SOVIET PHYSICS JETP VOLUME 16, NUMBER 1 JANUARY, 1963

ON THE THEORY OF THE OPTICAL POTENTIAL

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Submitted to JETP editor March 4, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 319-326 (July, 1962)

Scattering of nucleons on density fluctuations of nuclear matter within the nucleus is considered. The scattering coefficient defined by the imaginary part of the optical potential is expressed in terms of the nucleon-nucleon scattering amplitude and the spectral distribution of the space-time correlation function for the nuclear matter density fluctuations. As examples, the following nuclear models are considered: degenerate Fermi gas, superconducting nucleon gas, and a Fermi liquid. It is shown that the presence of a gap in the density fluctuation spectrum of even-even nuclei leads to a decrease in the imaginary part of the optical potential of even-even nuclei, compared with the imaginary part of the optical potential of odd nuclei.

1. It is known that the optical potential, which describes the refraction and absorption of a nucleon wave within the nucleus, can be determined by assuming the refraction and absorption to be brought about by multiple scattering which occurs as a result of interaction between the wave and individual nucleons of the nucleus. The optical potential is expressed in this case in terms of the scattering amplitude of the nucleon wave by individual nucleons and in terms of the nucleon distribution density in the nucleus. Neglecting the contribution of many-particle forces, the optical potential for a nucleon with momentum k inside the nucleus can be represented in the form [1]

$$U(r) = -\frac{1}{2\pi^2 M}\int d\mathbf{k}' n(\mathbf{k}' - \mathbf{k}) f(\mathbf{k}, \mathbf{k}') e^{i(\mathbf{k}'-\mathbf{k})\mathbf{r}}, \qquad (1)$$

where $f(\mathbf{k}, \mathbf{k}')$ is the nucleon-nucleon scattering amplitude outside the energy surface and $n(q)$ is the Fourier component of the nucleon density in the nucleus:

$$n(q) = \int d\mathbf{r}\, n(r) e^{-i\mathbf{q}\mathbf{r}}.$$

Let us consider the motion of a nucleon within the nucleus, assuming the latter to be sufficiently large ($A \gg 1$) and neglecting surface effects. Assuming also uniform distribution of the nuclear matter, $n(r) = n_0$, we obtain from (1) for the optical potential the expression

$$U_0 = -4\pi n_0 f(0)/M \qquad (2)$$

[$f(0)$ is the amplitude of elastic scattering through zero angle]. The wave function describing the state of the nucleon inside a nucleus with definite momentum has in this case the form of a plane wave

$$\Psi_{\mathbf{k}}(\mathbf{r}, t) = e^{i(\mathbf{k}\mathbf{r}-Et)}, \qquad E = k^2/2M - U_0. \qquad (3)$$

Writing down the potential (2) in the form $U_0 = -(V_0 + iW_0)$ we have for the refraction potential V_0 and the absorption potential W_0

$$V_0 = \frac{4\pi n_0}{M}\operatorname{Re} f(0), \qquad W_0 = \frac{4\pi n_0}{M}\operatorname{Im} f(0). \qquad (4)$$

The imaginary part of the optical potential can be determined directly by considering the scattering of a nucleon wave on random density fluctuations of the nuclear matter. Indeed, as a result of random density fluctuations of the nuclear matter, scattering similar to Rayleigh scattering of light [2] is possible when the nucleon moves in the nucleus. This effect of scattering of the nucleon waves can be evaluated by substituting in (1) the density n in the form $n = n_0 + \delta n$, where δn are the random density fluctuations, which depend both on the coordinates and on the time.

Assuming that $|\delta n| \ll n_0$, we can easily calculate the scattering of the nucleons on the density fluctuations by using perturbation theory. By statistical averaging over the fluctuations we ultimately obtain the following expression for the differential cross section for the scattering of nucleons, referred to unit volume (differential scattering coefficient):

$$d\Sigma = (2/\pi M k)\,|f(\mathbf{k}, \mathbf{k}')|^2 \langle \delta n^2 \rangle_{q\omega}\, d\mathbf{k}'. \qquad (5)$$

Here $\mathbf{k}'$ and E' are the momentum and the energy of the nucleus after scattering, $\mathbf{q} = \mathbf{k}' - \mathbf{k}$, $\omega = E' - E$, and $\langle \delta n^2 \rangle_{q\omega}$ is the spectral distribution of the space-time correlation function of the density fluctuations of nuclear matter:

$$\langle \delta n^2 \rangle_{q\omega} \equiv \int d\mathbf{r}\, dt e^{-i\mathbf{q}(\mathbf{r}-\mathbf{r}')+i\omega(t-t')} \langle \delta n(\mathbf{r}, t)\, \delta n(\mathbf{r}', t') \rangle. \quad (6)$$

Integrating (5) with respect to $d\mathbf{k}'$, we can determine the total coefficient Σ of the scattering of the nucleon in the nuclear matter. This scattering coefficient Σ can be related to the imaginary part of the optical potential W, using the well known relation

$$W = k\Sigma/M. \quad (7)$$

We thus have

$$W = \frac{2}{\pi M^2} \int d\mathbf{k}' \, |f(\mathbf{k}, \mathbf{k}')|^2 \langle \delta n^2 \rangle_{q\omega}. \quad (8)$$

This formula determines in general form the imaginary part of the optical potential W from the known nucleon-nucleon scattering amplitude $f(\mathbf{k}, \mathbf{k}')$ and the density fluctuations $\langle \delta n^2 \rangle_q$ of the nucleons in the nucleus. Unlike (4), formula (8) makes it possible to take into account the dependence of the optical potential both on the Pauli principle, which governs the nucleons within the nucleus, and on the interaction between the nucleons in the nucleus.

The real part of the optical potential, which takes into account the Pauli principle and the interaction between nucleons, can be determined from the imaginary part (8) by using the Kramers-Kronig relation for the refractive index of the nucleon wave in nuclear matter $N^2(E) = 1 - U(E)/E$. This relation gives

$$V(E) = \frac{2}{\pi} \int_0^\infty \frac{EW(E')}{E'^2 - E^2} dE'. \quad (9)$$

2. Let us determine the scattering coefficient of the nucleon in the nuclear matter, Σ, using by way of a nuclear model a degenerate Fermi gas at zero temperature. The spectral distribution of the density fluctuations in an ideal Fermi gas is determined by the following general formula

$$\langle \delta n^2 \rangle_{q\omega} = \frac{1}{2\pi^2} \int d\mathbf{p}\, n_{\mathbf{p}} (1 - n_{\mathbf{p}-\mathbf{q}})\, \delta(\omega - E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{q}}), \quad (10)$$

where $n_{\mathbf{p}}$ and $n_{\mathbf{p}-\mathbf{q}}$ are the Fermi distribution function ($E_{\mathbf{p}}$ is the energy of a particle with momentum $\mathbf{p}$). The integration in (10) can be readily carried out if the Fermi-gas temperature is zero. In this case we have

$$\langle \delta n^2 \rangle_{q\omega} = \frac{3\pi}{4} \frac{n_0}{E_F} \begin{cases} \dfrac{k_F}{q} \dfrac{|\omega|}{E_F}, & -\dfrac{q}{k_F}\left(2 - \dfrac{q}{k_F}\right) < \dfrac{\omega}{E_F} < 0 \\[2mm] \dfrac{k_F}{q}\left[1 - \dfrac{1}{4}\left(\dfrac{q}{k_F} + \dfrac{k_F}{q}\dfrac{\omega}{E_F}\right)^2\right], & -\dfrac{q}{k_F}\left(\dfrac{q}{k_F} + 2\right) < \dfrac{\omega}{E_F} < -\dfrac{q}{k_F}\left|\dfrac{q}{k_F} - 2\right| \end{cases}; \quad (11)$$

E_F is the limiting Fermi energy ($E_F = k_F^2/2M$).

We assume for simplicity that the nucleon-nucleon scattering amplitude is constant ($f(k, k') = f_0$), and then the total scattering coefficient Σ can be represented in the form

$$\Sigma = 4\pi n_0 |f_0|^2 P, \quad (12)$$

where P is a dimensionless coefficient characterizing the fluctuating properties of the nuclear matter:

$$P = \frac{3}{4Mkk_F^3} \int_{|\mathbf{k}+\mathbf{q}|>k_F} d\mathbf{q} \langle \delta n^2 \rangle_{q\omega}. \quad (13)$$

Noting that $\omega = (q^2 + 2\mathbf{k}\cdot\mathbf{q})/2M$, we can reduce the integration with respect to $d\mathbf{q}$ in (13) to integration with respect to dq and $d\omega$:

$$P = \frac{3\pi}{2k^2k_F^3} \int_{-(k^2-k_F^2)/2M}^{0} d\omega \int_{k-\sqrt{k^2+2M\omega}}^{k+\sqrt{k^2+2M\omega}} dq\, q \langle \delta n^2 \rangle_{q\omega}. \quad (14)$$

Using (11), we obtain for a degenerate Fermi gas at zero temperature

$$P(\varepsilon) = \begin{cases} 1 - 7/5\varepsilon + 2(2-\varepsilon)^{5/2}/5\varepsilon, & \varepsilon < 2 \\ 1 - 7/5\varepsilon, & \varepsilon > 2 \end{cases}, \quad (15)$$

where ϵ is the ratio of the energy of the incident nucleon to the limiting Fermi energy ($\epsilon = E/E_F$). Expression (15) characterizes the reduction in the scattering coefficient of the nucleon in nuclear matter, due to the influence of the Pauli principle.

The differential nucleon scattering cross section is given by formula

$$\frac{1}{\Sigma} \frac{d\Sigma}{d\varepsilon' do} = \frac{3}{4\pi P(\varepsilon)} \sqrt{\frac{\varepsilon'}{\varepsilon}}\, \varphi(\varepsilon', \vartheta);$$

$$\varphi(\varepsilon', \vartheta) = \frac{1}{2\sqrt{(A\varepsilon, \varepsilon')}} \times \begin{cases} \varepsilon - \varepsilon', & A(\varepsilon, \varepsilon') > B(\varepsilon, \varepsilon') \\ 1 - B(\varepsilon', \varepsilon)/A(\varepsilon, \varepsilon'), & B(\varepsilon, \varepsilon') > A(\varepsilon, \varepsilon') > B(\varepsilon', \varepsilon), \end{cases}$$

$$A(\varepsilon, \varepsilon') = \varepsilon + \varepsilon' - 2\sqrt{\varepsilon\varepsilon'} \cos\vartheta,$$

$$B(\varepsilon, \varepsilon') = \varepsilon(\sqrt{\varepsilon} - \sqrt{\varepsilon'} \cos\vartheta)^2, \quad (16)$$

where ϑ is the scattering angle of the nucleon and $\epsilon' = E'/E_F$.

Formulas (15) and (16) agree with the results of Goldberger [3] and Hayakawa, Kawai, and Kikuchi [4], who started out from the microscopic picture of successive collisions between the incoming nucleon and the Fermi-distributed intranuclear nucleons.

Substituting (12) in (7) and using the optical theorem

$$\int do\,|f_0|^2 = \frac{4\pi}{k}\,\mathrm{Im}\,f(0),$$

we obtain for the imaginary part W of the optical potential

$$W = (4\pi n_0/M)\ \mathrm{Im}\,f(0)\,P(\varepsilon). \qquad (17)$$

This formula differs from (4) for W_0 in the presence of a factor $P(\epsilon)$, which takes the Pauli principle into account.

Using (8) we obtain for the real part V of the optical potential, unlike (4), the following expression

$$V = (4\pi n_0/M)\ \mathrm{Re}\,f(0)\,R(\varepsilon),$$

$$R(\varepsilon) = \frac{1}{\pi}\Big\{\ln\frac{\varepsilon+1}{\varepsilon-1} + \frac{7}{5\varepsilon}\ln(\varepsilon^2-1) + \frac{2}{5\varepsilon}\Big[(2+\varepsilon)^{5/2}\ln\frac{\sqrt{2+\varepsilon}+1}{\sqrt{2+\varepsilon}-1} - 8\sqrt{2}\ln\frac{\sqrt{2}+1}{\sqrt{2}-1} - 4\varepsilon^2\Big]\Big\} + \frac{4}{5\pi\varepsilon}\begin{cases}{}^1/_2\,(2-\varepsilon)^{5/2}\ln[(1+\sqrt{2-\varepsilon})/(1-\sqrt{2-\varepsilon})], & \varepsilon<2\\ (\varepsilon-2)^{5/2}\,\mathrm{arc\,tg}\,[\sqrt{\varepsilon-2}]^{-1}, & \varepsilon>2\end{cases}. \qquad (18)^*$$

Figure 1 shows the dependence of the coefficients P and R on the energy of the incoming nucleon $\epsilon = E/E_F$.

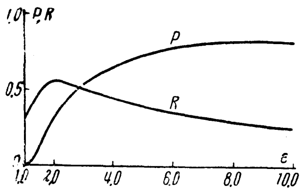

FIG. 1

3. We now evaluate the influence of the interaction between nucleons on the magnitude of the scattering coefficient Σ. As is well known[5,6], the presence of short-range pair interaction between nucleons leads to the possibility of formation of a superconducting state in the nuclear matter, characterized by a gap in the energy spectrum. Such a gap appears in the energy spectra of even-even nuclei and not in odd nuclei. Since the presence of a gap manifests itself in the spectral distribution of the nuclear-density fluctuations, the scattering coefficients (and consequently also the optical potentials) should, according to (13), be different for even-even and for odd nuclei.

The spectral distribution of the density fluctuations of the nucleon gas in the superconducting state at zero temperature is determined by the expression

$$\langle\delta n^2\rangle_{q\omega} = \frac{1}{8\pi^2}\int d\mathbf{p}\,\{1 - (\xi_{\mathbf{p}}\xi_{\mathbf{p-q}} - \Delta^2)/\widetilde{E}_{\mathbf{p}}\widetilde{E}_{\mathbf{p-q}}\} \times \delta(\omega + \widetilde{E}_{\mathbf{p}} + \widetilde{E}_{\mathbf{p-q}}), \qquad (19)$$

where $\widetilde{E}_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2 + \Delta^2}$ is the energy of a quasi particle with momentum $\mathbf{p}$, $\xi_{\mathbf{p}}$ is the nucleon energy reckoned from the Fermi surface, and Δ is the width of the gap in the quasi-particle spectrum.

Since the energy of the quasi-particle always exceeds the width of the gap Δ, the density fluctuation spectrum will, according to (19), contain only frequencies $\omega < -2\Delta$. Consequently integration in formula (14), which determines the scattering coefficient for nuclear matter in the superconducting state, should be carried out in practice only in the interval of frequencies $-(k^2 - k_F^2)/2M < \omega < -2\Delta$.

Using formula (19) for the spectral distribution of the fluctuations, one cannot separate the factor P characterizing the scattering coefficient in explicit form. The influence of the gap in the spectrum on the scattering coefficient Σ can be accounted for approximately by using formula (11) for the spectral distribution of the fluctuations, but confining oneself to integration in (14) only over the frequency range $-(k^2 - k_F^2)/2M < \omega < -2\Delta$. We thus obtain for the factor P in the scattering coefficient Σ, in place of (17), the following expression

$$P(\varepsilon,\delta) = \begin{cases}1 - \varepsilon^{-1}[1 + \delta + {}^2/_5\,(1-\delta)^{5/2}] + 2\,(2-\varepsilon)^{5/2}/5\varepsilon, & 1+\delta<\varepsilon<2,\\ 1 - \varepsilon^{-1}[1 + \delta + {}^2/_5\,(1-\delta)^{5/2}], & \varepsilon>2,\end{cases} \qquad (20)$$

where δ is the ratio of double the gap width to the limiting Fermi energy ($\delta = 2\Delta/E_F$).

Comparing (20) with (15) we see that the presence of a gap in the spectrum of the nucleus leads to a reduction in the scattering coefficient Σ, and consequently also in the imaginary part of the optical potential W. The relative decrease in the imaginary part of the optical potential is

$$\delta W/W = [\delta + {}^2/_5\,(1-\delta)^{5/2} - {}^2/_5]/\varepsilon P(\varepsilon). \qquad (21)$$

Putting $2\Delta = 3$ MeV, $E_F = 33$ MeV, and $V_0 = 40$ MeV at incoming-nucleon energies of 1, 2, and 5 MeV (ϵ = 1.24, 1.27, and 1.36) we obtain for δW/W values of 0.25, 0.15, and 0.1.

Figure 2 shows the dependence of the coefficient P on the energy ϵ at different values of δ.

*arctg = $\tan^{-1}$.

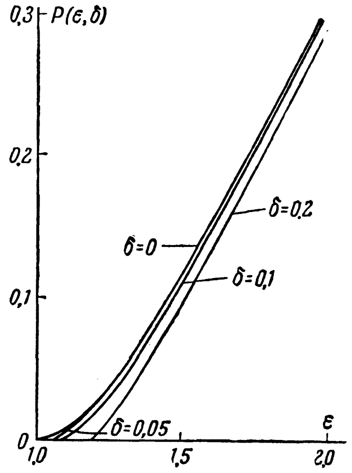

FIG. 2

We note that in accordance with (20) the difference between the imaginary parts of the even-even and odd nuclei is significant only at low incoming-nucleon energies.

Evidence that a difference exists between the imaginary parts of the optical potentials of even-even and odd nuclei is offered by Klyucharev's [7] data on the scattering of protons by separated isotopes. Thus, in those cases when calculations in the optical model in [7] are in good agreement with the experimental data, the imaginary parts of the optical potentials of even-even nuclei turn out to be somewhat smaller than the imaginary parts of the optical potentials of odd nuclei. For example, at incoming-proton energy 5.4 MeV, the imaginary parts of the optical potentials for the nuclei Cu^{65} and Ni^{64} are, in accord with [7], 6 and 5.5 MeV, respectively. These values agree with (21), although the quantitative agreement should be regarded as rather accidental, in view of the far-reaching simplifying assumptions made in the derivation of (21).

4. So far we have started from the assumption that $f(\mathbf{k}, \mathbf{k}')$ is constant. Actually, the nucleon-nucleon scattering amplitude is dependent on the energy, and this dependence explains in particular the reduction of the imaginary part of the optical potential with increasing energy of the incoming nucleon after the potential reaches a certain maximum value.

If $v(r)$ is the potential of the pair interaction of the nucleons, then the scattering amplitude outside the energy surface is given by the expression

$$f(\mathbf{k}, \mathbf{k}') = -\frac{M}{4\pi}\int d\mathbf{r} e^{-i\mathbf{k}'\mathbf{r}} v(r) \psi_{\mathbf{k}}(\mathbf{r}), \tag{22}$$

where the function $\psi_{\mathbf{k}}$ is the solution of the equation

$$\{-M^{-1}\Delta + v(r) - k^2/M\}\psi_{\mathbf{k}}(\mathbf{r}) = 0.$$

(We note that in (22), generally speaking, $k \neq k'$.)

Describing the interaction between the incoming nucleon and the nucleon of the nucleus by the nonlocal Yamaguchi potential [8], 1)

$$(\mathbf{p}|v|\mathbf{p}') = -\frac{\lambda}{M}\frac{1}{\beta^2+p^2}\frac{1}{\beta^2+p'^2}, \tag{23}$$

where λ and β are parameters that depend on the spin state of the nucleons, we can find the scattering amplitude outside the energy surface in explicit form

$$f(\mathbf{k}, \mathbf{k}') = \frac{\beta^2+k^2}{\beta^2+k'^2}\left\{-ik + \left[\frac{(\beta^2+k^2)^2}{2\pi^2\lambda} + \frac{\beta^2+k^2}{2\beta} - \beta\right]\right\}^{-1}. \tag{24}$$

With the aid of this expression we can readily calculate, in accordance with (8), the imaginary part of the optical potential W. Using formula (11) for the spectral distribution of the density fluctuations and ascribing to the triplet and singlet states equal weights, $3/2$, we obtain ultimately the following expression for the imaginary part of the optical potential:

$$\frac{W}{E_F} = \frac{8}{\pi}\left\{\left(\varepsilon + \frac{1}{4}\left[\frac{\varepsilon-\varkappa}{\sqrt{\varkappa}} + \frac{(\varepsilon+\varkappa)^2}{2\pi^2\Lambda_t}\right]^2\right)^{-1} + \left(\varepsilon + \frac{1}{4}\left[\frac{\varepsilon-\varkappa}{\sqrt{\varkappa}} + \frac{(\varepsilon+\varkappa)^2}{2\pi^2\Lambda_s}\right]^2\right)^{-1}\right\}\sqrt{\varepsilon}P(\varepsilon);$$

$$\begin{aligned} P(\varepsilon) = \frac{(\varepsilon+\varkappa)^2}{\varepsilon}\Big\{&\frac{1}{1+\varkappa} - \frac{1}{\varepsilon+\varkappa} \\ &+ \frac{1}{\varepsilon+\varkappa-1}\Big[4 + 3(\varepsilon+\varkappa-1)^{3/2}\operatorname{arc\,tg}\frac{1}{\sqrt{\varepsilon+\varkappa-1}} \\ &- 3(\varepsilon+\varkappa) - \frac{1}{1+\varkappa}\Big] \\ &+ \frac{1}{\varepsilon+\varkappa-1}\Big[\frac{(2-\varepsilon)^{5/2}}{1+\varkappa} + (4\varepsilon+3\varkappa-5)\sqrt{2-\varepsilon} \\ &- 3(\varepsilon+\varkappa-1)^{3/2}\operatorname{arc\,tg}\frac{\sqrt{2-\varepsilon}}{\sqrt{\varepsilon+\varkappa-1}}\Big]\theta(2-\varepsilon)\Big\}, \end{aligned} \tag{25}$$

where $\theta(x)$ is the Heaviside function ($\theta = 1$ when $x > 0$ and $\theta = 0$ when $x < 0$); $\kappa = \beta^2/k_F^2$ ($\beta = \beta_t = \beta_s$), $\Lambda_t = \lambda_t/k_F^S$ and $\Lambda_s = \lambda_s/k_F^S$.

On the left side of Fig. 3 is shown the dependence of the imaginary part of the optical potential W on the energy ϵ. The values of the parameters β, λ_t, and λ_s are chosen in accord with [8] as follows: $\beta = 1.45 \times 10^{13}$ cm^{-1}, $\lambda_t = 0.41 \times 10^{39}$ cm^{-3}, and $\lambda_s = 0.29 \times 10^{39}$ cm^{-3}; $E_F = 33.4$ MeV.) In the right half of Fig. 3 is shown the dependence of the

1)The nonlocal interaction between nucleons was first used to determine the optical potential by Verlet and Gavoret.[9] The interaction between nucleons in states with $l \neq 0$ is the subject of a paper by Dabrowski and Sobiczewski.[10]

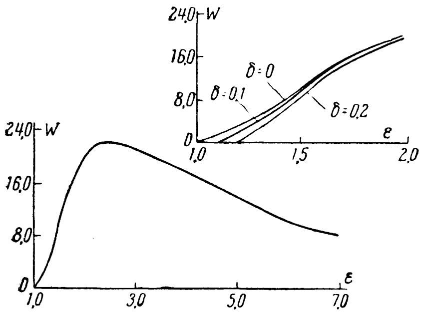

FIG. 3

imaginary part W of the optical potential on the magnitude of the gap δ in the density fluctuation spectrum for the superconducting state of nuclear matter.

The differential scattering cross section of the nucleons is determined in this case by the expression

$$\frac{1}{\Sigma}\frac{d\Sigma}{d\varepsilon'\,do} = \frac{3}{4\pi P(\varepsilon)}\sqrt{\frac{\varepsilon'}{\varepsilon}}\left(\frac{\varkappa+\varepsilon}{\varkappa+\varepsilon'}\right)^2 \varphi(\varepsilon', \vartheta). \tag{26}$$

Figure 4 shows the energy distribution of the scattered nucleons at different values of the scattering angle for an incoming nucleon energy $\epsilon = 2$.

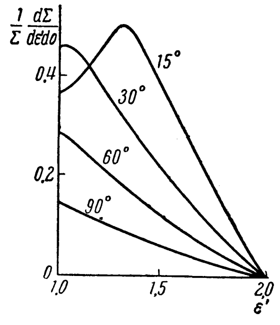

FIG. 4

5. Account of the long-range interaction between nucleons also leads to a change in the character of the spectral distribution of the density fluctuations. In particular, the presence of such an interaction causes additional maxima, corresponding to the natural frequencies of the collective degrees of freedom of the nucleus, to appear in the density fluctuation spectrum.

By way of a very simple example, which takes into account such collective degrees of freedom, one can use for the nucleus the Fermi-liquid model. The theory of the Fermi liquid has been developed in the papers of Landau [11,12]. According to Abrikosov and Khalatnikov [13], we have for the spectral distribution of the correlation function of the density fluctuations of a Fermi liquid at $q \ll k_F$

$$\langle \delta n^2 \rangle_{q\omega} = \frac{3\pi}{2}\frac{n_0}{E_F}\left\{\frac{u\theta(1-u)}{\left[1 + F_0\left(1 - \frac{u}{2}\ln\frac{1+u}{1-u}\right)\right]^2 + \frac{\pi^2}{4}F_0^2u^2} + \frac{2\eta(\eta^2-1)}{F_0(F_0+1-\eta^2)}\delta(u-\eta)\right\},$$

$$u = \frac{1}{2}\frac{k_F}{q}\frac{|\omega|}{E_F}, \tag{27}$$

where F_0 is a dimensionless amplitude, characterizing the interaction energy of the Fermi-liquid quasi-particles, and η is the root of the dispersion equation for zero sound

$$1 + F_0\left\{1 - \frac{\eta}{2}\ln\left|\frac{1+\eta}{1-\eta}\right|\right\} = 0. \tag{28}$$

The second term in (27) characterizes the density fluctuations connected with the possibility of existence of collective motion in the nuclear matter (zero sound). The relative weight of these fluctuations increases with increasing F_0. Figure 5 shows the dependence of the relative weight

$$g(F_0) = \int_1^\infty \langle \delta n^2 \rangle_{q\omega}\, du \Big/ \int_0^\infty \langle \delta n^2 \rangle_{q\omega}\, du$$

on the value of F_0. Thus, even when $F_0 \sim 2$ the relative weight of the collective fluctuations is already approximately four times larger than the weight of the low-frequency fluctuations ($u < 1$).

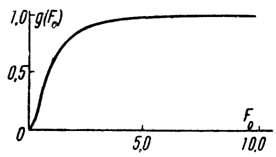

FIG. 5

Using (27), we obtain for the differential cross section of the scattering of the nucleon on the density fluctuations of the Fermi liquid[2)]

$$d\Sigma = 3n_0|f(\mathbf{k}, \mathbf{k}')|^2\left\{\frac{u\theta(1-u)}{[1 + F_0(1 - {}^1\!/\!_2\, u \ln((1+u)/(1-u))]^2 + {}^1\!/\!_4\,\pi^2F_0^2u^2} + \frac{2\eta(\eta^2-1)}{F_0(F_0+1-\eta^2)}\delta(u-\eta)\right\}\sqrt{\frac{\varepsilon'}{\varepsilon}}\,d\varepsilon'\,do. \tag{29}$$

We note that when a nucleon is scattered in nuclear matter and zero sound is excited, the scattering angle of the nucleon is a single-valued function of the energy transfer. The presence of such a connection can make it possible in principle to observe scattering with excitation of collective degrees of freedom in nuclei.

An account of the attenuation of the zero-sound oscillations reduces to replacing $\delta(u - \eta)$ in (29)

2)The scattering of neutrons on density fluctuations in liquid helium was considered by A. Akhiezer, I. Akhiezer, and Pomeranchuk.[14]

by $\pi^{-1}\gamma/[(u-\eta)^2+\gamma^2]$, where γ is the damping coefficient of zero sound[14].

A study of the continuous spectrum of nucleons arising upon interaction between the nucleons and nuclei can yield information on the interaction between the nucleons in nuclei, particularly on the collective degrees of freedom of nuclei. One must point out, however, that the results obtained merely serve as a model, since the finite dimensions of the nucleus were not taken into consideration in the calculations.

I am sincerely grateful to A. I. Akhiezer and A. B. Migdal for a useful discussion of many problems, and also to V. K. Tartakovskiĭ for carrying out the numerical calculations.

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Translated by J. G. Adashko
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