EFFECT OF THE DIMENSIONS AND PURITY ON THE ELECTRICAL RESISTANCE OF METALS IN A LONGITUDINAL MAGNETIC FIELD AT HELIUM TEMPERATURES

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A strong dimensional effect at helium temperatures was found in very pure samples of Sn, Zn, and Al in longitudinal magnetic fields. Estimates of the electron momentum, made using available theory, gave reasonable values for Sn, Zn, and Al. A considerable anisotropy of the resistance rise in a longitudinal field was observed in Sn, Zn, and Cd; this rise was several times greater in massive samples with perpendicular orientation than in samples with parallel orientation. Kohler's diagram was plotted for six metals of various purities, and it was found that for small changes in the resistance ($\Delta R/R \leq 0.2$) the curves are linear and have the same tangent of the slope angle (equal to ≈ 1.43) while in strong effective fields all the curves exhibit saturation.

RECENTLY much attention has been given to measurements of the low-temperature resistance of metals of the highest possible purity in a transverse magnetic field. These studies are interesting because with their help the general topology of the Fermi surface of metals can be determined, and details of this surface can then be found by other physical methods (high-frequency, ultrasonic and magnetic methods). However, analogous galvanomagnetic studies in longitudinal magnetic fields have only rarely been carried out. The limited interest in measurements of the resistance in a longitudinal magnetic field is related to the fact that the effect itself was until recently considered insignificant compared with the effect in transverse fields, and to the absence of a rigorous theory of the effect in thin samples.

Nowadays studies in longitudinal magnetic fields should attract interest as well as studies in transverse fields, because of the availability of various very pure metals in which the resistance rises considerably (by a factor of 2—6) in longitudinal fields. For example, using measurements of the resistance of samples of various dimensions in a longitudinal magnetic field, one could estimate the mean free path of electrons λ and their momentum on the Fermi surface,[1] provided a suitable theory were available. Moreover, such studies, as well as studies in transverse fields, could give some information on the topology of the Fermi surface (open directions).[1)]

[1)]The possibility of determining the direction along which the Fermi surface of a given metal is open, using its magnetoresistance in a longitudinal field, was pointed out by M. Ya. Azbel' and will form the subject of a separate communication.

A theory is now available for the resistance of thin films[2] and cylindrical wires[3] in a longitudinal magnetic field. For wires Chambers[3] developed a theory from the free-electron model and this theory does not give any dependence of the resistance of a bulk metal sample on the magnetic field. Nevertheless it follows from this theory that at a sample diameter $d \leq \lambda$ the resistance should decrease with increase of the magnetic field intensity, which is in qualitative agreement with experiment. A quantitative comparison of Chambers's theory with experiment and its use to estimate the electron momentum are possible only when the magnetoresistance of a massive sample is small compared with the dimensional effect.

Experimental investigations in a longitudinal field have been concentrated mainly on cylindrical samples of several metals. For example, one of the earlier papers reported measurement of the resistance at 4.2 and 20.4°K in Zn and Cd in transverse and longitudinal fields up to H = 15—18 kOe[4] and in Ga in longitudinal fields.[5] Similar measurements have been carried out on polycrystalline copper wires of 0.2 mm diameter at 297, 78, and 4.2°K in fields up to 100 kOe,[6] on bulk samples of Na and Rb at 4.2°K in 25 kOe fields,[7] and on In at 2 and 4.2°K in fields up to H = 12 kOe,[8] or pulsed fields up to H = 80 kOe.[9]

Very recently the discovery was made of the monotonic decrease of the relative change in the resistance of a thin In wire,[8] on increase of a longitudinal magnetic field beyond some value H_{max}. In Na[3,10] this decrease was so great that the resistance of a thin wire in a longitudinal field became smaller than its resistance without a field.

A similar effect was found also on application of transverse fields to Na.[10,11] The nature of the dependence of the resistance on the longitudinal magnetic field intensity, observed in Sb[12] and Bi[13,14] at various temperatures, is obviously also related to the dimensional effect.

The main purpose of the present work was the extension of the experimental studies of the dimensional effect in longitudinal magnetic fields in very pure metals Sn, Zn, Al, and Cd, and elucidation of the effect of the purity of these metals, and of Pb and In, on their magnetoresistance.

EXPERIMENTAL DETAILS

The dimensional effect in a magnetic field was studied on the same samples of the very pure metals Sn, Zn, Cd, and Al as had been used earlier to study the dimensional effect without a field.[15] The amounts of impurities in these metals, determined from the dependence of the residual resistance on the impurity content, were: 1.5×10^{-4}% for Sn, 2×10^{-4}% for Al, 5×10^{-6}% for Zn, and 6×10^{-6}% for Cd. The technique of preparing single crystals of various orientations, the method used to reduce the sample diameter, and the etching, assembly and annealing of samples were described in detail earlier.[15] The potentiometer circuit used to measure the resistance and the error of a single measurement were also dealt with in the earlier paper. The measurements in magnetic fields on all samples (especially massive samples) were repeated many times in order to increase the reliability of the reported curves. The plotted curves are thus the results of averaging of several measurements. The errors of a single measurement are given only for the curves representing massive samples of $Zn_{\parallel}$ and $Cd_{\parallel}$.[2)] These errors could be reduced only slightly by repetition of the measurements because the value of R for $Zn_{\parallel}$ and $Cd_{\parallel}$ rose very little in magnetic fields.

To find the effect of the purity of the metals on their magnetoresistance, we also used some less pure samples, most of which were used in earlier work on the effect of impurities on the residual resistance of Sn, In, Pb, Al, Zn, and Cd.[16] In the case of the less pure samples, all wires were polycrystals with grain dimensions comparable with the sample diameter, with the exception of the purest and largest single crystals and also the Sn samples (which were mostly $Sn_{\perp}$).

The electrical connections of the samples were such that the potential contacts were welded to the same generatrix of a cylindrical wire and were separated from the current contacts by 10—15 mm, i.e., by 3—6 diameters (for wires of large diameters d = 2—4 mm), or by 10—15 diameters (for thin wires with $d \leq 1$ mm). The difference between the diameters of the current leads and the sample was greatest (by a factor of 8—4) in the case of thick wires (d = 4—2 mm); this difference was very small in the case of thinner wires. These precautions in the electrical connections were necessary to avoid possible errors during measurement of the resistance, such as were noted earlier in galvanomagnetic[17] and usual measurements of the resistance of Bi at helium temperatures.[15]

The considerable peculiarities detected in measurements of R of Bi (with and without magnetic fields) are probably due to the low electron density in this metal. Similar effects in metals with high electron densities (Sn, In, Zn, Cd, Al, Pb) should be very small. To check this the resistance was measured for an Sn sample of 3 mm diameter which had soldered current leads of d = 0.9 mm and potential leads welded first to the current leads and then at a distance of 10 and 15 sample diameters from the current leads. In contrast to Bi, these differences in the potential lead positions produced no difference, within the experimental error of ≈2%, in the magnitude of the relative resistance $\delta_T = R_T/R_{293}$[3)] of Sn at T = 4.2°K. Consequently it was assumed for all six metals that in measurements of the resistance it was quite sufficient to keep the distance between the current and potential leads at not less than three sample diameters (even when the sample diameter differed considerably from that of the current leads) in order to avoid all errors due to assembly.

In the case of some thin samples (d < 1 mm) it was necessary to allow for the effect of the magnetic field of the current flowing through the sample. In such cases the ratio $\Delta R/R$ was determined as follows: ΔR was taken to be $R_H - R'$, where R_H and R′ are, respectively, the resistance with and without the applied field for a sufficiently strong current, and R is the resistance without the field extrapolated to the zero current. For sufficiently thin samples R′ = R. Although the correction described here was small it was nevertheless applied.

Magnetic fields were produced in a solenoid of 700 mm length, with constant 53 Oe/A on its axis and with the internal and external diameters ≈140

[2)]The sign ∥ or ⊥ always denotes that the principal axis of the crystal is parallel or perpendicular to the wire axis.

[3)]R_T and R_{293} represent the sample resistance at T and 293° K respectively.

and 240 mm respectively. In the central part of the solenoid there was a region of uniform field (accurate to within 1.2—1.5%) of 200 mm length. The necessary measurements at $H \leq 2.25$ kOe were carried out in this region. The solenoid could be fixed at angles of up to 6—7° to the vertical. This was very important since a small departure from parallelism of the field and current vectors produced a considerable (quadratic) rise of the resistance due to the effect of the transverse field component. Since a transverse field affects the resistance of metals more strongly than a longitudinal field, such nonparallelism of the field and current vectors produced a contribution of the transverse field which increased with H and in strong fields masked completely the effect of the longitudinal field. To avoid the effect of the transverse component it was necessary to incline the solenoid along some direction so that the quadratic rise of the resistance, very noticeable in fields of H = 1.25—2.25 kOe, was eliminated.

The nonparallelism of the field and current vectors affected least the measurements on Al samples, somewhat more those on $Sn_{\perp}$, even more those on Zn and very strongly those on Cd. For this reason the dependence $\Delta R/R = f(H, d)$ was not determined for Cd because thin and short samples could not be aligned exactly along the field in the solenoid.

Measurements of the resistance at 4.22°K and below this temperature were carried out in liquid helium at normal and low pressures. The resistances of Sn and In samples at T = 1.65°K and H = 0, at which these metals are superconducting, were estimated as follows. For massive samples the relative resistance δ_T, proportional to the resistivity, was calculated at T = 1.65°K using the following formulas for samples of 2.75 mm diameter

$$\delta_T(Sn_{\parallel}) = (0.93 + 5 \cdot 10^{-4} T^5) \cdot 10^{-5},$$
$$\delta_T(Sn_{\perp}) = (1.26 + 6 \cdot 10^{-4} T^5) \cdot 10^{-5}$$

and a different formula for a sample of 3 mm diameter[16]:

$$\delta_T(In) = (4 + 2.94 \cdot 10^{-3} T^5) \cdot 10^{-5}$$

Then from the known value of R_{293} the quantities $R_T = \delta_T R_{293}$ were calculated for the three samples. For thinner Sn wires it was assumed that on cooling below 4.22°K the slope of the linear dependence $\delta_{4.2}(1/d)$ falls only a little and that this can be neglected, i.e., it was assumed that $(\delta_{1.65})_{thin} = (\delta_{4.2})_{thin} - \Delta\delta_{thick}$, where $\Delta\delta_{thick} = [\delta_{4.2} - \delta_{1.65}]_{thick}$. Judging from the similar straight lines for Cd, obtained at T = 4.22 and 1.65°K, this change in the slope is indeed small.[15] However, how small this change is for Sn is not known exactly. In this connection we may take it that the dimensional effect given in Figs. 1 and 2 for Sn in a magnetic field at 1.65°K differs a little (it is too high) from the true value.

Owing to the reduction of the slope of the straight line dependence on cooling from 4.22 to 1.65°K the resistance of a thin (d < 1 mm) Sn wire at T = 1.65°K estimated as indicated above is too high by an amount γ. Consequently the ratio

$$\Delta R/R = [R_H - (R + \gamma)]/(R + \gamma)$$
$$\approx (\Delta R/R)_{true} - (R/\gamma + 1)^{-1}$$

is lower by $(R/\gamma + 1)^{-1}$ compared with the true

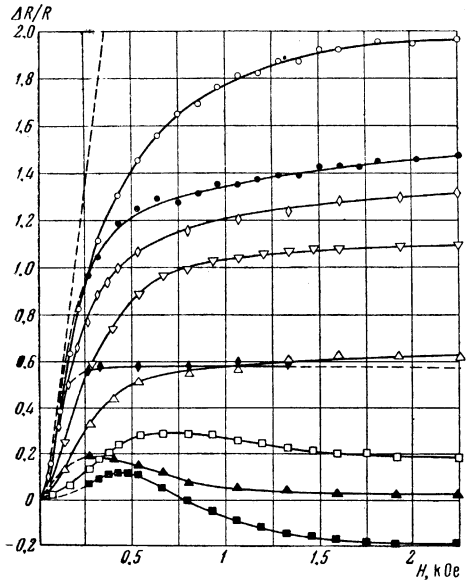

FIG. 1. Change of the electrical resistance in a magnetic field for $Sn_{\perp}$ samples of various diameters at 4.22° K (open symbols) and 1.65° K (black symbols): o, ● – d = 2.75 mm; ◇, ◆ – d = 1.39 mm; ▽ – d = 0.71 mm; △, ▲ – d = 0.49 mm; ■, □ – d = 0.2 mm.

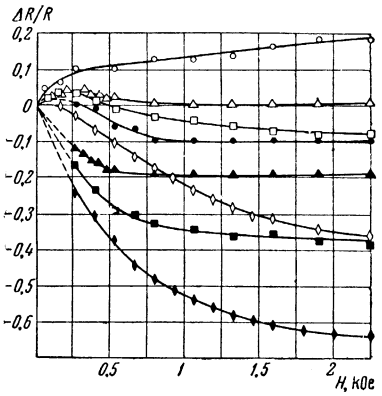

FIG. 2. Change of the electrical resistance in a magnetic field for $Sn_{\parallel}$ samples of various diameters at 4.22° K (open symbols) and 1.65° K (black symbols): o, ● – d = 2.21 mm; △, ▲ – d = 0.9 mm; □, ■ – d = 0.45 mm; ◇, ◆ – d = 0.17 mm.

value. Since $(\Delta R/R)_{1.65} > 0$ for $Sn_{\perp}$ and $(\Delta R/R)_{1.65} < 0$ for $Sn_{\parallel}$, we may say that in determination of $R_{d,H=0}$ the ratio $\Delta R/R$ is too low in the first case and too high in the second. Clearly with decrease of the wire diameter the value of γ rises and consequently the error in determination of $|\Delta R/R|$ increases.

RESULTS

1. Effect of dimensions and orientation. The results of measurements of the magnetoresistance at 4.22 and 1.65°K are given in Figs. 1 and 2 for $Sn_{\perp}$ and $Sn_{\parallel}$, respectively, in the form of the dependence of the ratio $\Delta R/R$ on H; here R is the resistance of a wire without a field. Similar curves for $Zn_{\perp}$ and $Zn_{\parallel}$ at T = 4.22°K are given in Figs. 3 and 4. Values of R_{293} and δ_T for all samples of Sn, Zn, and Al at H = 0 are listed in Table I. From the curves in Figs. 1—4 and Fig. 5 (Cd) it is clear that when the current and field vectors

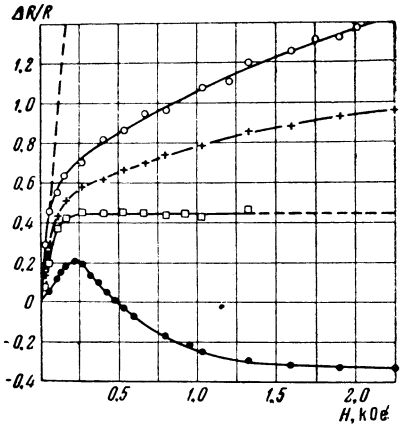

FIG. 3. Change of the electrical resistance in a magnetic field for $Zn_{\perp}$ samples of various diameters at 4.22° K: o – d = 4.1 mm; + – d = 1.9 mm; □ – d = 0.9 mm; ● – d = 0.3 mm.

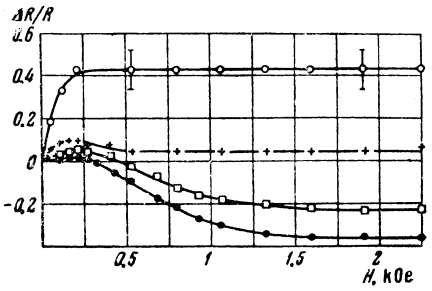

FIG. 4. Change in the electrical resistance in a magnetic field for $Zn_{\parallel}$ samples of various diameters at 4.22° K: o – d = 4.1 mm; + – d = 1.65 mm; □ – d = 0.42 mm; ● – d = 0.23 mm.

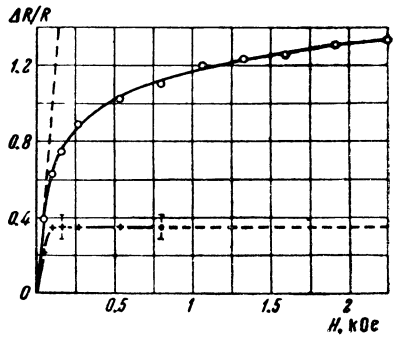

FIG. 5. Change in the electrical resistance in a magnetic field for Cd at 4.22° K: o – $Cd_{\perp}$, d = 3.5 mm, $\delta_{4.2} = 3.5 \times 10^{-5}$; + – $Cd_{\parallel}$, d = 3.4 mm, $\delta_{4.2} = 2.5 \times 10^{-5}$.

lie in the basal plane the greatest effect is obtained for Sn, Zn, and Cd, with the resistance rising by a factor of 2—3 in the maximum field. When the current and field vectors coincide with the principal axis of the crystal the resistance in the same maximum fields rises only by 20—40%. Nevertheless even in "thick" wires of Sn (d ≈ 2.2 mm), Zn and Cd (d ≈ 4 mm) the dimensional effect is still large. If that effect were eliminated then the resistances of $Sn_{\perp}$, $Zn_{\perp}$, and $Cd_{\perp}$ would

Table I

Sample	d, mm	$10^3 R_{293}$, Ω	$10^5\delta_{4.2}$	Sample	d, mm	$10^2 R_{293}$, Ω	$10^5\delta_{4.2}$	$10^5\delta_{1.65}$
$Zn_{\parallel}$	∞	—	1.4	$Sn_{\parallel}$	∞	—	1.45	0.79
	4.1	0.852	1.93		2.21	0.73	1.68	1.06
	1.65	1.94	2.22		0.9	0.963	1.89	1.3
	0.42	8.6	4.95		0.45	1.41	2.2	1.6
	0.23	27	7.5%		0.17	7.57	4.0	3.4
$Zn_{\perp}$	∞	—	1.6	$Sn_{\perp}$	∞	—	1.55	0.77
	4.1	0.863	2.63		2.75	0.342	2.32	1.5
	1.9	2.05	3.55		1.39	0.463	2.44	1.65
	0.9	2.45	5.9		0.71	0.955	3.3	—
	0.3	18.2	13.4		0.49	1.66	4.5	3.7
Al	∞	—	3.4		0.2	4.63	7.75	6.85
	3.6	0.7	3.8					
	1.18	1.47	5.1					
	0.7	2.5	6.1					
	0.6	3.6	6.1					
	0.3	7.0	9.5					
	0.2	7.9	13.5					

increase by a factor of 5—6 in a field of 2.25 kOe (Fig. 11). Variation of the resistance with the applied field for infinitely thick samples is shown in Figs. 1, 3, and 5 by a dashed line close to the ordinate axis; the method of obtaining this line is described below.

For $Sn_{||}$, $Zn_{||}$, and $Cd_{||}$ there is a characteristic rapid rise of the resistance in weak fields and a very small rise in strong fields (this was not found for $Zn_{||}$ and $Cd_{||}$ because of the insufficient accuracy in measurement of $R_{4.2}$). Thus for Sn, Zn and Cd we have a strong anisotropy of the quantity $\Delta R/R$ reaching a factor of 10 or more. It is necessary to point out that a similar anisotropy in the magnetoresistance at 4.2°K was observed earlier by Lazarev, Nakhimovich, and Parfenova [4] in a longitudinal field, but they used samples of Zn and Cd that were much less pure (by a factor of 10—100). However, surprisingly enough, these workers observed the greatest increase of the resistance for parallel orientations of both metals and not for normal orientations.

The experimental curves for Al are shown in Fig. 6, where the continuous curves represent the results for samples made from the same thick sample, and the dashed and chain curves represent thin samples made from another thick sample. Both the original thick samples had the same δ_0 and could differ only in their orientation.

It also follows from Figs. 1—4, 6, and 11 (see below) that if the wire diameter is reduced, the rise of the ratio $\Delta R/R$ with rising field intensity slows down (compared with the rise of the resistance of an infinitely thick wire in a magnetic field), and this slow-down is relatively greater in $Sn_{\perp}$ and $Zn_{\perp}$ than in $Sn_{||}$ and $Zn_{||}$. For the thinnest samples the value of $\Delta R/R$ becomes negative, i.e., the resistance in a magnetic field becomes smaller than the resistance without a field. This is related to the free path λ of electrons (which have a velocity component normal to the wire axis) due to bending of their trajectories by the magnetic field, while without the field the value of λ is limited by the sample surface. In Zn, Sn, and Al at T = 4.22°K the mean free path λ is, respectively, 2, 0.6, and 0.6 mm, [13] but since λ increases with increase of temperature, the dimensional effect in the magnetic field also increases. This is particularly clear in the case of Sn (Figs. 1 and 2) for which the resistance at T = 1.65°K is approximately half the resistance at T = 4.22°K (correspondingly λ increases by a factor of 2). Similar behavior could not be observed in Zn and Al since for these metals $\delta_{4.2} \approx \delta_0$. In all the figures there is at least one curve with a maximum at certain field and diameter values; this is due to the influence of two factors: increase of the resistance with increase of the magnetic field intensity and the simultaneous fall of the resistance on reduction of the diameter.

If the Fermi surface of the metal is closed along the given crystallographic direction, the saturation observed for thin wires in strong fields (this is particularly clear in the case of curves with maxima) should obviously occur when the δ_T values at H_{sat} are the same for bulk and thin samples. If the Fermi surface is open, then δ_T for a thin wire should differ considerably from δ_T for bulk metal (see footnote [1]). The relevant comparisons at T = 4.22°K are given in Table II which shows a satisfactory agreement between the numerical estimates for $Zn_{\perp}$ and $Sn_{\perp}$. We must

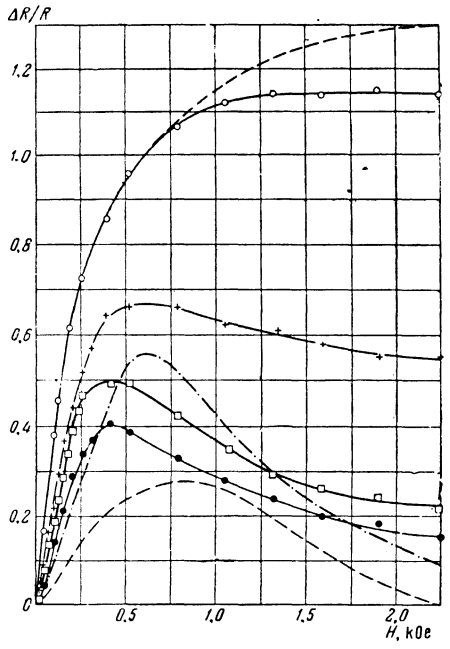

FIG. 6. Change in the electrical resistance in a magnetic field for Al samples at 4.22° K: o – d = 3.6 mm. + – d = 1.18 mm; □ – d = 0.7 mm; ● – d = 0.6 mm; chain curve – d = 0.3 mm; dashed curve – d = 0.2 mm.

Table II

	d, mm	ΔR/R	$10^5\rho_{4.2}$ H = 0	$10^5\rho_{4.2}$ H_{sat} = 2.25 kOe
$Zn_{||}$	0.23	−0.37	7.5	4,75
	4.1	+0.43	1.93	2.76
$Sn_{||}$	0.17	−0,4	4	2,4*
	2.21	+0,2	1.68	2.0*
$Zn_{\perp}$	0.3	−0.33	13.4	9
	∞	+4.0	1.6	9
$Sn_{\perp}$	0.2	+0,19	7.75	9,2
	∞	+5,2	1.55	9.6

*H = 2.75 – 3 kOe.

remember that the values of $\Delta R/R$ for $Sn_\perp$ and $Zn_\perp$ taken from Fig. 11 for infinitely thick samples are only approximate (particularly for Zn).

From the results for $Sn_\parallel$ and $Zn_\parallel$ (Table II) we may conclude that the samples of 2.21 and 4.1 mm diameters are not sufficiently thick (in a lesser degree this also applies to $Zn_\perp$ and $Sn_\perp$ of the same diameters), because $\delta_{H,thick} < \delta_{H,thin}$ due to the lower value of $\Delta R/R$ for thick samples. The latter circumstance is in agreement with earlier work.[15] Knowing $(\Delta R/R)_{thin}$ for a thin wire at H_{sat} and $(\Delta R/R)_\infty$ for a very thick wire at the same value of H, as well as $\delta_{d,H=0}$, we can determine the relative resistance of an infinitely thick sample in the absence of a field: $\delta_{\infty,H=0}$. In this case we should have the following equality

$$\delta_{\infty,\,H=0} = \delta_{d,\,H=0}\left\{1 - \left[\left(\frac{\Delta R}{R}\right)_\infty - \left(\frac{\Delta R}{R}\right)_{thin}\right]\right\},$$

since $\Delta R/R \equiv \Delta\delta/\delta$. The relative comparison is given in Table III for Sn, Al, and Zn at T = 4.22°K; this shows satisfactory agreement (the differences are less than 20%) for all samples, apart from $Sn_\parallel$ of 0.17 and 0.45 mm diameters, for which saturation was not yet reached, and for the $Zn_\perp$ sample of 0.3 mm diameter.

Table III

	d, mm	$\left(\frac{\Delta R}{R}\right)_\infty - \left(\frac{\Delta R}{R}\right)_{thin}$	$10^5\rho_d$ (H_{sat})
$Zn_\parallel$	0.23	0.8	1.5
	0.42	0.67	1.65
	1.65	0.4	1.33
	∞	0	1.4*
$Sn_\parallel$	0.17	0.54	1.84
	0.45	0.26	1.63
	0.9	0.18	1.55
	∞	0	1.45*
$Zn_\perp$	0.3	4.3	2.5
	0.9	3.55	1.3
	∞	0	1.6*
$Sn_\perp$	0.2	5	1.3
	∞	0	1.55*
Al	0.7	1.1	3.0
	∞	0	3.4*

*H = 0.

As pointed out above, MacDonald's theory[7] is not fully applicable to the experimental results, since the resistance rise in a field is very considerable. Nevertheless an estimate of the electron momentum can be obtained for Sn and Zn by the use of the theoretical dependence $\rho/\rho_0 = f(d/2r_0, k)$ given by Chambers;[3] here $k = d/\lambda$, $r_0 = mvc/eH$, and ρ, ρ_0 are the resistivities in a magnetic field of thin and infinitely thick samples, respectively. To estimate the electron momentum $p = mv$ we used the curves for $Sn_\parallel$ and $Zn_\parallel$ samples at T = 4.22°K for which the effect of a magnetic field on thick samples is considerably less than for $Sn_\perp$ and $Zn_\perp$. The resultant values of the momentum ($p \approx 10^{-19}$ cgs esu) are quite reasonable.

An order-of-magnitude value of p can also be obtained from very general considerations using measurements carried out on very thin and sufficiently thick wires in a longitudinal field $H \approx H_{sat}$ (see footnote [1]). Since at $H = H_{sat}$ we have $\delta_d \approx d_\infty$, which is true mainly when $2r_0 \approx d$, we can estimate p by taking $H = H_1$ close to H_{sat}. Estimates of p obtained in this way using $H_1 = 1.5$ kOe give reasonable momentum values for the three metals.

We must mention also the resistance decrease in thin wires of $Zn_\parallel$ on increase of the measuring current passing through them (see Fig. 7). This phenomenon, which was always observed only in $Zn_\parallel$ samples,[15] is obviously also related to the dimensional effect in resistance, but here the transverse magnetic field (the current field) is active.

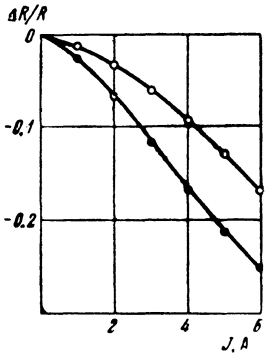

FIG. 7. Magnetoresistance of $Zn_\parallel$ as a function of the measuring current at 4.22° K: o – d = 0.42 mm; • – d = 0.23 mm.

The fall of the resistance of $Zn_\parallel$ samples on increase of the measuring current was greater for smaller thicknesses.

Finally we should note that the reduction of the resistance in a longitudinal magnetic field observed in thin (compared with λ) wires should be ascribed to the dimensional effect in the field and should not be related to the "quadratic" component of the Hall effect as asserted by several authors.[14] This is supported by the presence of the dimensional effect in sufficiently thick and in very thin wires: the resistance in a field always decreases with reduction of the diameter and the positions of the current and potential leads for samples with $d \leq 2$ mm prevent the appearance of the interfering "quadratic" components of the Hall effect.

2. Effect of the metal purity. The change of resistance in a magnetic field is shown in Figs. 8–10 for Sn, Al, and In as a function of purity. The dashed curves in Figs. 8 and 9 represent infinitely thick and purest samples. From these figures it

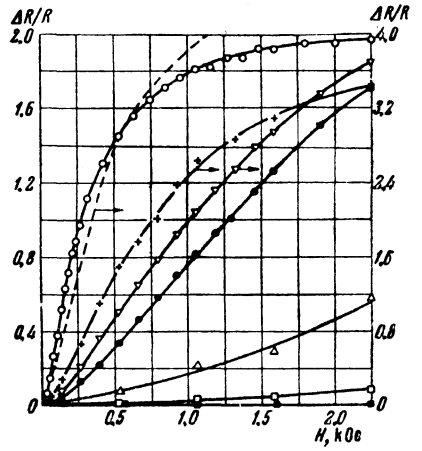

FIG. 8. Dependence of $\Delta R/R$ on the magnetic field intensity for Sn samples of various purities at 4.22° K: o – d = 2.75 mm, $\delta_{4.2} = 2.3 \times 10^{-5}$, $\delta_{4.2,\infty} = 1.55 \times 10^{-5}$, $\delta_{0,\infty} = 0.76 \times 10^{-5}$ ($Sn_{\perp}$ sample); + – d = 3 mm, $\delta_{4.2} = 4.45 \times 10^{-5}$, $\delta_{4.2,\infty} = 4 \times 10^{-5}$, $\delta_{0,\infty} = 3.1 \times 10^{-5}$ ($Sn_{\perp}$ sample); $\nabla - \delta_{4.2} = 6.73 \times 10^{-5}$, $\delta_{4.2,\infty} = 6.2 \times 10^{-5}$, $\delta_{0,\infty} = 5.25 \times 10^{-5}$ ($Sn_{\perp}$ sample); $\bullet - \delta_{4.2} = 1.59 \times 10^{-4}$, $\delta_{4.2,\infty} = 1.53 \times 10^{-4}$, $\delta_{0,\infty} = 1.41 \times 10^{-4}$ ($Sn_{\perp}$ sample); $\Delta - \delta_0 \approx \delta_{4.2} = 5.05 \times 10^{-4}$; $\square - \delta_0 = \delta_{4.2} = 1.46 \times 10^{-3}$; $\blacksquare - \delta_0 = 1.42 \times 10^{-2}$.

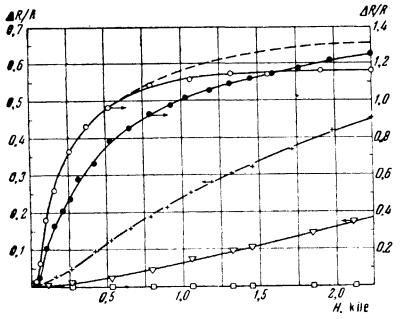

FIG. 9. Dependence of $\Delta R/R$ on the magnetic field intensity for Al samples of various purities at 4.22° K: o – d = 3.6 mm, $\delta_0 = 3.8 \times 10^{-5}$; $\bullet$ – d = 2.9 mm, $\delta_0 = 7.15 \times 10^{-5}$; + – $\delta_0 = 6.8 \times 10^{-4}$; $\nabla - \delta_0 = 1.94 \times 10^{-3}$; $\square - \delta_0 = 5 \times 10^{-2}$.

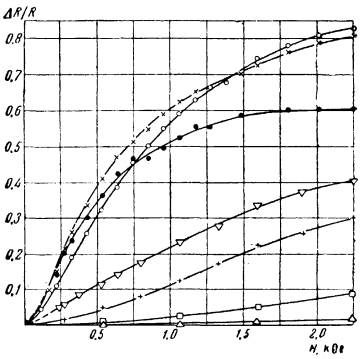

FIG. 10. Dependence of $\Delta R/R$ on the magnetic field intensity for In samples of various purities: o, x, $\bullet$ – d = 3 mm, $\delta_{4.2} = 7.85 \times 10^{-5}$, $\delta_{4.2,\infty} = 7.3 \times 10^{-5}$, $\delta_{0,\infty} = 3.3 \times 10^{-5}$, where o denotes T = 4.22° K, x – T = 3.4° K, $\bullet$ – T = 1.65° K; +, ∇ – d ≈ 2 mm, $\delta_{4.2} = 3.56 \times 10^{-4}$, $\delta_{4.2,\infty} = 3.5 \times 10^{-4}$, $\delta_{0,\infty} = 2.86 \times 10^{-4}$ where + denotes T = 4.22° K, ∇ – T = 1.65° K; $\square$ – $\delta_{4.2,\infty} = 1.07 \times 10^{-3}$, $\delta_0 = 1 \times 10^{-3}$; $\Delta - \delta_0 = \delta_{4.2} = 3.7 \times 10^{-3}$.

follows that with increase of the metal purity the resistance rise in the field increases at a constant temperature T = 4.22°K and the curve becomes steeper and steeper. This continues until λ becomes comparable with the wire diameter (until the metal becomes sufficiently pure). Then the dimensional effect limits the resistance rise with increase of the field. This has been discussed in detail above, but the behavior is also clear from Figs. 8 and 9, where the dashed curves differ considerably from the continuous ones from the thickest and purest wires. The same can be said about a single sample of In of 3 mm diameter (Fig. 10) at different temperatures T = 4.22, 3,4, and 1.65°K: with increase of λ ($\lambda_{4.2} = 0.2$ mm, $\lambda_{3.4} = 0.31$ mm, and $\lambda_{1.65} = 0.43$ mm [15,18]) the curve first becomes steeper but at 1.65°K it is less steep again. We should note that the dependence of $\Delta R/R$ on H for this sample of In at T = 4.22°K agrees very well with the analogous curve obtained by Olsen [8] for a wire of the same purity and 2 mm diameter.

To obtain a general picture of the change of resistance in a longitudinal field the results of measurements for all samples were plotted in Kohler's coordinates (Fig. 11). The curve for Pb was plotted from the data for four samples having the following values of $\delta_{4.2}$: 3×10^{-3}, 3.15×10^{-4}, 2.61×10^{-4} and 1.5×10^{-4}; for all the other metals the curves of Figs. 8–10 were used. To avoid

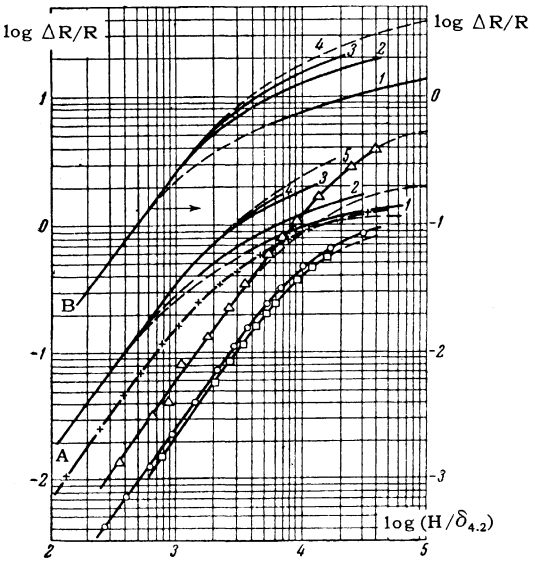

FIG. 11. Kohler's diagram for samples of Sn (Δ), In (o), Pb ($\square$), Al (+), Cd (A) and Zn (B). Part A: curve 1) d = 3.5 mm, $\delta_{4.2} = 3.5 \times 10^{-5}$, $\delta_{4.2,\infty} = 2.7 \times 10^{-5}$, $Cd_{\perp}$; curve 2) d ≈ 2 mm, $\delta_{4.2} = 7.73 \times 10^{-5}$; curve 3) $\delta_{4.2} = 1.7 \times 10^{-4}$; curve 4) $\delta_{4.2} = 3.96 \times 10^{-4}$; curve 5) d = ∞, pure $Cd_{\perp}$. Part B: curve 1) d = 4.1 mm, $\delta_{4.2} = 2.63 \times 10^{-5}$, $\delta_{4.2,\infty} = 1.4 \times 10^{-5}$, $Zn_{\perp}$; curve 2) d ≈ 3.5 mm, $\delta_{4.2} = 5.8 \times 10^{-5}$; curve 3) $\delta_{4.2} = 1.26 \times 10^{-4}$; curve 4) d = ∞, pure $Zn_{\perp}$.

overloading Fig. 11 only some of the experimental points are shown and for Zn and Cd only the smooth curves are given.

From the Kohler diagram it follows that all the points for a given metal can be fitted by the same continuous curve up to the point where the sample dimensions begin to have an effect. When that happens there is a departure from the general curve toward reduction of $\Delta R/R$. This is clearly seen in the dashed curves next to the continuous curves for Sn and Al, plotted for the purest and thickest single crystals, and in curves 1—4 for Cd and 1—3 for Zn. For the Pb sample with $\delta_{4.2} = 1.5 \times 10^{-4}$ a dimensional effect in the magnetic field could not be found up to the diameter of 0.5 mm, while for $Sn_{\perp}$ having $d \approx 3$ mm and $\delta_{0,\infty} = 3.1 \times 10^{-5}$ the effect of dimensions is quite strong (this is not shown in Fig. 11). Using the curves of Fig. 11 we can estimate the change $\Delta R/R$ in a field for very pure and infinitely thick samples: these estimates are shown by curves 4 and 5 for Zn and Cd respectively and by the dashed extensions of the continuous curves for $Sn_{\perp}$ and Al. Curves 4 and 5 are very approximate especially as they were plotted from measurements on polycrystals and they should apply only to $Zn_{\perp}$ and $Cd_{\perp}$.

If the dimensional effect is excluded then all the curves of Fig. 11 show saturation in high effective fields which is in general agreement with the results of Luthi.[19] However, if a recalculation is made replacing $HR_{293}/R_{4.2}$ with $HR_{\theta}/R_{4.2}$ (θ is the Debye temperature), it is then clear that good agreement is obtained only for In in strong fields and for Zn. There is poorer agreement for Sn and considerable differences for Al and Pb; there are no data which could be used to compare with the results for Sn. In weak effective fields all the curves have the same slope of $\approx 55°$, and the tangent of this angle is ≈ 1.43. Thus for all six metals $\log(\Delta R/R) = A + 1.43\, H/\delta_{4.2}$ when $\Delta R/R \leq 0.2$. This experimental relationship is difficult to discuss in the absence of a satisfactory theory.

Finally a comparison of the saturation values of $\Delta R/R$ for In, Al, and Sn, taken from Fig. 11, with similar values obtained from measurements in a transverse field (in the crystallographic direction of the minimum change of resistance),[20,21] indicates that these quantities do not differ very greatly. Once again we see that the longitudinal effect in pure metals has now become comparable with the transverse effect.

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216