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TEMPERATURE EQUILIBRATION RATE FOR CHARGED PARTICLES IN A PLASMA

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We consider temperature relaxation in a plasma containing charged particles of one species and in a plasma containing two species taking account of the interaction of particles with plasma waves. It is shown that the temperature relaxation time in a highly nonisothermal plasma can be much smaller than that which is computed neglecting the interaction between particles and waves.

1. INTRODUCTION

THE temperature equilibration of charged particles in a plasma has been treated extensively [1-3] by means of the Fokker-Planck equation with a collision integral in the form given by Landau; [1] in this work only the Coulomb interaction between particles at distances smaller than the Debye radius has been considered. The relaxation time for the temperature of an electron gas is given by the expression [2]

$$\tau_{ee} = \nu_{ee}^{-1} = \frac{5}{8}\sqrt{\frac{mT_e^3}{\pi}}\frac{1}{e^4LN_e} \tag{1}$$

(L is the Coulomb logarithm).

Electron-electron collisions play a decisive role in the relaxation of the electron temperature. Similarly, temperature relaxation in an ion gas is determined by ion-ion collisions[1)]

$$\tau_{ii} = \nu_{ii}^{-1} = \frac{5}{8}\sqrt{\frac{MT_i^3}{\pi}}\frac{1}{e_i^4LN_i}. \tag{2}$$

Finally, we may consider the time for an ion gas and an electron gas to come to an equilibrium temperature when the particle velocity distributions for each species remains an equilibrium distribution during the relaxation process; this relaxation time is given by the expression

$$\tau_{ei} = \nu_{ei}^{-1} = \frac{3}{8}\frac{(mT_i + MT_e)^{3/2}}{\sqrt{2\pi mM}}\frac{1}{e^2e_i^2L(N_e+N_i)}. \tag{3}$$

The formula above applies only when the following conditions are satisfied:

$$\tau_{ee} \ll \tau_{ei}, \qquad \tau_{ii} \ll \tau_{ei}. \tag{4}$$

The first of these conditions is satisfied for any T_e and T_i; the second reduces to the requirement

$$T_i / T_e \ll (M/m)^{1/3} \approx 10. \tag{5}$$

If this condition is not satisfied, i.e., if the ion temperature is ten times the electron temperature or greater, using the kinetic equation with the Landau collision integral we find that when the electron and ion gases interact the ion distribution function does not relax and can differ considerably from the equilibrium function.

The expressions given above for the temperature relaxation time in a plasma (1)—(3) take account only of the near Coulomb interactions. Actually, however, in a true plasma the interaction distance can be appreciably greater than the Debye radius. The interaction mechanism in this case is associated with weakly damped waves that propagate in the plasma. This particle interaction will be called the remote interaction. It is evident that remote interactions will be important in relaxation processes only when the plasma can support weakly damped waves with phase velocities smaller than the particle thermal velocities. A large fraction of the plasma particles participate in the radiation and absorption of these slow waves (particles whose velocities are greater than the phase velocities of the waves) so that the role of remote particle interactions can be enhanced.

When weakly damped waves cannot propagate in the plasma the effect of remote particle interactions on relaxation processes becomes negligible. As shown in [4-6], taking account of the remote particle interactions in an isotropic plasma in equilibrium that supports weakly damped waves with phase velocities greater than the thermal

1)Kogan[2] has obtained Eqs. (1) and (2) by an analysis of the relaxation between the longitudinal temperature $T_{\parallel}$ and the transverse temperature $T_{\perp}$ of the gas. These equations then represent the time required for the particle distribution function to become isotropic.

velocities yields only a small correction (of order $1/L$).

In a nonisothermal plasma that supports weakly damped slow waves, however, it can be shown that certain relaxation processes are very sensitive to remote particle interactions, in contrast with an isothermal plasma. For example, a nonisothermal plasma in which the electrons are hotter than the ions $T_e \gg T_i$ can support weakly damped slow waves whose phase velocities lie between the ion thermal velocity and the electron thermal velocity [7] $v_{T_i} \ll \omega/k \ll v_{T_e}$. The dispersion relation for these waves is

$$\omega^2 = \omega_{Li}^2 k^2 v_{T_e}^2 / (\omega_{Le}^2 + k^2 v_{T_e}^2), \tag{6}$$

where ω_{Le} and ω_{Li} are the electron and ion Langmuir frequencies respectively. Almost all of the plasma electrons interact strongly with these waves. Thus in a nonisothermal plasma in which $T_e \gg T_i$ the remote interactions can be of considerable importance in relaxation processes within the electron gas (electron-electron collisions).[2)] On the other hand, the interaction of the main mass of ions with these waves is a weak one. Hence, the role of the remote interactions due to these waves is found to be insignificant for relaxation processes within the ion gas and also for relaxation between the electron and ion gases in a plasma.

In the limiting case of a nonisothermal plasma in which $T_i \gg MT_e/m$ the weakly damped slow waves are described by

$$\omega = \omega_{Le}. \tag{7}$$

The phase velocity of these waves is between the thermal velocities of the electrons and ions $v_{T_e} \ll \omega/k \ll v_{T_i}$. Hence when $T_i \gg MT_e/m$ these waves have an important effect on relaxation processes in the ion gas but do not affect relaxation processes within the electron gas nor between the electron and ion gases.

To investigate relaxation processes in a plasma in the absence of strong fields we start with the Fokker-Planck equation with a collision integral that takes account of both the near Coulomb interaction as well as the remote wave interaction.

[2)] A short presentation of a theory of transport phenomena that takes account of the wave interaction in a plasma of this kind has been given by Silin and Gorbunov.[8] In this work the authors determined the conductivity, the diffusion coefficient, the thermal diffusion ratio, the electron thermal conductivity, and the viscosity for the case in which the remote interaction due to the plasma oscillations is stronger than the shielded Coulomb interaction.

This kinetic equation is of the following form (cf. [9] and the literature cited therein):

$$\frac{\partial f_\alpha}{\partial t} = \frac{\partial}{\partial p_i}\left\{ D_{ij} \frac{\partial f_\alpha}{\partial p_j} - A_i f_\alpha \right\}, \tag{8}$$

where

$$D_{ij} = \frac{e_\alpha^2}{2\pi} \int_{-\infty}^{+\infty} d\omega \int d\mathbf{k} \frac{k_i k_j}{k^4} \delta(\omega - \mathbf{k}\mathbf{v}_\alpha) \frac{F(\omega, \mathbf{k})}{|\varepsilon(\omega, \mathbf{k})|^2},$$

$$A_i = -\frac{e_\alpha^2}{2\pi^2} \int_{-\infty}^{+\infty} d\omega \int d\mathbf{k} \frac{k_i}{k^2} \delta(\omega - \mathbf{k}\mathbf{v}_\alpha) \frac{\operatorname{Im} \varepsilon(\omega, \mathbf{k})}{|\varepsilon(\omega, \mathbf{k})|^2},$$

$$F(\omega, \mathbf{k}) = \sum_\alpha 4\pi e_\alpha^2 \int d\mathbf{p}\, \delta(\omega - \mathbf{k}\mathbf{v}_\alpha) f_\alpha,$$

$$\varepsilon(\omega, \mathbf{k}) = \frac{k_i k_j}{k^2} \varepsilon_{ij}(\omega, \mathbf{k}) = 1 + \sum_\alpha \frac{4\pi e_\alpha^2}{k^2} \int d\mathbf{p} \frac{1}{\omega - \mathbf{k}\mathbf{v}_\alpha} \mathbf{k} \frac{\partial f_\alpha}{\partial \mathbf{p}}, \tag{9}$$

$\epsilon_{ij}(\omega, \mathbf{k})$ is the plasma dielectric tensor.

The kinetic equation (8) takes account of remote particle interactions due to longitudinal plasma waves. To avoid a divergence in (9) the integration over k must be cut off at some maximum $k = k_{max}$; this procedure corresponds to cutting off the particle interaction at some minimum distances at which, for example, the classical analysis no longer holds. The integration need not be cut off at small k, i.e., an upper limit on the interaction is not needed. There is no divergence at small values of k in (9) because we have taken account of polarization effects in the plasma. If we write $|\epsilon(\omega, \mathbf{k})| = 1$ in (9) then (8) assumes the form of the Fokker-Planck equation with the Landau collision integral. In this case the integration over k must evidently be cut off at both high and low values.

It will be shown below that remote particle interactions due to slow electromagnetic waves in a nonisothermal plasma in which $T_e > 10^2 T_i$ cause a considerable reduction in the temperature relaxation time in the electron gas (the time for the distribution to become isotropic) as compared with the expression given in (1); the reduction factor is approximately $10^{-2} T_e/T_i$. Taking account of the remote wave interactions a nonisothermal plasma in which $T_i \gg MT_e/m$, leads to a reduction in temperature relaxation time within the ion gas (the time for the distribution to become isotropic) as compared with that given in (2); the reduction factor is approximately $10^{-2} T_i/T_e$. However, taking account of the remote interactions between particles has only a small effect on temperature relaxation time between the electron and ion gases as compared with that given in (3); the reduction factor is approximately 10%.

2. TEMPERATURE RELAXATION IN A GAS OF CHARGED PARTICLES OF ONE SPECIES

We consider the time required for an isotropic distribution function to be reached in a gas of charged particles of one species; we consider the relaxation of a small difference between the longitudinal and transverse temperatures $T_{\alpha\perp} - T_{\alpha\parallel}$, that is to say, the particle distribution function for the α particles is taken in the form[3)]

$$f_\alpha = \frac{N_\alpha}{(2\pi m_\alpha)^{3/2}\sqrt{T_{\alpha\parallel}T_{\alpha\perp}^2}} \exp\left\{-\frac{m_\alpha v_\parallel^2}{2T_{\alpha\parallel}} - \frac{m_\alpha v_\perp^2}{2T_{\alpha\perp}}\right\}. \tag{10}$$

The temperature difference $T_{\alpha\perp} - T_{\alpha\parallel}$ decreases with time and, in accordance with the kinetic equation (8),

$$d\,(T_{\alpha\perp} - T_{\alpha\parallel})\,/\,dt = -\nu_{\alpha\alpha}\,(T_{\alpha\perp} - T_{\alpha\parallel}), \tag{11}$$

where

$$\nu_{\alpha\alpha} = \frac{3}{2}\,\frac{m_\alpha}{N_\alpha\,(T_{\alpha\perp} - T_{\alpha\parallel})}\int d\mathbf{p}\cdot v_\parallel^2\,\frac{\partial}{\partial p_i}\left\{D_{ij}\,\frac{\partial f_\alpha}{\partial p_j} - A_i f_\alpha\right\}. \tag{12}$$

In the limit of small differences $(T_{\alpha\perp} - T_{\alpha\parallel})$ the quantity $\tau_{\alpha\alpha} = \nu_{\alpha\alpha}^{-1}$ is the time required for the distribution function for particles of the α species to become isotropic.

We now consider temperature relaxation within an electron gas. It is assumed that the electron velocity distribution is given by (10) and that the ion distribution is Maxwellian. As already noted, a strong departure from (1) is to be expected in a nonisothermal plasma in which $T_e \gg T_i$. Although the electron relaxation time in such a plasma is smaller than that given by (1), the ion relaxation time given by (2) still holds. Hence, the assumption that the ion distribution function is an equilibrium function is completely incorrect.

Electron-electron collisions play a decisive role in the relaxation of the electron temperature. For this reason we retain only the electron terms in the expressions for $F(\omega, \mathbf{k})$ and $\mathrm{Im}\,\epsilon(\omega, \mathbf{k})$. We must, however, take account of the ion terms in $\epsilon(\omega, \mathbf{k})$ because the spectrum of slow waves, given by the zeros in $\epsilon(\omega, \mathbf{k})$, depends sensitively on the ion motion. Substituting these expressions in (12), after some simple manipulation we obtain (for small values of the difference $T_{e\perp} - T_{e\parallel}$)

$$\nu_{ee} = \frac{8\sqrt{\pi}}{5}\,\frac{e^4 N_e}{\sqrt{m T_e^3}}\,\{\ln(k_{max} r_D) + \sqrt{2} I_{ee}\}, \tag{13}$$

where r_D is the Debye radius, $r_D^{-2} = \Sigma\omega_{L\alpha}^2 m_\alpha / T_\alpha$ and

$$I_{ee} = -\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-x^2}dx\left\{\frac{1}{2}\ln(A^2 + B^2) + \frac{A}{B}\left(\frac{\pi}{2} - \operatorname{arc\,tg}\frac{A}{B}\right)\right\},$$

$$A = 1 - \left(1 + \left|\frac{e_i}{e}\right|\frac{T_e}{T_i}\right)^{-1} \times \left\{\mathrm{Re}\,J_+(x) + \left|\frac{e_i}{e}\right|\frac{T_e}{T_i}\,\mathrm{Re}\,J_+\left(x\sqrt{\frac{T_e M}{T_i M}}\right)\right\}, \tag{14*}$$

$$B = \sqrt{\frac{\pi}{2}}\,x\left(1 + \left|\frac{e_i}{e}\right|\frac{T_e}{T_i}\right)^{-1} \times \left\{e^{-x^2/2} + \left|\frac{e_i}{e}\right|\left(\frac{T_e}{T_i}\right)^{3/2}\sqrt{\frac{M}{m}}\exp\left(-\frac{M}{2m}\frac{T_e}{T_i}x^2\right)\right\}$$

The function $J_+(x)$ is given by

$$J_+(x) = xe^{-x^2/2}\int_{i\infty}^{x} d\tau\, e^{\tau^2/2} = -i\sqrt{\frac{\pi}{2}}\,xW\left(\frac{x}{\sqrt{2}}\right).$$

The function $W(x)$ has been tabulated.[11]

The first terms in the curly brackets in (13) corresponds to the result obtained with the kinetic equation with the Landau collision integral while I_{ee} gives the correction due to the interaction of electrons with plasma waves. In the case of interest to us, high values of T_e/T_i, it is a simple matter to obtain an analytic expression for I_{ee} by making use of the asymptotic form of the function $J_+(x)$ (cf. for example, [7]). As a result we have

$$I_{ee} \approx \frac{1}{2}\,\frac{T_e}{T_i}\left|\frac{e_i}{e}\right|\ln^{-1}\left\{\frac{e_i^2}{e^2}\,\frac{M}{m}\,\frac{T_e^3}{T_i^3}\right\} \text{ for } \frac{T_e}{T_i} \gg 1. \tag{15}$$

We note, that the basic contribution to the integral comes from small values of x; specifically,

$$\frac{m}{M}\,\frac{T_i}{T_e}\ln\left\{\frac{e_i^2}{e^2}\,\frac{M}{m}\,\frac{T_e^3}{T_i^3}\right\} < x^2 < \frac{m}{M}.$$

In the table we give values of I_{ee} obtained by numerical integration of (14) and by evaluating the approximate formula (15) for $|e_i/e| = 1$ and $M/m = 1840$ (hydrogen plasma). It is evident that the approximate formula (15) gives reasonably accurate values of I_{ee} when $T_e > 10^2 T_i$. Furthermore, at this temperature ratio I_{ee} is larger than the Coulomb logarithm [the first term in (13)] which, under typical conditions, is approximately 10.

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3)It should be emphasized that our analysis applies only for relaxation of a small difference in the temperatures $T_{\alpha\perp}$ and $T_{\alpha\parallel}$ in a bounded plasma; if this requirement is not satisfied an instability develops[10] and the temperature can relax via other mechanisms. Using the instability criterion $\Delta T_\alpha > k^2c^2T_\alpha/\omega_{L\alpha}^2$ we find the critical plasma dimension $L < (c/\omega_{L\alpha}^2)\sqrt{T_\alpha/\Delta T_\alpha}$ for which instabilities can not arise. On the other hand, if the particle wave interactions considered below are to be important the following inequality must be satisfied.

$$L > (v_{T\alpha}/\omega_{L\alpha})_{min}\,|\ln[(e_i^2/e^2)(T_e^3/T_i^3)(M/m)]|^{1/2}.$$

It is easy to show that in the case of interest here, a nonisothermal plasma, these requirements are satisfied up to temperature differences $\Delta T_\alpha \sim T_\alpha$.

*arc tg = $\tan^{-1}$.

T_e/T_i	0	1	10	10^2	10^3	10^4	10^5	10^6
$\sqrt{2}\, I_{ee}$ from (14)	0.3	−0.335	0.832	3.61	26.1	196	1670	$1.48 \cdot 10^4$
$\sqrt{2}\, I_{ee}$ from (15)	—	—	0.490	3.33	25.0	201	1680	$1.48 \cdot 10^4$

Thus, when $T_e > 10^2 T_i$ the electron relaxation time is determined primarily by the interaction of electrons with slow plasma waves and is given by

$$\tau_{ee} = \nu_{ee}^{-1} \approx \frac{5\sqrt{2}}{8}\left|\frac{e}{e_i}\right| \sqrt{\frac{mT_eT_i^2}{\pi}}\frac{1}{e^4 N_e} \ln\left\{\frac{e_i^2}{e^2}\frac{M}{m}\frac{T_e^3}{T_i^3}\right\}. \quad (16)$$

In the opposite limit, $T_e < 10^2 T_i$, the correction for the interaction with electromagnetic waves in the plasma is small; when $T_e \ll T$ the correction approaches the asymptotic value $I_{ee} \approx 0.3$.

It must be strongly emphasized that everything given above refers to the time required for the electron distribution to become isotropic. The time required for the isotropic electron distribution function to become Maxwellian is given with good accuracy by (1), just as in the case of the kinetic equation with the Landau collision integral. Taking account of remote collisions in this case leads to a correction of the same form as (14) except that the function e^{-x^2} in the integral is replaced by the function $x^2 e^{-x^2}$. As a result there is a sharp reduction in the contribution to the integral at small values of x and the entire correction becomes small compared with the Coulomb logarithm. Specifically, as has been kindly pointed out by L. M. Gorbunov, when $T_e > 10 T_i$ the following relation provides high accuracy:

$$I_{ee}^{(1)} = -\frac{1}{\sqrt{2\pi}}\int_0^\infty dx \cdot x^2 e^{-x^2}\left\{\frac{1}{2}\ln(A^2+B^2) + \frac{A}{B}\left(\frac{\pi}{2} - \operatorname{arc\,tg}\frac{A}{B}\right)\right\} \approx \frac{1}{4\sqrt{2}}\ln\left(\frac{T_e}{T_i}\left|\frac{e_i}{e}\right|\right). \quad (17)$$

Hence, the time required for the isotropic distribution to become Maxwellian is given by a theory in which wave effects are neglected provided the Coulomb logarithm is large compared with $I_{ee}^{(1)}$. This situation will always be the case for the plasma parameters encountered in practice.

In a nonisothermal plasma in which the electrons are hotter than the ions the time for the electron distribution function to become isotropic is appreciably smaller than the time for it to become Maxwellian. It is of interest, therefore, to generalize (16) to the case of arbitrary electron distributions. In this case the ion distribution function can evidently be assumed to be an equilibrium distribution. If we take T_e to be the mean electron energy

$$T_e = \frac{1}{3}\int d\mathbf{p}\, mv^2 f_e,$$

the time for the electron distribution function to become isotropic, which does not differ greatly from that of an isotropic distribution, can be written in the form ($T_e > 10^2 T_i$)

$$\tau_{ee} = a\frac{5\sqrt{2}}{8}\left|\frac{e}{e_i}\right| \sqrt{\frac{mT_eT_i^2}{\pi}}\frac{1}{e^4 N_e}\ln\left\{\frac{e_i^4}{e^4}\frac{MN_i^2}{(2\pi)^3 m^4 T_i^3 f_e^2(0)}\right\}, \quad (18)$$

where a is a numerical coefficient of order unity that depends weakly on the electron distribution function.

We now consider temperature relaxation in an ion gas. As indicated, one expects the ion temperature relaxation time in a highly nonisothermal plasma in which $T_i \gg MT_e/M$ to differ from that given in (2). Inasmuch as the electron-temperature relaxation time is appreciably smaller than the ion-temperature relaxation time, in this case the electron distribution function may be assumed to be an equilibrium function throughout the relaxation process. However, the ion distribution function is taken in the form given by (10) and the difference between the longitudinal and transverse temperatures is assumed to be small. Finally, since the ion-temperature relaxation (time required to reach an isotropic distribution) occurs much more rapidly than the temperature relaxation between the electron and ion gases, we can neglect this latter relaxation time. The basic mechanism in the relaxation of the ion temperature is the ion-ion collision. As a result we have

$$\nu_{ii} = \frac{8\sqrt{\pi}}{5}\frac{e_i^4 N_i}{\sqrt{MT_i^3}}\{\ln(k_{max} r_D) + \sqrt{2}\, I_{ii}\}. \quad (19)$$

The following asymptotic expression holds for I_{ii} in the case being considered here, in which $T_i \gg MT_e/m$,

$$I_{ii} = \frac{1}{2}\frac{T_i}{T_e}\left|\frac{e}{e_i}\right| \ln^{-1}\left\{\frac{e^2}{e_i^2}\frac{m}{M}\frac{T_i^3}{T_e^3}\right\}. \quad (20)$$

At high values of T_i/T_e, I_{ii} is appreciably greater than the Coulomb logarithm [first term in Eq. (19)]; this result indicates that the relaxation of the ion temperature in a nonisothermal plasma in which

$T_i \gg MT_e/m$ is due primarily to the interaction of ions with slow plasma waves. The relaxation time (time to achieve an isotropic distribution) in this case is

$$\tau_{ii} = \nu_{ii}^{-1} = \frac{5\sqrt{2}}{8}\left|\frac{e_i}{e}\right|\sqrt{\frac{MT_iT_e^2}{\pi}}\frac{1}{e_i^4 N_i}\ln\left\{\frac{e^2}{e_i^2}\frac{m}{M}\frac{T_i^3}{T_e^3}\right\}. \quad (21)$$

3. EQUILIBRATION OF THE ELECTRON AND ION TEMPERATURES IN A PLASMA

We have shown in the preceding section that in a highly nonisothermal plasma that supports weakly damped plasma waves with phase velocities smaller than the thermal velocities of the electrons or ions the relaxation processes are affected, and sometimes determined by, contributions due to the interaction of particles with these slow plasma waves. The situation is different, however, for the relaxation of the temperature difference between the electron and ion gases in the plasma. As is well known, weakly damped waves with phase velocities smaller than the thermal velocities of both the electrons and ions cannot propagate in a plasma. Thus, either the electrons or the ions, but not both, interact with the plasma waves; as a result the role of the particle-wave interaction in the equalization of the electron and ion temperatures is always a small one and Eq. (3) applies to within a small correction. Using (8) and (9) we obtain the relation that characterizes the equilibration of the electron and ion temperatures in the plasma

$$\frac{d}{dt}(T_e - T_i) = -\nu_{ei}(T_e - T_i), \quad (22)$$

where

$$\nu_{ei} = \frac{8}{3}\sqrt{2\pi mM}\frac{e^2 e_i^2(N_e + N_i)}{(mT_i + MT_e)^{3/2}}\{\ln(k_{max}r_D) + I_{ei}\}. \quad (23)$$

The quantity I_{ei}, which characterizes the contribution due to the interaction of particles with plasma waves in the electron-ion collision frequency, is given by the expressions

$$I_{ei} = -\frac{1}{\sqrt{2\pi}}\int_0^\infty dx \cdot x^2 e^{-x^2/2}\left\{\frac{1}{2}\ln(A^2 + B^2) + \frac{A}{B}\left(\frac{\pi}{2} - \operatorname{arc\,tg}\frac{A}{B}\right)\right\},$$

$$A = 1 - \left(1 + \left|\frac{e}{e_i}\right|\frac{T_e}{T_i}\right)^{-1}\left\{\operatorname{Re} J_+\left(\frac{x}{\sqrt{1+a}}\right) + \left|\frac{e}{e_i}\right|\frac{T_e}{T_i}\operatorname{Re} J_+\left(\frac{x\sqrt{a}}{\sqrt{1+a}}\right)\right\},$$

$$B = \sqrt{\frac{\pi}{2}}\left(1 + \left|\frac{e}{e_i}\right|\frac{T_e}{T_i}\right)^{-1}\frac{x}{\sqrt{1+a}}\left\{\exp\left[-\frac{x^2}{2(1+a)}\right] + \left|\frac{e}{e_i}\right|\frac{T_e}{T_i}\sqrt{a}\exp\left[-\frac{x^2 a}{2(1+a)}\right]\right\}, \quad (24)$$

where $a = MT_e/mT_i$.

Values of I_{ei} for various values of T_e/T_i are given below.

T_e/T_i:	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1	10^1	10^2	10^3	10^4	10^5
I_{ei}:	1.5	0.85	−0.06	−0.45	−0.45	−0.17	0.71	1.3	1.6	1.7	1.8

It is thus evident that I_{ei} is small compared with the Coulomb logarithm which, under usual conditions, is of the order of 10. The contribution due to the particle-wave interaction in the plasma is 10–15% in the electron-ion collision frequency. The highest values of I_{ei} are reached in a nonisothermal plasma in which either $T_e \gg T_i$ or $T_i \gg MT_e/m$, that is to say, in a plasma in which there are weakly damped slow waves that interact strongly either with electrons or with ions. In the temperature region $m/M < T_e/T_i < 1$, however, I_{ei} is not important since the uncertainty in the upper limit k_{max} can be of the same order of magnitude as I_{ei} itself.

It should be noted that when $T_i > T_e(M/m)^{1/3}$ [(5) is not satisfied], the quantity E_i is no longer meaningful; just as in (3), the ion distribution function cannot relax and differs markedly from an equilibrium distribution. The values of I_{ei} in Table II for this temperature region are given only to illustrate the role of the remote interactions in collisions between electrons and ions.

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