NUCLEAR BREAKUP REACTIONS

Yu. A. BEREZHNOĬ, A. P. KLYUCHAREV, Yu. N. RANYUK, and N. Ya. RUTKEVICH

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The breakup of colliding carbon nuclei into α particles ($C^{12} + C^{12} \to 6\alpha$) is studied by irradiating nuclear emulsions with 115-MeV carbon ions. The angular and energy distributions of the α particles and the excitation function are compared with calculations based on the statistical model of direct nuclear breakup.

1. INTRODUCTION

MANY light nuclei can be regarded as consisting of subgroups of strongly bound nucleons such as α particles. In some nuclei these subgroups are only weakly bound to each other, so that these nuclei are more likely to decay by emitting subgroups than by ejecting individual nucleons. Collisions between these nuclei can produce three or more particles resulting from the partial or complete breakup of the system into α particles. The present work is an experimental and theoretical investigation of one such reaction, $C^{12} + C^{12} \to 6\alpha$.[1]

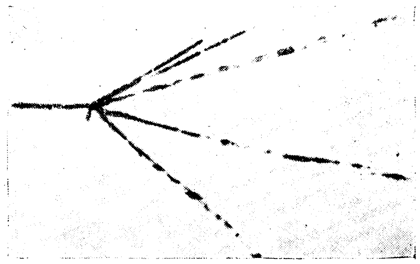

FIG. 1. Star from the reaction $C^{12} + C^{12} \to 6\alpha$.

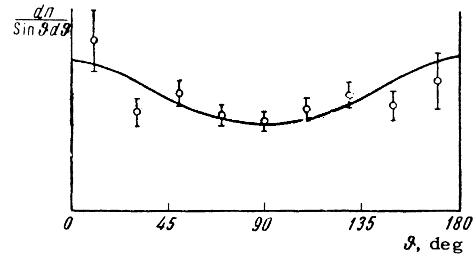

FIG. 2. Angular distribution of α particles from the reaction $C^{12} + C^{12} \to 6\alpha$.

2. EXPERIMENTAL RESULTS

Nuclear emulsions were irradiated with 115-MeV carbon ions from the linear accelerator of multicharged ions at the Khar'kov Physico-technical Institute. Among the various nuclear reactions induced by the heavy ions in the emulsions we detected a considerable number of reactions with light nuclei, in which a system consisting of an incoming ion and a target nucleus breaks up completely into particles possessing more than one unit of charge. A typical multipronged star is seen in Fig. 1.

Complete measurements of the star parameters were possible on the discriminative emulsions (NIKFI type D) 300—400 μ thick used in this work. Measurements and calculations were performed on 100 stars. The kinematics of the interacting particles supported the hypothesis that the observed reaction was a complete breakup of the system into α particles.

Our experimental procedure was unchanged from that used in other work.[1] The angular distribution of α particles in the center-of-mass system is shown in Fig. 2, where the abscissas are the angles ϑ of particle emission and the ordinates are the numbers of α particles per unit solid angle. The statistical errors are indicated. The distribution is symmetric around $\vartheta = 90°$, which was to be expected in collisions between particles having equal masses.

The histogram in Fig. 3 is the c.m. energy distribution of α particles in all the measured six-pronged stars. The spectrum is seen to be continuous.

We also measured the excitation function (Fig. 4) from the threshold (denoted by a triangle) to 115 MeV. The cross section increases steeply far above the threshold, attaining 10% of the geometric cross section at 100 MeV. Our theoretical investigation was based on the direct breakup model.

3. STATISTICAL MODEL OF DIRECT NUCLEAR BREAKUP

The theoretical account of direct breakup is analogous to Fermi's treatment of the multiple production of mesons.[2] When two nuclei collide

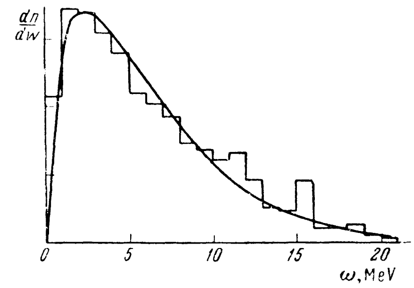

FIG. 3. Energy distribution of α particles from the reaction $C^{12} + C^{12} \to 6\alpha$.

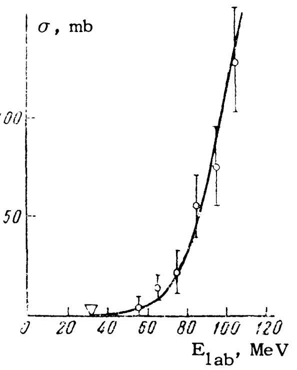

FIG. 4. Cross section for the reaction $C^{12} + C^{12} \to 6\alpha$ as a function of carbon ion energy in the laboratory system.

their kinetic energy is instantaneously released into a relatively small effective volume surrounding the nucleus and is divided statistically among groups of strongly bound nucleons rather than among individual nucleons. The breakup parameters, such as the angular and energy distributions of the products, will obviously depend on the statistical distribution of energy, momentum, and angular momentum among the particles within the effective volume. The statistical description of breakup reactions improves in accuracy with an increasing total number N_0 of produced particles.

Two colliding nuclei ordinarily form a system possessing high angular momentum, since rigorously central collisions are of low probability. The angular-momentum conservation law requires that the entire angular momentum be distributed among the outgoing α particles. As a result of the fact that angular momentum is conserved in addition to energy, the c.m. angular distribution of α particles is not spherically symmetric.

The number of particles in an element **dpdr** of phase space is given by

$$dn = C\,(2\pi\hbar)^{-3}\, e^{-\alpha w + \beta m_z}\, d\mathbf{p}\,d\mathbf{r},$$
$$d\mathbf{p} = p^2 \sin\theta\, dp\, d\theta\, d\varphi, \quad d\mathbf{r} = 2\sqrt{R^2 - \rho^2}\,\rho\, d\rho\, d\psi, \tag{1}$$

where the z axis is perpendicular to the plane of the reaction; C is a normalizing constant; $w = p^2/2\mu$ is the α-particle energy; $m_z = p\rho \sin\theta \cos(\psi - \varphi)$ is the angular momentum of an α particle; θ and φ are the angles of the momentum vector **p**; ρ lies in the reaction plane; the effective volume is taken to be a sphere of radius R. The coordinate angles are represented in Fig. 5; the colliding nuclei move along the y axis in the c.m. system. Equation (1) differs from the corresponding formula given by Fermi;[2] because of the relatively low energies the Bose-Einstein distribution is here replaced by a Gibbs distribution, and the effective volume is considered as a sphere since Lorentz contraction is not involved.

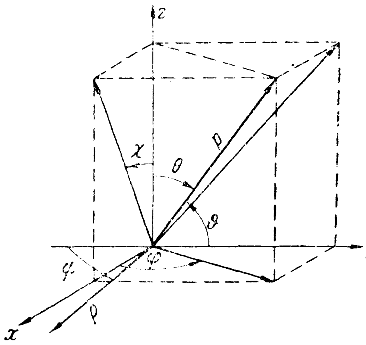

FIG. 5

The constants C, α, and β were selected to make the total number N_0 of particles, the total energy E_0, and the total angular momentum M_0 fit the experimental values. In the given coordinate system these quantities are defined by

$$N_0 = \int dn, \qquad E_0 = \int w\, dn, \qquad M_0 = \int m_z\, dn. \tag{2}$$

Calculating N_0 from (2), we obtain

$$C = \sqrt{\pi} N_0 \beta^3 \hbar^3 e^{-u} / 2\Gamma(^3/_2, u), \tag{3}$$

where $u = \mu\beta^2 R^2/2\alpha$, μ is the α-particle mass, and the incomplete gamma function is defined by

$$\Gamma(a, b) = \int_0^b e^{-x} x^{a-1}\, dx.$$

From (1), (2), and (3) we obtain

$$E_0 = \frac{N_0 u \Gamma(^1/_2, u)}{2\alpha\Gamma(^3/_2, u)}, \quad M_0 = \frac{2N_0 u}{\beta}\left[1 - \frac{\Gamma(^5/_2, u)}{u\Gamma(^3/_2, u)}\right]. \tag{4}$$

Equation (4) defines the relationship between the constants α and β, the total number N_0 of particles, the total energy E_0, and the total angular momentum M_0.

We now obtain the energy distribution of α particles resulting from the complete breakup of the colliding nuclei. Noting that the angle ϑ measured from the direction of nuclear motion is related to the angle θ measured from the normal to the reaction plane (Fig. 5) by the formula $\sin^2\theta = 1 - \sin^2\vartheta \cos^2\chi$ and that for the solid angle element $d\Omega_{(\theta,\varphi)} = d\Omega_{(\vartheta,\chi)}$, we obtain from (1) an expression for the number of α particles emitted in a solid angle element $d\Omega_{(\vartheta,\chi)} = \sin\vartheta d\vartheta d\chi$:

$$\frac{dn}{d\Omega_{(\vartheta,\chi)}} = \frac{N_0 e^{-u}}{4\pi\Gamma(^3/_2, u)} \int_0^u \exp\{x(1-\sin^2\vartheta\cos^2\chi)\} x^{1/2} dx. \quad (5)$$

This result shows that the angular distribution of α particles exhibits maxima in the forward and backward directions ($\vartheta = 0$ and π) and is symmetric around the angle $\vartheta = \pi/2$. If $u \ll 1$ (low angular momentum) the right-hand side of (5) is independent of the angles.

A comparison with experiment is facilitated by averaging (5) over χ. The angular distribution of α particles is then given by

$$\frac{dn}{2\pi\sin\vartheta d\vartheta} = \frac{N_0 e^{-u}}{\sqrt{2\pi}\Gamma(^3/_2, u)} \times \int_0^{u/2} \exp\{x(1+\cos^2\vartheta)\} I_0(x\sin^2\vartheta) x^{1/2} dx. \quad (6)$$

If $u \gg 1$ (high angular momentum) the right-hand side of (6) is proportional to $\sin^{-1}\vartheta$.

The energy distribution of α particles is obtained by integrating (1) over the angles and over ρ, replacing momentum with energy ($p = \sqrt{2\mu w}$):

$$\frac{dn}{dw} = \frac{2\alpha N_0 u e^{-u}}{\sqrt{\pi}\Gamma(^3/_2, u)} e^{-\alpha w} \int_0^{\pi/2} \mathrm{sh}\left(\sqrt{\frac{w}{\varepsilon}}\sin x\right)\cos^2 x dx, \quad (7)^*$$

where $\epsilon = \frac{1}{4}\alpha u$. If $u \ll 1$ (low angular momentum) the α particles have the Maxwellian distribution $dn/dw \sim w^{1/2} e^{-\alpha w}$.

4. COMPARISON OF EXPERIMENT WITH THEORY

The constants α and u appear in the angular and energy distributions and are determined from (4) if N_0, E_0, and M_0 are known. The energy is $E_0 = E_1 + E_2 - Q$, where E_1 and E_2 are the c.m. kinetic energies of the colliding particles and Q is the reaction energy. The angular momentum M_0 is evaluated as follows. In a collision of two nuclei with the impact parameter ρ the system possesses the angular momentum ρp, where the momentum of one of the nuclei is

$$p = [2m_1 m_2 (m_1 + m_2)^{-1}(E_1 + E_2 - B)]^{1/2},$$

m_1 and m_2 are the masses of the two nuclei, and B is the Coulomb barrier height. The probability that in a collision of two nuclei having radii R_1 and R_2 the impact parameter is between ρ and $\rho + d\rho$ is given by

$$dw = 2(R_1 + R_2)^{-2}\rho d\rho. \quad (8)$$

The average of M_0 is given by

$$M_0 = \int_0^{(R_1+R_2)} p\rho dw. \quad (9)$$

From (8) and (9) we obtain

$$M_0 = \tfrac{2}{3}(R_1 + R_2)[2m_1 m_2(m_1 + m_2)^{-1}(E_1 + E_2 - B)]^{1/2}. \quad (10)$$

For the reaction $C^{12} + C^{12} \rightarrow 6\alpha$ the constants have the following values: $N_0 = 6$, $E_0 = 36$ MeV, $M_0 \approx 15\hbar$, and R = 5 F. From (4) we obtained $1/\alpha = 2.3$ MeV, $1/\beta = 1.2\hbar$, and u = 2. The curve in Fig. 2 was computed from (6) for the given values of α and u. The experimental points at the angles $\vartheta = 30°$ and 150° do not fit the angular distribution (6), and can possibly indicate minima of the angular distribution associated with another reaction mechanism.

In Fig. 3 the experimental histogram is accompanied by the α-particle energy distribution computed from (7) for the same values of α and u. Figures 2 and 3 indicate that there is qualitative agreement between the experimental results and the theory.

The excitation function was calculated from Sachs' formula[3] for the cross section as a function of energy and the number of outgoing particles. For the breakup of an excited system into six particles this relation has the form

$$\sigma(W) = KW^3(1 - Q/W)^{13/2}(r_0 + a\hbar/\sqrt{2mW})^{12}, \quad (11)$$

where $W = E_1 + E_2$ is the total kinetic energy of the colliding particles, r_0 and a are constants characterizing the interaction volume, and K is a constant. The interaction volume, following [4], has the form $V = \frac{4}{3}\pi\lambda\!\!\!^{-}(R + a\lambda\!\!\!^{-})^2$, where $\lambda\!\!\!^{-}$ is the wavelength of one of the colliding nuclei in the c.m. system.

Figure 4 shows a curve computed from (11) for $r_0 = 1.06$ F and a = 0.85. Satisfactory agreement with experiment is seen in the investigated energy region.

The experimental points agree with calculations based on the theory of direct breakup. However, the same qualitative results can be obtained from a suitably modified compound-nucleus model in which α particles replace the elementary particles. This latter model seems very artificial, although it cannot be rejected decisively on the basis of the experimental results. A model could be selected uniquely on the basis of an experiment investigating a reaction between non-α-correlated nuclei (e.g., $N^{14} + B^{10} \rightarrow 6\alpha$). A comparison of the reaction cross section with that for $C^{12} + C^{12} \rightarrow 6\alpha$ would enable us to determine whether nucleons regroup to form α particles at the instant

*sh = sinh.

of collision or whether the breakup into α particles is due to the α-particle structure of the colliding nuclei.

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