PROPERTIES OF THE GROUND AND EXCITED STATES OF STRONGLY DEFORMED NUCLEI

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Calculations of the energies of even-even nuclei and of the β-transition probabilities for even-A and odd-A nuclei in the ranges $154 \leq A \leq 188$ and $225 \leq A \leq 255$ are presented. The superfluid model of the nucleus was used, and properties of the ground and excited states of strongly deformed nuclei derived from this model are discussed. Experiments are suggested, with indications of the most favorable conditions for revealing three- and four-particles states populated via the relevant β transitions. It is pointed out that an experimental verification of all levels of even-even nuclei predicted by the model is necessary. It is shown that an experimental determination of the degree of F-forbiddenness of β decays will indicate the effect that residual forces neglected in the superfluid model have on the properties of the ground and excited states. All strongly deformed even-A nuclei are analyzed and the cases most favorable for determining the degree of F-forbiddenness of β transitions are indicated.

1. INTRODUCTION

THE superfluid model of the nucleus [2] has been applied successfully to the investigation of the ground- and excited-state properties of strongly deformed nuclei. [1] This model, which is based on the independent-particle model, takes into account the short-range portion of residual nucleon-nucleon forces within nuclei. The model has been used to calculate two-quasi-particle energy levels of even-even nuclei and the relative values of $\log ft_r$ for β transitions in even-A and odd-A nuclei in the ranges $154 \leq A \leq 188$ [3-5] and $225 \leq A \leq 255$. [6] All parameters used in these calculations were determined from experimental data on the single-particle levels of odd-mass nuclei and from pairing energies determined through mass differences. No new parameter was used in calculating the properties of even-even nuclei, since all parameters had been determined in the investigation of odd-mass nuclei. Therefore the study of even-even nuclei is especially valuable for testing the fundamental assumptions of the superfluid model.

We shall first compute the number of parameters used to calculate nuclear properties in the ranges $154 \leq A \leq 188$ and $225 \leq A \leq 255$. Corrected levels of Nilsson's scheme [7] were taken as average-field levels; these were determined from 20 parameters characterizing both Nilsson's scheme and its modifications. From a comparison with the experimental pairing energies it was found [6,8] that the neutron and proton pairing interaction constants G_N and G_Z obey a $1/A$ law in both ranges of strongly deformed nuclei:

$$G_N = 26A^{-1}\,\text{MeV}, \qquad G_Z = 29A^{-1}\,\text{MeV}. \tag{1}$$

The calculations employed 22 parameters, which were determined from 58 pairing energies and 205 separate data on the ground and excited states of odd-mass nuclei. Thus values of 22 free parameters were established to account roughly for 263 experimental facts concerning single-particle levels of odd-mass nuclei and pairing energies.

Values of the pairing interaction constants (1) were obtained by summing over 36 average-field levels in the basic equations of the model. It has been shown [6,8] that the superfluid properties of a system do not depend on a cutoff at energies greater than 3—5 MeV both above and below the K level. It is unnecessary to introduce a cutoff constant to limit summations in the basic equations of the model, since the pairing interaction constant G is normalized with this cutoff taken into account.

The energy difference between ground and excited two-quasi-particle states was calculated with an error of the order 10%; in the most unfavorable cases the error does not exceed 20%. The error results from, first, insufficient knowledge of the behavior of average-field levels and because of their fluctuations from nucleus to nucleus, and, secondly, inaccuracy of the mathematical method. The introduction of experimental data on the levels of odd-mass nuclei diminished the errors associated with the behavior of average-field levels. However, we intentionally neglected the fluctuations of average-field levels for different nuclei, in order to determine the correctness of the fundamental assumptions of the superfluid model without introducing a large number of parameters. Therefore the accuracy of the calculation can be somewhat improved if for each even-even nucleus we take its own set of average-field levels, such that the levels of neighboring odd-mass nuclei calculated herefrom will provide a good description of the experimental findings. A comparison of the calculations with experiment indicates that the accuracy of the former is limited to a large degree by the fluctuations of the average-field levels.

It would be very difficult to determine the accuracy of the approximate method rigorously; rough estimates show that the error should not exceed 10%. In [9] the accuracy of the method was investigated using a model that considered a superconductivity type of interaction between six particles on five average-field levels. A comparison between the approximate solutions and the exact model solution shows that in the calculations based on the superfluid model the same level sequence is obtained as in the exact solution, and a better agreement with the exact solution is obtained on the whole than in calculations based on the original treatment of pair correlations.[10,11] Since the calculations yield a correct sequence of energy levels and are based on experimental pairing energies, it can be concluded that the accuracy of the method has been effectively improved. This is confirmed by a study of the aforementioned model. Thus errors in energy differences obtained by the approximate method as compared with the exact calculations are reduced one-half if the calculations are performed for the same pairing energy rather than for the same value of the pairing interaction constant G.

It has been shown[3-5,8] that the superfluid model of the nucleus describes correctly the ground and several excited states of strongly deformed nuclei and can serve as a basis for further investigations.

In the present work, in addition to the foregoing results of previous investigations, we consider the properties of the ground and excited states of strongly deformed nuclei that are determined from the superfluid model as well as the experiments required to check these properties. We also suggest experiments for determining how the properties of the ground and excited levels are affected by the residual forces not taken into account in the superfluid model.

We shall not consider interactions leading to collective effects, which are especially important for 2^+, 0^+, and 0^- states, and shall not take account of coupling with rotational states. Although these effects are of considerable magnitude in some instances, they do not appreciably change the investigated properties of strongly deformed nuclei. Our investigation can also furnish additional information regarding the cases in which these effects must be taken into account.

2. CHARACTER OF THE GROUND AND EXCITED STATES OF ODD NUCLEI

The superfluid model gives a one-quasi-particle structure of the ground state and some excited states, and a three-quasi-particle structure of higher excited states. Mottelson and Nilsson's analysis[12] of experimental data on the levels of strongly deformed odd-mass nuclei shows that the spins and parities of these states are unambiguously consistent with Nilsson's scheme, while the $\log ft_e$ values for β transitions are classified according to selection rules based on asymptotic quantum numbers. This analysis leads to a one-quasi-particle structure for the ground and low-lying excited states of odd-mass nuclei.

It has been shown[3] that pair correlations of the superconductivity type have an important effect on β-decay probabilities and lead to the necessity of systematizing the values of $\log ft_e R\eta$ (where R is the superfluid correction and η is a statistical factor) instead of $\log ft_e$, as in [12]. This systematization takes the form

$$\begin{aligned} 4.0 < \log ft_e R\eta < 4.7, &\quad au, \\ 5.5 < \log ft_e R\eta < 6.5, &\quad ah, \\ 5.5 < \log ft_e R\eta < 6.5, &\quad 1u. \end{aligned} \tag{2}$$

The distribution of $\log ft_e R\eta$ for all experimental data on β transitions in odd-mass nuclei, which is shown in Figs. 1a and 2b, demonstrates the feasibility of the systematization (2).

Figure 1a shows that there are two groups, au and ah, of allowed transitions, whose clear separation is evidence for the fulfillment of selection rules based on asymptotic quantum numbers. Figures 2a and 2b are histograms of first-forbidden

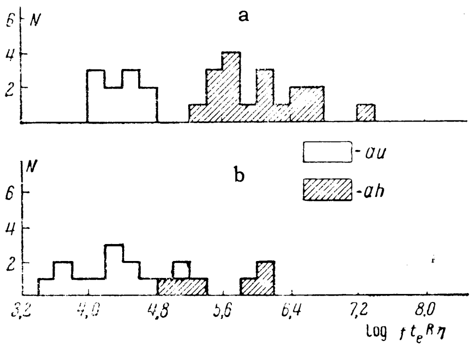

FIG. 1. Allowed β transitions. a – odd-A nuclei; b – even-A nuclei.

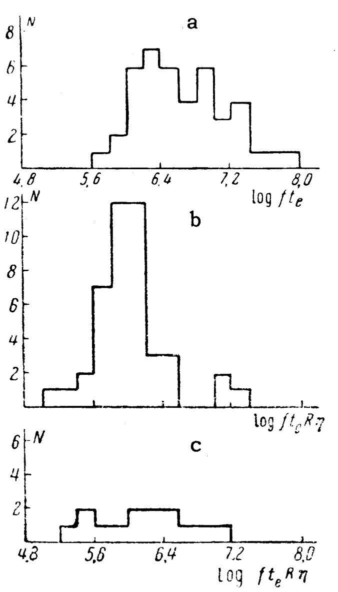

FIG. 2. 1u β transitions. a, b – odd-A nuclei; c – even-A nuclei.

unhindered β transitions (1u). The range of log $ft_eR\eta$ values is seen to be narrower than that of log ft_e and is shifted toward smaller values. A comparison of the histograms shows the important role played by the superfluid corrections in β-transition probabilities. We note that all three cases of log $ft_eR\eta \approx 7.2$ in Fig. 2b belong to transitions between the 402↑ and 512↓ states,[8] 1) and that several β transitions in the transuranium region with log $ft_eR\eta \leq 5.6$ are not well determined experimentally. The spread of log $ft_eR\eta$ is associated with fluctuations of the average-field levels and with experimental inaccuracies. The probabilities of hindered ah and 1h β transitions are more sensitive to fluctuations of average-field levels than in the case of unhindered transitions.

Three-particle levels should be found in odd-mass nuclei insofar as the basic hypotheses of the superfluid model are correct. The three-particle states should be of two kinds, those where all three quasi-particles are either neutrons or protons [(3n) and (3p)], and those consisting of a combination of different quasi-particles [(2n, p) and (2p, n)].

The lowest (3n) and (3p) states should be found at 1.5 MeV and higher; the lowest (3n) states can be found at energies of the order 1 MeV only in Dy^{161}. Since β transitions from the ground state of an even-system parent nucleus to (3n) and (3p) states are F-forbidden, it would be extremely difficult to detect the latter through β transitions. Such states can be found experimentally either by means of Coulomb excitation or by investigating γ spectra in transitions from high-lying excited states, as in [13]. The detection of these states and the determination of the degree of forbiddenness for β transitions to them are extremely important for our model.

The three-particle (2n, p) and (2p, n) states should be well populated through β transitions. Abstracting from interactions among the three quasi-particles, the probabilities of β transitions to these states should be the same as for transitions to excited states of even-even nuclei. We shall now consider in which nuclei it would be easiest to observe these states experimentally. Since the lowest-lying (2n, p) and (2p, n) states are in the range 1–2 MeV, the energy Q released in a ground-state β transition should be fairly large and the β transitions should not be greatly hindered, i.e., they should be of the types au, ah, and 1u, in order to be observable at low decay energies. Table I gives β transitions to (2n, p) and (2p, n) states satisfying these requirements. The second column gives the configuration of the parent state, and the fourth column gives the configurations of three-particle daughter states. Here n and p denote quasi-neutrons and quasi-protons, K is the last filled level in the independent-particle model, K + 1 is the first particle level, K – 1 is the first hole level etc. The energies of (2n, p) and (2p, n) levels given in the fifth column were calculated roughly neglecting inter-particle interactions. The seventh column gives the classification of the β transitions, while the eighth column gives Q for ground-state transitions.

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1)Average-field states are defined in a notation based on the following asymptotic quantum numbers: N – the total number of oscillator quanta, n_z – the number of oscillator quanta along an axis perpendicular to the nuclear axis of symmetry, Λ – the projection of the angular momentum on the nuclear symmetry axis, Σ – the projection of nucleon spin on this axis, $K = \Lambda \pm \Sigma$, and π-parity. States are denoted as $K\pi[Nn_z\Lambda]$ or $Nn_z\Lambda\uparrow$ when $K = \Lambda + \Sigma$, and as $Nn\Lambda\downarrow$ when $K = \Lambda - \Sigma$; $\hbar\omega_0^0 = 41A^{-1/3}$ MeV.

Table I. Three-quasi-particle states (2n, p) and (2p, n)

Parent nucleus	State	Daughter nucleus	State	E, MeV	$K\pi$	Classification of β-decay	Q, MeV
$_{68}Er^{161}$	$n521\uparrow$	$_{67}Ho^{161}$	$n521\uparrow K$,	1.6—1.7	$15/2^-$	—	>2.0
			$n523\downarrow K+2$,		$9/2^-$	$2\Lambda\,(au)$	
			$p523\uparrow K$		$5/2^-$	au 4.8	
					$1/2^-$	au 4.8	
			$n521\uparrow K$,	~1.5	$15/2^+, 9/2^+$	—	
			$n642\uparrow K+1$,		$5/2^+$	$1h$	
			$p523\uparrow K$		$1/2^+$	$1h$	
			$n521\uparrow K, n521\uparrow K$,	1.8—2.0	$1/2^+$	$1u$	
			$p411\downarrow K+1$				
			$n521\uparrow K$,	1.8—2.0	$9/2^-, 7/2^-$	$2\Lambda\,(ah)$	
			$n642\uparrow K+1$,		$3/2^-$	$a\Lambda(ah)$	
			$p411\downarrow K+1$		$1/2^-$	ah	
$_{69}Tm^{165}$	$p411\downarrow$	$_{68}Er^{165}$	$p411\downarrow K+1$,	~1.3	$13/2^+, 11/2^+$	—	
			$p523\uparrow K, n523\downarrow K$		$3/2^+$	au	
					$1/2^+$	au	
$_{69}Tm^{163}$	$p411\downarrow$	$_{68}Er^{163}$	$p411\downarrow K+1$,	~1.3	$13/2^+, 11/2^+$	—	
			$p523\uparrow K$,		$3/2^+$	au	
			$n523\downarrow K+1$		$1/2^+$	au	
$_{71}Lu^{169}$	$p404\downarrow$	$_{70}Yb^{169}$	$p404\downarrow K+1, p523\uparrow$	~2	$7/2^-$	$1u$	1.970
			$\uparrow K-1, n633\uparrow K$				
					$21/2^-$	—	
			$p404\downarrow K+1$,	1.5—1.7	$9/2^-$	$1u$	
			$p411\downarrow K$		$7/2^-$	$1u$	
			$n521\downarrow K+1$		$5/2^-$	$1u$	
	$p514\uparrow$		$p514\uparrow K+1$,	1.5—1.7	$11/2^+$	$1u$	
			$p411\downarrow K$,		$9/2^+$	$1u$	
			$n521\downarrow K+1$		$7/2^+$	$1u$	
$_{73}Ta^{173}$	$p404\downarrow$	$_{72}Hf^{173}$	$p404\downarrow K+1$,	≈1.7	$9/2^-$	$1u$	2.800
			$p411\downarrow K, n521\downarrow K$		$7/2^-$	$1u$	
					$5/2^-$	$1u$	
			$p404\downarrow K+1$,	1.5—2.2	$23/2^+$	—	
			$p514\uparrow K$,		$9/2^+$	au	
			$n514\downarrow K+2$		$5/2^+$	$a\Lambda\,(au)$	
$_{72}Hf^{173}$	$n521\downarrow$	$_{71}Lu^{173}$	$n521\downarrow K$,	1.3	$15/2^-, 13/2^-$	—	
			$n512\uparrow K+1$,		$5/2^-$	2	
			$p514\uparrow K+1$		$3/2^-$	ah	
			$n521\downarrow K$,	1.8	$17/2^-, 15/2^-$	—	
			$n514\downarrow K+2$,		$3/2^-$	au	
			$p514\uparrow K+1$		$1/2^-$	au	
$_{73}Ta^{175}$	$p404\downarrow$	$_{72}Hf^{175}$	$p404\downarrow K+1$,	~1.9	$17/2^-$	—	1.830
			$p402\uparrow K+2$,		$7/2^-$	$1u$	
			$n512\uparrow K$		$3/2^-$	$1^*\Lambda\,(1u)$	
			$p404\downarrow K+1$,	1.4—1.8	$23/2^+$	—	
			$p514\uparrow K$,		$9/2^+$	au	
			$n514\downarrow K+1$		$5/2^+$	au	
$_{73}Ta^{177}$	$p404\downarrow$	$_{72}Hf^{177}$	$p404\downarrow K+1$,	1.0—1.2	$23/2^+$	—	1.160
			$p514\uparrow K, n514\downarrow K$		$9/2^+$	au	
					$5/2^+$	au	
$_{75}Re^{179}$	$p402\uparrow$	$_{74}W^{179}$	$p402\uparrow K+1$,	~1.3	$21/2^+, 11/2^+$	—	2.615
			$p514\uparrow K, n514\downarrow K$		$7/2^+$	au	
					$3/2^+$	$a\Lambda\,(au)$	
			$p402\uparrow K+1$,	1.4—1.7	$21/2^-$	—	
			$p404\downarrow K-1$,		$9/2^-$	$1^*\Lambda\,(1u)$	
			$n514\downarrow K$		$5/2^-$	$1u$	
$_{75}Re^{181}$	$p402\uparrow$	$_{74}W^{181}$	$p402\uparrow K+1$,	~1,3	$23/2^-, 13/2^-$	—	1.670
			$p514\uparrow K, n624\uparrow K$		$5/2^-$	$1u$	
			$p402\uparrow K+1$,	~1.4	$7/2^+$	ah	
			$p404\downarrow K-1$,		$3/2^+$	ah	
			$n624\uparrow K$		$21/2, 11/2^+$	—	
$_{91}Pa^{237}$	$3/2\, p530\uparrow$	$_{92}U^{237}$	$p530\uparrow K$,	~1.7	$7/2^+$	—	2.3
			$p523\downarrow K+2$,		$5/2^+$	1^*u	
			$n631\downarrow K$		$3/2^+$	$1\,(1^*u)$	
			$p530\uparrow K, p642\uparrow$	1.8—2.1	$11/2^-, 9/2^-$	—	
			$\uparrow K+1, n622\uparrow K+1$				
					$1/2^-$	ah	
			$p530\uparrow K, p523\downarrow$	1.9—2.2	$11/2^+, 9/2^+$	—	
			$\downarrow K+2, n622\uparrow K+1$		$1/2^+$	$1u$	

A very favorable β decay for observing a (2n, p) level is $Er^{161} \to Ho^{161}$; here an au transition is possible with $\log ft_r = 4.8$ according to our calculation. It is entirely possible that the 1700- and 1830-MeV states of Ho^{161} discovered in [14] are three-particle states with $K\pi$ equal to $5/2^-$ and $1/2^-$, respectively.

An analysis of the data given in Table I shows that in some cases three-particle states lead to a different interpretation of $7/2^-$ and $9/2^-$ levels observed in the range 1—15 MeV.[15] While for W^{181}, Hf^{177}, and Hf^{175} the $7/2^-$ and $9/2^-$ levels can undoubtedly be treated as the single-particle states $7/2^-[503]$ and $9/2^-[505]$, this is improbable for Yb^{169}, which should exhibit three-particle $9/2^-$ and $7/2^-$ states at 1.5—1.6 MeV and 1u β transitions from Lu^{169}. Therefore for Yb^{169} the $7/2^-$ state at 1.465 MeV and the $9/2^-$ state at 1.452 MeV are more correctly regarded as three-particle states, because the experimental energies are much too low for their identification as $7/2^-$[503] and $9/2^-$[505] states.

Table I shows that there are several favorable possibilities for the experimental observation of (2n, p) and (2p, n) states. One of the clearest criteria for these states is the detection of allowed unhindered β transitions (au) going to them whenever there are no transitions to single-particle levels. Table I contains as examples several β transitions to (2n, p) and (2p, n) levels, in order to attract the attention of experimenters. The existence of (2p, n) and (2n, p) levels follows directly from the superfluid model, and their absence would be at least strange.

The investigation of very high excited states in odd-mass nuclei is very important for determining the limits of excitation energy with which excited-state structures of one, three, five, and more quasi-particles are preserved in odd-mass nuclei.

One-particle levels of odd-mass nuclei give information regarding average-field energy levels, which are required in calculating the energies of even-even nuclei, in analyzing β-transition probabilities etc. The experimental search for these levels is therefore of extreme interest. For example, the analysis of two-particle levels in $Gd^{154,156,158}$, Dy^{158}, and other nuclei is greatly hampered by uncertainty regarding levels in neutron systems with N = 89—95. It would therefore be extremely useful to determine $K\pi$ for the ground states of $Gd^{153,159}$ and $Dy^{155,157,159}$, and the spins and parities of higher excited states of $Gd^{155,157}$, $Dy^{161,163,165}$, and $Er^{163,165}$, which are also important for determining the energy difference between the average-field levels 523↓ and 633↑. In the transuranium region it would be very important to have information on one-particle levels of odd-mass nuclei in order to confirm the existence of a subshell at N = 152, indications of which have been derived from spontaneous fission and α decay.

3. TWO-QUASI-PARTICLE STRUCTURES OF EXCITED STATES OF EVEN-EVEN NUCLEI

The superfluid model leads naturally to a two-quasi-particle treatment of many levels of even-even nuclei, the correctness of which has been confirmed experimentally.[5] The superfluid model is an independent-particle model, although it takes into account the influence of unpaired particles on the superfluid properties of the system; this has been called the blocking effect. A comparison between theory and experiment with regard to the lowering of the (K, K+1) state below the formal gap has shown[3,4,8] that the blocking effect is very important in strongly deformed nuclei.

The two-particle structure of some excited states of even-even nuclei is confirmed by experimental β-transition probabilities, as shown by histograms in Figs. 1 and 2. It is here seen that the range of log $ft_eR\eta$ for β transitions is approximately the same for both even- and odd-mass nuclei. The large spread of log $ft_eR\eta$ is associated with the fact that we have neglected interactions between quasi-particles and fluctuations of the average-field levels, and also with the insufficient accuracy and reliability of the available experimental data.

A comparison of the calculated and experimental energy levels of even-even nuclei shows that the great majority of the calculated lowest-lying two-particle levels have been observed experimentally; these levels should be populated rapidly via β decays. We are left with the problem of observing all the calculated levels or of proving the nonexistence of some. We must pass from checking the correctness of the basic hypotheses of the model to an investigation of all levels in even-even nuclei in order to determine departures from the simple picture given by the superfluid model. If the assumed scheme of single-particle average-field levels is correct, we should observe the rapid population via β decay of the following levels: the 1^+ proton level with E > 1.4 MeV in W^{182}, which should be populated by ah β decay of 13-hr Re^{182} with log ft_r = 6.5; the 1^- proton level with E ~ 1.3 MeV in Hf^{178} from 1u β decay of 9.3-min Ta^{178}; the 4^- neutron level with E ~ 1.7 MeV in Yb^{172}, which should be populated by ah β decay of Lu^{172}; and the 5^- proton level with E > 1.4 MeV in Dy^{160}, which should be populated by 1u β decay of Ho^{160}.

Gallagher and Soloviev[5] have reported the spin splitting of some states subject to the rule of somewhat lower energies for states having antiparallel spins (Σ = 0) than for those having parallel spins (Σ = 1). The spin splitting results from inter-particle interactions and indicates the need for additional terms in the Hamiltonian. However, the presently available information regarding spin splitting is extremely meager and should be increased. For example, two 4^- levels of Er^{168} should be observed: a neutron level below 1.1 MeV and a proton level below 1.5 MeV. Beta decay to these states from the 3^+ state of Tm^{168} is Λ-forbidden and is classified as 1Λ (1u).

It would be very interesting to find the states of even-even nuclei to which β decay is F-forbidden. These levels can be observed in γ transitions from higher excited states.

The superfluid model takes into account (but only approximately) merely a fraction of the residual forces between nucleons within the nucleus. It would therefore be interesting to investigate how strongly the neglected residual interactions influence the ground and excited levels of strongly deformed nuclei. In investigations of the influence of pair correlations on β-decay probabilities it has been shown [3,4] that β transitions belonging to a third group (denoted as F-forbidden in [5]) are strictly forbidden in the superfluid model. The experimental determination of the degree of F-forbiddenness of β decays would be extremely important both in determining the role of the neglected residual forces and in ascertaining how correctly and accurately the superfluid model treats the ground- and excited-state properties of strongly deformed nuclei.

An analysis of the experimental data has shown [5] that not a single F-forbidden β transition has been firmly established. Table II gives several transitions which are most suitable for determining the degree of F forbiddenness. For example, the β decay of Ta^{182} in the 3^- state with the configuration $p404\downarrow - n510\uparrow$ to the 2^- proton state of W^{182} with the configuration $514\uparrow - 402\uparrow$ is F-forbidden. The energy 1.289 MeV is obtained for this state, which is well populated via the β decay of 13-hr Re^{182}; this is in good agreement with the calculated energy 1.3 MeV. Log ft_e = 8.2 is indicated by data for this aF β transition. However, the spin of 112-day Ta^{182}, [16] the configuration of this state, and the value of log ft_e are not entirely reliable. Assuming that our treatment is correct, the forbiddenness would slow down the rate of β decay by a factor of about 100, which is too small to be reconciled easily with all the available data. Another good example is the β transition from the 2^+ state of 7.7-hr Tm^{166} with the configuration $p411\downarrow - n642\uparrow$ to the 1^- neutron state of Er^{166} with the configuration $523\downarrow - 633\uparrow$ and energy 1.828 MeV, as determined from the β decay of 27-hr Ho^{166}. In addition to the transitions given in Table II, a large number of F-forbidden β transitions are reported in [5,8].

The relatively high excited levels of even-even nuclei should include four-particle as well as two-particle states. The former states are of two kinds: (4n) or (4p), and (2n, 2p). Beta transitions to the (4n) and (4p) states are F-forbidden, and these states should be filled via γ transitions from high-lying excited levels. The degree of F-forbiddenness in this case will evidently be greater than for β transitions to two-particle states. Superconductivity-type pair correlations will be absent from the majority of these states. The $(K-1, K, K+1, K+2)$ states have the lowest energy, which we calculate neglecting interparticle interactions. For example, in W^{182} this (4n) state has the energy ~ 3 MeV, spins 10, 9, 7, 6, 3, 2, 1, and 0, and negative parity; the corresponding (4p) state has energy above 3 MeV, spins 11, 10, 6, 5, 4, 3, 2, and 1, and negative parity. In Yb^{172} the (4n) state $(K-1, K, K+1, K+2)$ has ~ 2.5 MeV, spins 10, 9, 5, 4, 3, and 2, and negative parity; the (4p) state has 2.8 MeV, spins 12, 11, 5, 4, 3, and 2, and positive parity.

Table II. F-type β transitions

Odd-odd nuclei		Even-even nuclei		Energy, MeV	Class. of β transition	Remarks
Nucleus	State	Nucleus	State			
$_{63}Eu^{156}$	$1^- p\,413\downarrow - n\,521\uparrow$	$_{64}Gd^{156}$	$p1^-\,532\uparrow - 411\uparrow$	~ 1.7	aF	Gd^{158} should have the same level
$_{69}Tm^{166}$	$2^+ p\,411\downarrow - n\,642\uparrow$	$_{68}Er^{166}$	$n1^-\,523\downarrow - 633\uparrow$	1.828	$1F$	
			$n2^+\,523\downarrow - 521\downarrow$	1.7	aF	
			$3^+\,523\downarrow + 521\downarrow$	—	aF	$\Sigma = 1$
$_{71}Lu^{172}$	$4^- p\,404\downarrow + n\,521\downarrow$	$_{70}Yb^{172}$	$p3^+\,411\downarrow + 402\uparrow$	1.7	$1F$	$\Sigma = 0$
			$2^+\,411\downarrow - 402\uparrow$	—	$1F$	$\Sigma = 1$
			$p5^-\,411\downarrow + 514\uparrow$	1.8	aF	$\Sigma = 0$
			$4^-\,411\downarrow - 514\uparrow$	—	aF	$\Sigma = 1$
$_{73}Ta^{182}$	$3^- p\,404\downarrow - n\,510\uparrow$	$_{74}W^{182}$	$p2^-\,514\uparrow - 402\uparrow$	1.3	aF	
$_{93}Np^{240}$	$1^+ p\,642\uparrow - n\,624\downarrow$	$_{94}Pu^{240}$	$n2^+\,631\downarrow - 622\uparrow$	~ 1	aF	$\Sigma = 1$
			$n1^-\,743\uparrow - 622\uparrow$	~ 1.3	$1F$	

Table III. Four-quasi-particle states (2n, 2p)

Parent nucleus	State	Daughter nucleus	State	E, MeV	Kπ	Classification	Q, MeV
$_{75}Re^{182}$	$p402\uparrow, n624\uparrow$	$_{74}W^{182}$	$p402\uparrow K+1, p514\uparrow K,$	3,3	$15^+, 10^+$	—	
	7^+		$n624\uparrow K, n514\downarrow K-1$		8^+	au	~2.3
					6^+	au	
	2^+				3^+	au	~2.5
					1^+	au	
			$p402\uparrow K+1, p514\uparrow K,$	2,9	16^-	—	
	7^+		$n624\uparrow K, n624\uparrow K$		7^-	1u	
	2^+				2^-	1u	
$_{69}Tm^{166}$	$p411\downarrow, n642\uparrow$	$_{68}Er^{166}$	$p523\uparrow K, p402\uparrow K+1,$	3,7	$9^+, 8^+, 4^+$	—	2.7
	2^+		$n642\uparrow K-1, n523\downarrow K$		3^+	au	
					2^+	—	
					1^+	au	
			$p523\uparrow K, p402\uparrow K+1,$	3,1	$11^+, 10^-, 6^-$	—	
			$n624\uparrow K-1, n633\uparrow K+1$		$5^-, 4^-, 2^-$	—	
					1^-	1u	
					3^-	1u	
	$p411\downarrow, n523\downarrow$		$p523\uparrow K, p402\uparrow K+1,$	2,9	$10^+, 4^+, 3^+$	—	
	2^-		$n523\downarrow K, n633\uparrow K+1$		$9^+, 5^+$	—	
					2^+	1u	
$_{67}Ho^{160}$	$p523\uparrow, n521\uparrow$	$_{66}Dy^{160}$	$p411\uparrow K, p523\uparrow K+1,$	2,9	$9^+, 3^+$	—	
	5^+		$n521\uparrow K, n642\uparrow K+1$		$2^+, 1^+$	—	3,28
					6^+	ah	
					4^+	ah	
			$p413K-1\downarrow,$	3,3	$10^-, 7^-, 3^-$	—	
			$K-1p523\uparrow K+1,$		$2^-, 0^-$	—	
			$n521\uparrow K, n523\downarrow K+2$		5^-	1u	

In Er^{168} the neutron levels of this kind should have ~ 2.5 MeV, spins 9, 8, 4, 3, 2, and 1, and negative parity; the proton levels should be above 3 MeV with spins 10, 9, 7, 6, 3, 2, 1, and 0, and positive parity. The energies of some of these states can be somewhat lowered through inter-particle interactions. At somewhat higher energies four-particle states are found with different particle distributions in average-field levels. At excitation energies above 3 MeV the level density of even-even nuclei increases greatly due to four-particle states.

The superfluid properties of (2n, 2p) four-particle states are similar to those of the corresponding two-particle states. The former states should be well filled via β decays. Table III gives a number of possible β transitions to (2n, 2p) states, whose energies have been calculated without account of inter-particle interactions, which can result in the lowering of some levels. The table shows that in some cases (especially if the energy levels will be lowered greatly) these states can be observed experimentally in β decays. Four-particle states of both kinds should be observable in all strongly deformed even-even nuclei.

The higher excited levels of even-even nuclei should include six (or more)- particle states, although it is not clear up to what energies this treatment of the excited states will remain basically correct. This question can possibly be decided by neutron spectroscopy. Thus the observation of different γ-spectrum shapes or different forbiddennesses in transitions to low-lying levels from high excited levels with identical spins and very close energies could be evidence of different internal structures of these high-lying excited states.

We note in conclusion that the most important experiments for determining the properties of the ground and excited states of strongly deformed nuclei are, first, the determination of the degree of F-forbiddenness of β transitions, and, secondly, the detection of three- and four-particle excited states.

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