AZIMUTH ANGLE DISTRIBUTION OF SECONDARY PARTICLES PRODUCED IN HIGH-ENERGY INTERACTIONS

A. P. MISHAKOVA and B. A. NIKOL'SKIĬ

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The azimuth angle distributions of secondary particles produced in interactions of high-energy nucleons have been investigated by the angular correlation method. It has been found that the azimuth angle distribution is almost isotropic. The azimuth angle distribution of secondary particles in cosmic ray "jet" showers is analyzed to test the validity of a model in which an intermediate excited state is produced, and of the "fireball" model. The conclusion is that if the fireball model is valid, the direction of motion of the fireball coincides with that of the colliding particles. The model of an intermediate excited state (with a large angular momentum) disagrees with an isotropic azimuth angle distribution.

1. METHODS OF INVESTIGATING THE AZIMUTH ANGLE DISTRIBUTION

THE present article is devoted to the study of the azimuth angle distribution of secondary particles produced in the collisions of 10^{10}- to 10^{13}-eV protons with nucleons and nuclei. A direct determination of the shape of the azimuth angle distribution $F(\varphi)$ of the secondary particles is in practice impossible, since it would require the knowledge of the position of the origin of each interaction. This is statistically inaccurate due to the relatively small number of secondary particles per interaction. In the present work we have therefore used the method of pair angular correlations, i.e., we have studied the distribution of azimuth angles ϵ between pairs of secondary particles (see Fig. 1), which is independent of a physical point of reference and, consequently, has the advantage that the distributions $W(\epsilon)$ for separate interactions can be added together. Other methods of studying the azimuth distribution of secondary particles suitable for an analysis of the data on many interactions were proposed by Stern,[1] Chudakov,[2] and Friedlaender.[3]

The azimuth angles ϵ between the particles lie in plane Q (Fig. 1) and are determined as $\epsilon_{ik} = \varphi_i - \varphi_k$. The distribution $W(\epsilon)$ of the azimuth angles between the particles is related to the function $F(\varphi)$ in the following way:

$$W(\varepsilon) = \int_0^{2\pi} F(\varphi)\,[F(\varphi+\varepsilon) + F(\varphi-\varepsilon)]\,d\varphi,$$

$$0 \leqslant \varepsilon \leqslant \pi, \quad 0 \leqslant \varphi \leqslant 2\pi. \qquad (1)$$

From Eq. (1) it follows that for $F(\varphi)$ = const we have $W(\epsilon)$ = const also. If we represent $F(\varphi)$ by a series:

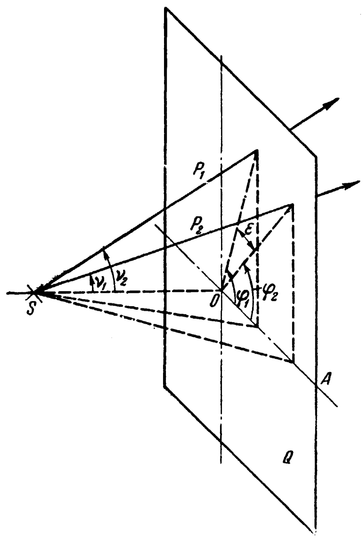

FIG. 1. Azimuth (φ_1, φ_2) and polar (ϑ_1, ϑ_2) angles of emission of secondary particles P_1 and P_2. SO – direction of motion of the primary particle, ϵ – relative azimuth angle between the particles P_1 and P_2.

$$F(\varphi) = \frac{1}{2\pi}\left(1 + \sum_{k=1}^{\infty} a_k \cos k\varphi\right), \qquad (2)$$

then, according to Eq. (1), we have

$$W(\varepsilon) = \frac{1}{\pi}\left(1 + \sum_{k=1}^{\infty} \frac{a_k^2}{2} \cos k\varepsilon\right). \qquad (3)$$

If $F(\varphi)$ is symmetric,

$$F(\varphi \pm \pi) = F(\varphi), \qquad (4)$$

then series (2) and (3) contain only terms with even values of k. As a first approximation, in order to estimate the azimuth angle anisotropy of $F(\varphi)$, let us put k = 2; we have then

$$f(\varphi) = \frac{1}{2\pi}(1 + a\cos 2\varphi) \qquad (5)$$

and, consequently

$$\omega(\varepsilon) = \frac{1}{\pi}\left(1 + \frac{a^2}{2}\cos 2\varepsilon\right). \qquad (6)$$

In order to find a value of a which determines the degree of asymmetry of $F(\varphi)$ [see Eq. (5)] we shall introduce the experimentally determined quantity

$$R = \frac{w_{\exp}(0-\pi/4;\, 3\pi/4-\pi) - w_{\exp}(\pi/4-3\pi/4)}{w_{\exp}(0-\pi/4;\, 3\pi/4-\pi) + w_{\exp}(\pi/4-3\pi/4)}, \qquad (7)$$

where $w_{\exp}(0-\pi/4;\, 3\pi/4-\pi)$ and $w_{\exp}(\pi/4-3\pi/4)$ are the numbers of particle pairs with angles $\epsilon = (0-45°;\, 135-180°)$ and $\epsilon = (45-135°)$, respectively.

From Eq. (6) it follows that

$$R = a^2/\pi, \text{ i.e., } a = \sqrt{\pi R}. \qquad (8)$$

The accuracy of the quantity a is determined by the error $\delta R: \delta a = \pi\delta R/2a$. In order to determine the statistical error δR we have carried out a Monte Carlo calculation of the quantities R for "showers" having distribution (5). The calculation showed that the error δR in the determination of R depends on the degree of anisotropy a, on the number of shower particles per interaction n_s, and the number of interactions r:

$$\delta R = f(a, n_s)/\sqrt{r}.$$

However, the analysis of numerous variants of the Monte Carlo calculation for different values of a, n_s, and r, shows that the statistical spread δR can be very satisfactorily interpolated by the relation $\delta R \approx \alpha\sqrt{N}$, where $N = rn_s(n_s - 1)/2$ (i.e., N is the total number of pairs in the group of interactions under consideration) and α is the factor reflecting the dependence of δR on a. Moreover,

$a =$	0	0.2	0.4	0.9
$\alpha = \delta R\sqrt{N} =$	1	1.5	2	3

The errors of the experimental values of R and a were determined according to the results of this calculation for the mean value of a for the whole shower group.

The distribution of the form (5), having the property (4), leads to a distribution $w(\epsilon)$ which is symmetrical about the angle $\epsilon = \pi/2$. The experimental proof of this fact will thus indicate the absence in (2) and (3) of terms with odd k. We shall characterize the symmetry of these quantities about the angle $\epsilon = \pi/2$ by the experimental quantity

$$\Delta = \frac{w_{\exp}(0-\pi/2) - w_{\exp}(\pi/2-\pi)}{w_{\exp}(0-\pi/2) + w_{\exp}(\pi/2-\pi)}, \qquad (9)$$

where the notation is the same as for (7).

The errors in the quantity Δ were also determined by a Monte Carlo calculation. It was shown that Δ is independent of a. As will be shown in the following, the magnitude of Δ is nearly zero for all types of interactions, which indicates that $W(\epsilon)$ is symmetric about the angle $\epsilon = \pi/2$ and, consequently, that $F(\varphi)$ is symmetric about the angle $\varphi = \pi$.

2. EXPERIMENTAL RESULTS

We have analyzed the azimuth angle distribution of secondary particles in: 1) interactions of 9-BeV protons with the emulsion nuclei, 2) pp interactions at 9-BeV incident proton energy, and 3) interactions of cosmic-ray protons with energies of 10^{10} -10^{13} eV with the emulsion nuclei.

The analysis was carried out by studying the distribution $W(\epsilon)$ with respect to the pair azimuth angle ϵ. The number of angles ϵ found in the analysis of N interactions of a given group is

$$\frac{1}{2}\sum_{i=1}^{N}[n_{si}(n_{si} - 1)],$$

where n_{si} is the number of relativistic secondary particles in the i-th interaction. In the study of interactions of protons with the emulsion nuclei, we have separated the interactions with the number of strongly ionizing particles $(n_h + n_g) \le 5$, and thus could consider the events as representing collisions of protons with peripherical nucleons of the nuclei.

Interactions of 9-BeV protons with the emulsion nuclei. The emulsion stack 5 cm × 10 cm × 20 cm in dimension was irradiated using the Joint Institute for Nuclear Research synchrotron. In the area scanning of the emulsion layers we have selected, among 180 interactions with $n_s \ge 5$, 46 interactions with $(n_h + n_g) \le 5$, and 134 with $(n_h + n_g) > 5$. The obtained experimental distributions $w(\epsilon)$ are shown in Fig. 2 where it can be seen that the distributions are isotropic within the limits of experimental errors. The experimental values are, respectively,

$$R = 0.027 \pm 0.061 \text{ for interactions with } (n_h + n_g) \leqslant 5,$$

$$R = 0.010 \pm 0.031 \text{ for interactions with } (n_h + n_g) > 5.$$

pp interactions at 9 BeV. In the analysis of the distribution w(s), for this group of interactions we have used the experimental data on the emission angles of secondary particles supplied by

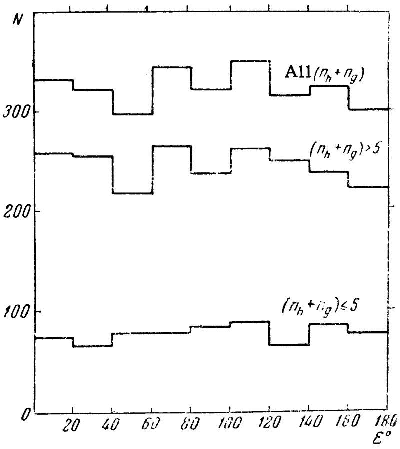

FIG. 2. The distribution $w(\epsilon)$ of secondary particles produced in the interaction of 9 BeV protons with emulsion nuclei ($n_h + n_g$ – number of black and grey tracks in a given interaction).

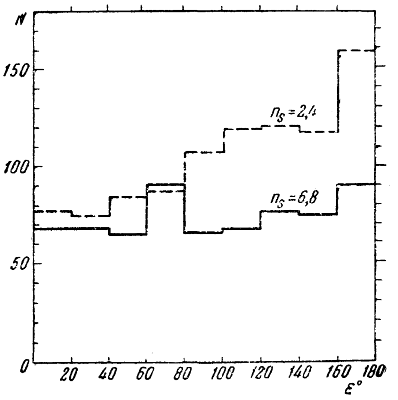

FIG. 3. The distribution $w(\epsilon)$ of secondary particles in pp collisions at 9 BeV with the number of secondary particles $n_s = 2.4$ and $n_s = 6.8$.

I. M. Gramenitskiĭ (JINR).[1)] We have analyzed 48 events with $n_s = 6.8$, and 167 events with $n_s = 2.4$. From the distribution $w(\epsilon)$ given in Fig. 3 we can see that for the emission of a small number of secondary particles ($n_s = 2.4$), the number of particle pairs with angles ϵ close to 180° is considerably greater than the number of pairs with small ϵ. Such an anisotropy of the distribution $w(\epsilon)$ can be explained by the momentum conservation law which leads to a fully determined angular correlation. Thus, for the elastic pp scattering where only two particles are produced, the azimuth angle of the pair is π. The distribution $w(\epsilon)$ for $n_s = 6.8$ is practically isotropic ($R = 0.020 \pm 0.061$).

Interactions of cosmic ray protons of $10^{10}-10^{13}$ eV with the emulsion nuclei. Experimental data on the emission angles of secondary particles in this group of interactions were obtained in our and other laboratories.[2)] For the analysis of the azimuth angle distribution of secondary particles we have selected 78 interactions produced by protons with $n_s \geq 5$ for $(n_h + n_s) \leq 5$.

The distributions $w(\epsilon)$ were considered as depending on the primary energy E_0, the number of shower particles n_s, and the shape of the angular distribution $f(\vartheta)$ (two-center and non-two-center distributions).

a) The dependence $w(\epsilon)$ on the shower energy E_0 is given in Fig. 4.

The quantities R and Δ, characterizing these distributions, are given in Table I. The quantity Δ characterizes the symmetry of the distribution $w(\epsilon)$ about the angle $\epsilon = \pi/2$. The increase in the number of events with angles $\epsilon > \pi/2$ ($\Delta < 0$) seen in Fig. 3 follows from the momentum conservation law, and is apparent for interactions with a small number of secondary particles. The increase in the number of events with angles $\epsilon < \pi/2$ ($\Delta > 0$) indicates that the shower axis differs from the direction of the primary particle. To avoid errors in this respect, we did not consider the showers produced by neutral particles that have an unknown direction of motion.

b) The dependence of the distribution $w(\epsilon)$ on the multiplicity n_s is illustrated by the values of R and Δ given in Table II. Analyzing these inter-

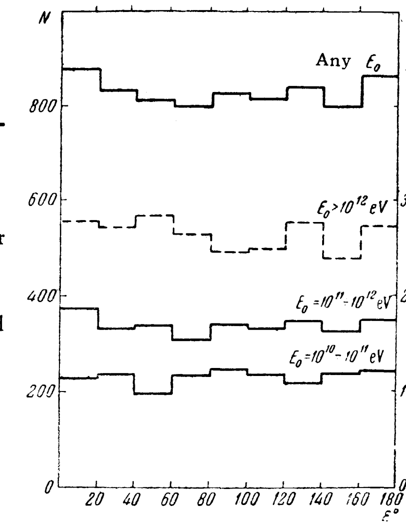

FIG. 4. The distribution $w(\epsilon)$ of secondary particles produced in cosmic ray showers of different energies E_0 for $(n_h + n_g) \lesssim 5$. The number of events N for the solid curves is indicated at the left, and for the dashed one at the right.

Table I. Values of R and Δ for cosmic-ray showers of various energies E_0

E_0, eV	Number of showers	Number of secondary particle pairs	R	Δ
$10^{10}-10^{11}$	33	2052	$+0.007 \pm 0.033$	$+0.023 \pm 0.022$
$10^{11}-10^{12}$	27	3012	$+0.022 \pm 0.027$	$+0.002 \pm 0.018$
$>10^{12}$	17	2379	$+0.017 \pm 0.031$	$+0.026 \pm 0.021$
Any E_0	77	7443	$+0.017 \pm 0.018$	$+0.003 \pm 0.012$

[1)]The method of selecting the pp interactions in the emulsion is described in [4].

[2)]We have used the data from the laboratories of J. Pernegr (Prague), M. Miesowicz (Cracow), and A. P. Zhdanov (Leningrad), which have been kindly supplied by the authors.

Table II. Values of R and Δ for cosmic-ray showers with various numbers of secondary particles

n_s	Number of showers	Number of secondary particle pairs	R	Δ
$n_s \leqslant 10$	40	1440	+0.017±0.039	−0.068±0.026
$10 < n_s \leqslant 25$	34	3701	+0.015±0.024	+0.023±0.016
$n_s > 25$	3	2302	+0.019±0.031	+0.014±0.021
Shower № 1 ($n_s = 37$; $\gamma_c = 12.6$)	1	666	+0.310±0.15	+0.006±0.04

actions, we have found an anomalous shower (No. 1 Table II) with a very anisotropic distribution w(ε) which is shown separately in Fig. 5. This interaction is clearly statistically different from the remaining 77 showers and, therefore, shower No. 1 is not included in the data of Tables I and II. The errors in the quantity R and a for shower No. 1 were determined according to a Monte Carlo calculation for a = 1 (R = 0.31).

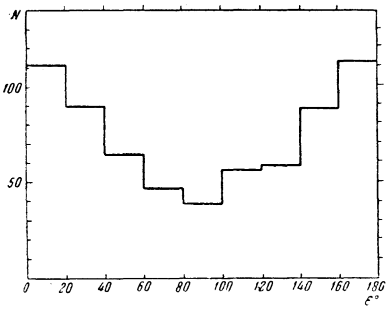

FIG. 5. The distribution w(ε) for shower No. 1.

c) The dependence of the distribution w(ε) on the shape of the angular distribution of secondary particles is illustrated in Fig. 6 and Table III which gives the values of the quantities R and Δ for showers with a two-center angular distribution. The latter were selected according to the shape of

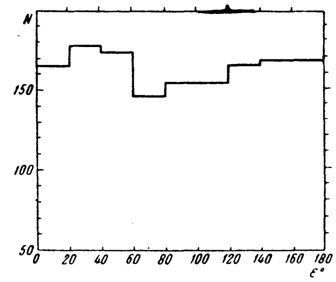

FIG. 6. The distribution w(ε) for showers with two-center angular distribution of secondary particles.

the angular distribution of shower particles in the variables $\ln[F/(1-F)] = f(\lambda)$,[5-7] where $\lambda = \ln \tan \vartheta$ and F is the integral angular distribution.

From the data given in Tables I—III and in Figs. 4 and 6, it is clear that the distribution w(ε) is almost isotropic for all types of interactions, excluding shower No. 1. A certain deviation from isotropy is revealed by two-center showers with a small number of secondary particles ($n_S \leq 10$) for which R = 0.180 ± 0.111; however, the statistical certainty of this conclusion is small. The values of a, characterizing the anisotropy of the azimuth angle distribution (5), calculated according to formula (8), are as follows:

Character of the showers	a
Two-center showers	$0.39^{+0.13}_{-0.21}$
Non-two-center showers	$0.17^{+0.13}_{-0.17}$
All showers	$0.23^{+0.10}_{-0.23}$
Shower No. 1	$0.99^{+0.21}_{-0.28}$

3. DISCUSSION OF RESULTS

The anisotropy of the azimuth angle distribution of secondary shower particles may possibly be explained by two models of multiple particle production; 1) the model of an intermediate excited state with a large angular momentum, and 2) the model of production of secondary particles from two excited centers (the fireball model).[8-11]

Table III. Values of R and Δ for two-center showers for various energies E_0 and numbers of secondary particles n_S

E_0, eV	n_s	Number of showers	Number of secondary particle pairs	R	Δ
10^{10}—10^{11}	Any n_s	3	239	−0.029±0.098	−0.038±0.065
10^{11}—10^{12}	Any n_s	5	275	+0.12±70.090	+0.029±0.060
$>10^{12}$	Any n_s	9	960	+0.146±0.048	+0.001±0.032
Any E_0	$n_s \leqslant 10$	5	183	+0.180±0.111	−0.049±0.074
Any E_0	$10 < n_s < 25$	12	1291	+0.030±0.042	+0.021±0.028
All two-center showers		17	1474	+0.049±0.039	+0.012±0.026

The pronounced anisotropy of the c.m.s. polar angle distribution of shower particles observed in the experiments (especially at $E_0 > 10^{12}$ eV)[1,2,13] can be attributed within the framework of the model of an intermediate excited state to a large angular momentum. It follows from such a model, however, that the shower particles should possess a similarly strongly anisotropic azimuth angle distribution. The absence of a considerable azimuth angle anisotropy of secondary particles in showers of all primary energies E_0, found in the present experiment, contradicts then the model of an intermediate excited state with large angular momentum.

Let us now consider the fireball model. According to this model, secondary particles are isotropically emitted by two excited centers in the rest system of each fireball. The model predicts a possible azimuth angle anisotropy in the angular distribution of secondary particles in the case where the two fireballs are emitted in directions differing by an angle α from the direction of motion of the primary particle. The degree of anisotropy increases with increasing α and with the velocity of motion of the fireballs. We have calculated the azimuth angle distributions $F(\varphi)$ of secondary particles emitted isotropically from the fireballs for different magnitudes of the angle α, of the Lorentz factor of the fireball in the c.m.s. and of the secondary particle momentum p. It was found that a change of the momentum p within the limits p = 0.2—2.0 BeV/c (in the rest system of the fireball) has little effect on the magnitude of the azimuth angle anisotropy. In calculating the distributions $W(\epsilon)$ for different values of the angle α and energy γ of the fireballs in the c.m.s., we have assumed therefore a mean value of the secondary particle momentum p = 0.5 BeV/c in the rest system of a fireball.

The values of R_{calc} characterizing the calculated distributions $W(\epsilon)$ of particles emitted from fireballs for different values of α and γ are given in Table IV. The values of γ were assumed equal to the c.m.s. Lorentz factors of the nucleon at l.s. energies $E_0 = 10^{10}$, 10^{11}, and 10^{12} eV. Evidently, γ depends on the degree of inelasticity of the collision, and cannot be directly correlated with the value of γ_c of the colliding nucleons. However, γ can be determined experimentally from the relation $\gamma \approx (\gamma_1 + \gamma_2)/2\sqrt{\gamma}$, where γ_1 and γ_2 are the Lorentz factors of the first and second fireballs, determined from the angular distribution in the variables $\ln[F/(1-F)]$ and λ for two-center showers. The mean value of γ for two-center showers given in Table III was found to be $\langle\gamma\rangle = 2.05$. The azimuth angle distribution of secondary particles emitted from the fireballs is furthermore characterized by the fact that the angular distribution of shower particles emitted from one fireball for $\alpha \neq 0$ is asymmetric about the angle $\varphi = \pi$. Small angles ϵ predominate therefore in the distribution $w_1(\epsilon)$ for one fireball, i.e., $\Delta_1 > 0$. The calculated values $\Delta_{1\,calc}$ which characterize the distributions $w_1(\epsilon)$ for different values of γ and α are also given in Table IV. The calculated values R_{calc} and $\Delta_{1\,calc}$ given in Table IV should be compared with the experimental values R and Δ_1 for two-center showers. The latter are given in Tables III and V. From the data of Table V it is clear that the experimental values $\Delta_{1\,exp}$ do not, within the limits of statistical errors, differ from zero: $\Delta_{1\,exp} = -0.031 \pm 0.039$ for all two-center showers ($\langle\gamma\rangle = 2.05$); $\Delta_{1\,exp} = 0 \pm 0.058$ for two-center showers with $\gamma > 1.9$ ($\langle\gamma\rangle = 2.7$) and $\Delta_{1\,exp} = 0.012 \pm 0.055$ for shower No. 1 ($\gamma = 2.56$). The values $\Delta_{1\,calc}$ calculated for $\gamma = 2.34$ which is close to the mean values $\langle\gamma\rangle = 2.05$ and $\langle\gamma\rangle = 2.7$ found for two-center showers in the experiment increase rapidly with increasing angle α, as can be seen in Table IV (thus, $\Delta_{1\,calc} = 0.107$ for $\alpha = 10°$). The comparison of the values $\Delta_{1\,exp}$ and $\Delta_{1\,calc}$ shows that the fireball model agrees with the azimuth angle distribution of secondary particles only if we assume that $\alpha \lesssim 5°$. This conclusion would be even more definite if we used all the data available at present on the angular distribution of secondary particles in two-center showers. The comparison of the values R and R_{calc} is in this respect less revealing because of the large error in the experimental value of R; the total value of R for all two-center showers is (see Fig. 3) R = 0.049 ± 0.039. This value of R, as follows from Table IV, also does not contradict

Table IV. Calculated values of R and Δ_1 for different values of γ and α

γ	α, deg	R_{calc}	$\Delta_{1\,calc}$
2.34	5	0.0002	0.028
	10	0.006	0.107
	30	0.148	0.493
7.11	5	0.032	0.251
	10	0.202	0.665
	30	0.815	1.000
22.4	5	0.463	0.928
	10	0.858	1.000
	30	1.000	1.000

Table V. Experimental values of R_1 and Δ_1, characterizing the distributions $w_1(\epsilon)$ for two-center shower particles emitted from one fireball ($\langle\gamma\rangle = 2.05$)

Shower energy E_0, eV	Number of showers	Number of secondary particle pairs	R_1	Δ_1
$10^{10}-10^{11}$	3	87	-0.126 ± 0.160	$+0.057\pm0.107$
$10^{11}-10^{12}$	5	115	$+0.200\pm0.130$	-0.009 ± 0.087
$>10^{12}$	9	446	$+0.029\pm0.071$	-0.054 ± 0.047
All two-center showers	17	648	$+0.039\pm0.058$	-0.031 ± 0.039
Shower No. 1	1	330	$+0.250\pm0.165$	$+0.012\pm0.055$
Two-center showers with $c\gamma>1.9$ ($\langle\gamma\rangle=2.7$)	9	294	$+0.027\pm0.087$	0 ± 0.058

the two-center model of shower production only for sufficiently small angles α.

It should be noted that the estimate of the angle α is of great interest since it permits us to determine experimentally the magnitude of the momentum transfer to the fireballs:

$$cp_{\perp} \approx M\gamma \sin\alpha,$$

where M is the fireball mass.

If we assume that

$$M = E_{\pi}\, 1.5\, n_s/2 \approx 12\cdot 0.5 \approx 6.0 \text{ BeV}$$

($1.5 n_s/2 \approx 12$ is the mean number of charged and neutral π mesons emitted from one fireball, and $E_\pi \approx 0.5$ BeV is the mean energy of π mesons in the rest mass of the fireball) then for $\langle\gamma\rangle = 2$ and $\alpha = 5$, $(pc)_{\perp} \approx 1$ BeV.

CONCLUSIONS

The following results have been obtained in the present investigation:

1. A method of investigating the azimuth angle distribution of secondary particles produced in multiple-production interactions has been proposed, consisting in studying the distribution $W(\epsilon)$ of the azimuth angles ϵ between particle pairs.

2. It has been shown that the distribution $W(\epsilon)$ of secondary particles produced in the interaction of 9 BeV protons with protons and nuclei is isotropic. From the isotropy of $W(\epsilon)$ there follows the isotropy of the azimuth angle distribution $F(\varphi)$. An exception are the events of production of a small number of secondary particles ($n_s = 2.4$) in pp collisions, where the function $F(\varphi)$, and consequently $W(\epsilon)$, are considerably affected by the momentum conservation law.

3. It has been shown that the distributions $W(\epsilon)$ of secondary particles in cosmic ray showers with energies $E_0 = 10^{10}-10^{13}$ eV are also almost isotropic. A relatively large anisotropy of the distribution $W(\epsilon)$ is found for so-called two-center showers, although the statistical certainty of such a conclusion is not great. The corresponding magnitudes of the coefficient a characterizing the azimuth angle distribution (5) for different showers are given in Sec. 2. The anisotropy $W(\epsilon)$ remains small, independently of the shower energy E_0 and the number of shower particles n_s.

4. Out of 78 analyzed cosmic-ray showers, one interaction (shower No. 1) has been found to possess a markedly anisotropic azimuth angle distribution. This interaction clearly differs from the remaining 77 showers. Shower number No. 1 is a two-center one.

5. The small anisotropy of the azimuth angle distribution of the secondary particles, and the accompanying pronounced anisotropy of the polar angle distribution in the c.m.s., contradict the model of secondary particle production via an intermediate excited state with large angular momentum.

6. The distribution $W_1(\epsilon)$ of pair azimuth angles of secondary particles emitted from one fireball of two-center showers was found to be isotropic. From the isotropy of $W_1(\epsilon)$ follows that the direction of motion of separate fireballs (if we assume that the model corresponds to reality) coincides in the c.m.s. to within 5° with the direction of motion of the colliding particles.

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