mathrm{Im}\,\alpha_{21}$ remained at the one-meson value. We found four solutions with $\chi^2 = 59$, 66, 63, and 78. Solution 1 yields, in the sequence corresponding to Table IV, phase shifts equal to 47.33 ± 0.44, 62.71 ± 0.60, 0.08, −3.57 ± 2.36, −6.20 ± 1.27, −1.95 ± 0.35, 4.59 ± 0.29, −7.89 ± 1.16, −4.17, 0.90, 5.84, and 0.50, respectively. Solution 2 differs only in the value of ϵ_1, which is equal to 11.31 ± 0.75. The remaining solutions have no analogs at the other energies. It was also noted that the value of $\delta_{^3D_1}$ did not change.

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239