inside the crystal and the normal to the plane of the end face. Measurement of the diameters confirms this relation.

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A CERTAIN INEQUALITY FOR THE EFFECTIVE MASS IN METALS WITH LOW CARRIER DENSITIES

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In pure metals with low carrier densities the number of negative carriers is equal to the number of positive ones. This is a direct consequence of the law of charge conservation. At sufficiently low carrier densities the distances between carriers are very great and the interaction between them is essentially of Coulomb type. It is known, however, [1] that in the attractive-field potential, which decreases as $1/r$ at infinity, there are always energy levels corresponding to the bound states; consequently positive and negative charges recombine into electrically neutral entities which cannot carry the current.

At high carrier densities the potential of each charge is screened by the Debye-Hückel cloud and is of short-range type. When the screening radius is sufficiently small there are no bound states at all in the corresponding potential well and transport of charge is thus possible. These ideas were first put forward by Mott. [2]

The order-of-magnitude condition which determines the importance of the Debye-Hückel screening is obtained by equating the mean distance between carriers ($\simeq N^{-1/3}$, where N is the number of carriers per cm^3) to the radius of the Debye-Hückel cloud r_D:

$$N^{-1/3} \sim r_D.$$

The value of r_D can easily be found from the relationship

$$\varkappa^2 = r_D^{-2} = 4\pi e^2 \sum_a \partial N_a / \partial \mu_a \sim e^2 \partial N / \partial \mu.$$

The chemical potential is $\mu \simeq \hbar^2 N^{2/3}/m^*$ (m^* is the mean effective carrier mass) and its derivative is

$$\partial\mu/\partial N \sim \hbar^2/m^* N^{1/3}.$$

Now we can easily write down the condition for metallic conduction:[1)] $N^{1/3} \gtrsim m^* e^2/\hbar^2$, i.e., at a given carrier density the effective mass m^* should satisfy the inequality

$$m^* \lesssim (\hbar/e)^2 N^{1/3}.$$

It is convenient to divide the above expression by the free-electron mass m. Allowing for the fact that the lattice parameter a is of the same order of magnitude as the Bohr radius $\hbar^2/me^2$, we can write

$$m^*/m \lesssim n^{1/3}, \qquad (1)$$

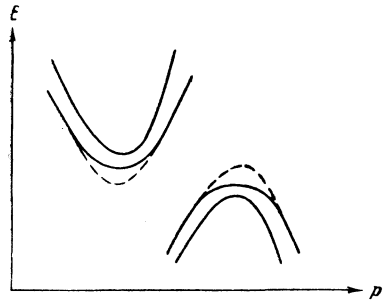
where $n = Na^3$ is the density of carriers per atom.

The formula obtained indicates that metals in which the effective carrier mass is of the order of the free electron mass should always have high carrier density. [3] Metals with low carrier density should always have small effective carrier mass.

In the case of graphite and bismuth the carrier density is $n \simeq 10^{-5}$ and the effective carrier masses are of the order of several hundredths of the free electron mass. Thus our relationship is fully satisfied qualitatively. Since graphite and bismuth have very complex and anisotropic electron spectra, we can expect only very rough quantitative agreement.

The inequality derived here is a selection rule. Materials in which the effective carrier masses do not satisfy the inequality cannot be metals with low carrier density.

It is necessary to make the following point. Let us assume that a nonmetallic crystal with a known electron spectrum is deformed so that the gap between the valence and conduction bands decreases. When the band overlap is small we should have a metal with low carrier density. However, the theorem proposed here indicates that band overlap is



prevented by formation of bound states. Then obviously the dependences of the energy on the quasi-momentum should be deformed in the way indicated in the adjoining figure: near the maximum and the minimum the curves are "flattened," i.e., the effective mass $m^* \simeq (\partial^2 E/\partial p^2)^{-1}$ increases owing to the effect considered here. This impedes the band overlap and the transition to metallic state with low carrier density.

We can assume that metals with low carrier densities can only have structures in which contact or overlapping of the valence and conduction bands are due to degeneracy governed by the crystal symmetry. Removal of the degeneracy, due to the Jahn-Teller effect, occurs with a weak distortion of the degenerate structure and formation of a metal with low carrier densities. Consideration of the crystal structure of known metals with low carrier density (graphite, bismuth, arsenic, antimony) indicates that they all have structures which are weak distortions of more symmetrical structures. The author plans later to publish an analysis of this effect.

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[1)] This formula was given by Mott,[3] but he used the free-electron mass instead of m*.

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CONCERNING ONE POSSIBILITY OF AMPLIFICATION OF LIGHT WAVES

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THE subject of the present note is a discussion of certain possibilities of amplification and generation of light waves in optically transparent crystals, the polarization of which depends in nonlinear fashion on the intensity of the electric field of the propagating wave. A nonlinearity of this type (it can obviously be regarded as the dependence of the dielectric constant on the field) was successfully utilized in several recently described experiments (see [1-3]) on the generation of optical harmonics. Naturally, this does not exhaust the possible nonlinear effects in such crystals. We show below that under certain conditions, in an optically transparent medium whose polarization depends quadratically on the intensity of the electric field, one can obtain parametric amplification of traveling light waves, obtained at the expense of the energy of an intense light wave (the so-called pumping) and that the condition for parametric amplification can be realized in uniaxial crystals.

As is well known (see, for example, [4,5] and also the review [6]), in the region of the fundamental parametric resonance the energy of the intense pumping oscillations of frequency ω_p, carrying out the modulation of the reactive parameters of a resonance circuit or of a transmission line, can be transferred to oscillations at frequencies ω_1 and ω_2, satisfying the condition

$$\omega_p = \omega_1 + \omega_2 \qquad (1)$$

(the particular case when $\omega_1 = \omega_2 = \omega_p/2$ is the so-called "degenerate" parametric interaction). To clarify the features of such an interaction space, it is necessary to consider a semi-bounded medium, the dielectric constant of which varies as[1)]

$$\varepsilon(t, x, \omega) = \varepsilon_0(\omega)\{1 + m[e^{i(\omega_p t - k_p x)} + e^{-i(\omega_p t - k_p x)}]\} \qquad (2)$$

(the x axis is perpendicular to the separation boundary).

Assume that the waves at frequencies ω_1 and ω_2 have components $E_y = E$; H_x; H; H_z, and assume that their wave vectors make angles θ_1 and θ_2 with the x axis. The electric field in the medium