ANGULAR CORRELATIONS OF LEPTONS IN K-MESON DECAYS

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The $e\nu$ and $\mu\nu$-lepton angular correlations in K_{e3} and $K_{\mu 3}$ decays have been calculated. It is convenient to compare these correlations with the experimental data if the probability of detection of an electron (or μ-meson) depends on its energy. The experimental data presently available are in agreement with the vector theory of the decay interaction and exclude the scalar and tensor theories.

A large number of papers have been devoted to the theoretical analysis of K_{e3} and $K_{\mu 3}$ decays. A detailed review of these papers has been given by Okun'.[1] We note the work of Pais and Treiman,[2] in which the angular correlations of pions and electrons (μ mesons) were calculated for a fixed pion energy. Comparison of these correlations with the experimental data does not depend on the assumptions regarding the form factors, since the latter depend only on the pion energy in the K-meson rest system. Such comparison, however, requires sufficiently large statistics; moreover, in an actual experiment it is difficult to ensure that the probability of recording electrons of different energy is the same, since only electrons of sufficiently low energy (in the laboratory system) can be identified. Thus, in the recent work of Luers et al,[3] where a sufficient number of K_{e3} decays was obtained for the first time, only electrons of energy $E_{e\,\mathrm{lab}} < 200$ MeV were identified. This leads to a strong distortion of the electron spectrum, so that the analysis of the experimental data is impossible without the introduction of corrections for the efficiency of recording electrons. Under such conditions, it is convenient to compare the experimental data with the angular correlations of the electron (μ meson) and neutrino, since in this case it is possible to carry out an analysis of the experimental data without the introduction of corrections for the above-mentioned distortion. This question will be considered in the present article.

If we are not interested in the polarization properties of the particles, then the probability dW for K_{e3} decay ($K \to \pi + e + \nu$) is a function of two variables. As these variables, we choose the electron energy E_e and $\cos\theta$, where θ is the angle between the directions of emission of the lepton having a mass (for the sake of brevity, we shall speak about the electron) and the neutrino. All quantities are taken in the K-meson rest system. Apart from constant factors, we have

$$dW(E_e, \cos\theta) = \overline{|m|^2}\, f(E_e, \cos\theta)\, dE_e\, d\cos\theta,$$

$$f = \frac{p_e E_e E_\nu^2}{M - E_e + p_e\cos\theta}, \qquad p_e = \sqrt{E_e^2 - m_e^2},$$

$$E_\nu = \frac{M(W_e - E_e)}{M - E_e + p_e\cos\theta}, \qquad W_e = \frac{M^2 - m_\pi^2 + m_e^2}{2M},$$

W_e is the maximum electron energy, M, m_π, and m_e are the masses of the K-meson, pion, and electron. The allowed region of variation of these variables is the interval $m_e \le E_e \le W_e$, $-1 \le \cos\theta \le 1$. The quantity $\overline{|m|^2}$ is the square of the matrix element modulus averaged over the spin states. If the K-meson spin is equal to zero, the matrix element can be represented in the form

$$m = \frac{1}{\sqrt{2}}\Big[g_S\bar{u}_e(1+\gamma_5)\,v_\nu - \frac{ig_{V1}}{M}\,k_\alpha\bar{u}_e\gamma_\alpha(1+\gamma_5)\,v_\nu + \frac{ig_{V2}}{M}\,(p_e+p_\nu)_\alpha\,\bar{u}_e\gamma_\alpha(1+\gamma_5)\,v_\nu + \frac{ig_T}{M^2}\,k_\alpha(p_e+p_\nu)_\beta\,\bar{u}_e\sigma_{\alpha\beta}(1+\gamma_5)\,v_\nu\Big],$$

$g_{S,V,T}$ are functions of $E_\pi = M - E_e - E_\nu$. It follows from the conservation of combined parity that they are real. The quantities γ_α are the Dirac Hermitian matrices; $\sigma_{\alpha\beta} = (\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)/2i$; k and p_ν are the four-momenta of the K meson and neutrino. Furthermore, we have

$$\begin{aligned}\overline{|m|^2} &= g_{V1}^2(1+\beta_e\cos\theta) + \tilde{g}_S^2(1-\beta_e\cos\theta) \\ &+ \frac{g_T^2}{M^2}\left\{(1-\beta_e\cos\theta)\left[(E_\nu - E_e)^2 - m_e^2\right] + \frac{2m_e^2E_\nu}{E_e}\right\} \\ &+ 2\tilde{g}_S g_{V1}\frac{m_e}{E_e} + \frac{2\tilde{g}_S g_T}{M}\left[(1-\beta_e\cos\theta)(E_\nu - E_e) + \frac{m_e^2}{E_e}\right] \\ &+ 2g_{V1}g_T\frac{m_e}{M}\left[\frac{E_e}{E_\nu} + \beta_e\cos\theta\right].\end{aligned}$$

Here $\beta_e = p_e/E_e$, $\tilde{g}_S = g_S - g_{V2}m_e/M$. We compare the quantity $dW/fdE_e = \overline{|m|^2} d\cos\theta$, and not dW, with the experimental data, since it is the former which depends on the form of the interaction.

We now consider the K_{e3} decay, for which

$$\overline{|m|^2} = g_{V1}^2(1+\cos\theta) + g_S^2(1-\cos\theta) + \frac{g_T^2}{M^2}(1-\cos\theta)(E_\nu - E_e)^2 + \frac{2g_S g_T}{M}(1-\cos\theta)(E_\nu - E_e).$$

We neglect the terms m_e/M, m_e/E_e and set $\beta_e = 1$. As seen from the expression above, $\overline{|m|^2}$ has a very simple dependence on $\cos\theta$ and does not depend on the electron energy for the scalar and vector theories if we consider g_i to be constants. The corresponding curves are shown in Fig. 1. It is characteristic for the tensor theory curves to go to zero for $\cos\theta = +1$.

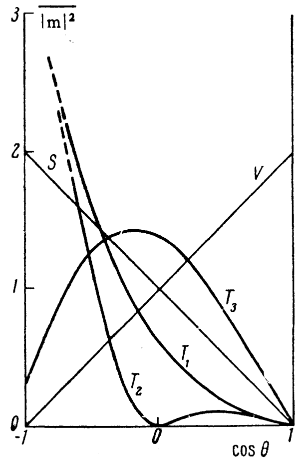

FIG. 1. Variation of $\overline{|m|^2}$ with $\cos\theta$. The curves S and V correspond to the scalar and vector theories and the curves T_1, T_2, and T_3 correspond to the tensor theory with energies $E_{e1} = 100$ MeV, $E_{e2} = 130$ MeV and $E_{e3} = 200$ MeV.

From the fact that $\overline{|m|^2}$ does not depend on E_e for the scalar and vector theories it follows that, for a comparison with the theoretical predictions, we can sum up the experimental data for the various electron energies. It is also clear that the dependence of dW/fdE_e on $\cos\theta$ does not change if this expression is multiplied by some function $\Phi(E_e)$ characterizing the efficiency for recording electrons as a function of E_e. In fact, the probability of registration depends on $E_{e\,\mathrm{lab}}$, but it is readily seen that this dependence can actually be formulated as a dependence on E_e. In fact, $E_{e\,\mathrm{lab}}$ is expressed as a function of E_e, the angle α (α is the angle between the directions of emission of the electron and K meson transformed to the K-meson rest system), and the K-meson velocity v. But dW does not depend on α or on v. Therefore $\Phi(E_e, \alpha, v)\,dW/fdE_e$ has the same dependence on $\cos\theta$ as dW/fdE_e. Allowance for the probability of recording the electrons leads only to the introduction of different statistical weights for electrons of different energy. This results only in a change of the dispersion (see Appendix). The foregoing remarks apply only to the scalar and vector theories with constant g_S and g_V. However, the weighted sum of the distributions with different E_e also cannot change the qualitative behavior of the curve for the tensor theory, i.e., the vanishing at $\cos\theta = +1$.

Figure 2a shows a histogram for $\overline{dW/fdE_e}$ (the bar denotes averaging) constructed from 142 K_{e3} decays given in [3]. The histogram has been normalized to the same area as the curves of Fig. 1. It is seen that the experimental data agree only with the vector interaction.

In order to show that g actually depends weakly on E_π, we constructed the distribution of $\overline{dW/fdE_e}$ separately for the cases $E_e < 100$ MeV and $E_e > 100$ MeV (62 and 80 cases, respectively). In the case of a strong, monotonic dependence of g on E_π, these distributions would be different, since the change ΔE_π of the pion energy as $\cos\theta$ changes from -1 to $+1$ depends on E_e. For $E_e < 100$ MeV, we have $\Delta E_\pi < 87$ MeV, while for 100 MeV $< E_e <$ 215 MeV, we have 87 MeV $< \Delta E_\pi <$ 130 MeV. It is seen from Figs. 2b and 2c that these distributions actually do not differ.

Figure 3 shows the values of the form factor g_{V1}^2 as a function of $\cos\theta$ averaged over $E_e > 100$ MeV. For comparison, g_{V1}^2 has been plotted as a function of $\cos\theta$ for $E_e = 100$ MeV, obtained from the theory with an intermediate boson,[4] where $g_{V1} \sim 1/(M_B^2 - M^2 - m_\pi^2 + 2mE)$; M_B is the boson mass. It is seen that the values $M_B \ll 600$ MeV are unlikely.

The foregoing conclusions agree with the results of Luers et al.[3] We note, however, that in our work these conclusions were obtained without introducing corrections for the efficiency of electron registration, and, moreover, the rejection of the scalar theory is not based on the assumption that the form factor is constant, for in our method of analysis the fact that the form factor weakly depends on E_π follows from the experimental data.

Thus far, we have considered cases in which any of the possible interaction theories is valid. If all three hold, then

$$\frac{dW}{fdE_e} = g_{V1}^2(1+\cos\theta) + \left[g_S + \frac{E_\nu - E_e}{M} g_T\right]^2 (1-\cos\theta).$$

If g_i are constants, then it follows from comparison with the experimental curve that

$$g_{V1} \neq 0, \quad \overline{\left[g_S + \frac{E_\nu - E_e}{M} g_T\right]^2} \approx 0, \quad \text{i.e.,}\ g_S \approx 0, \quad g_T \approx 0.$$

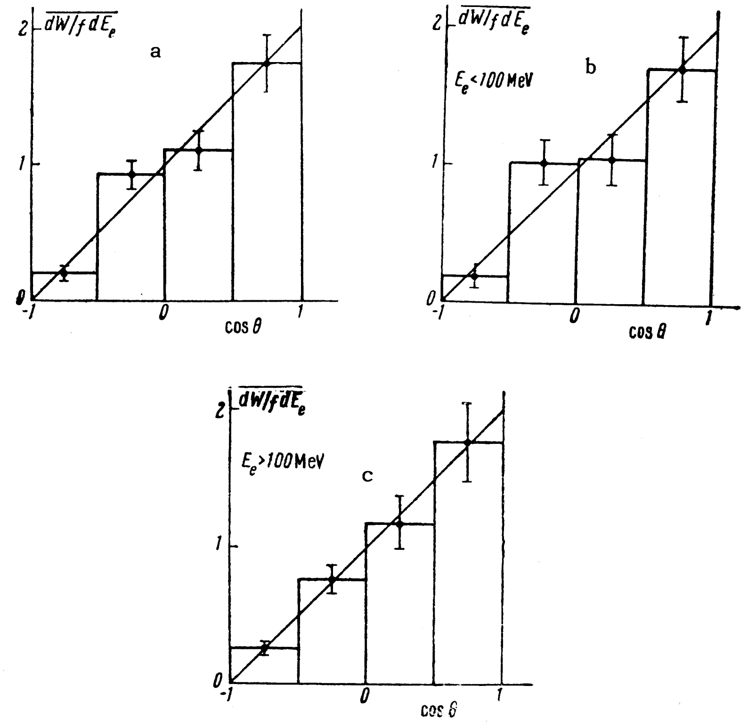

FIG. 2.

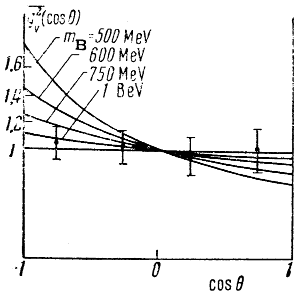

FIG. 3.

In conclusion, we note that the method used here to analyze the experimental data can also be applied to $K_{\mu 3}$ decay, although in this case $\overline{|m|^2}$ depends on E_e, since $\beta_e \neq 1$:

$$\frac{dW}{fdE_e} = g_{V1}^2(1 + \beta_e \cos\theta) + \frac{m_e}{M} g_{V2}^2 (1 - \beta_e \cos\theta) - 2 g_{V1} g_{V2} \frac{m_e}{E_e}.$$

The distribution summed over E_e will have the same shape as the weighted mean values $\overline{\beta_e}$ and (m_e/E_e) if g_{V1} and g_{V2} are constants. Constructing $\overline{dW/fdE_e}$ as a function of $\cos\theta$, we can choose one of the two possible solutions for the ratio g_{V2}/g_{V1} which are obtained if we take into account the approximate equality of W_{e3} and $W_{\mu 3}$ (see [1]). In fact, if $g_{V2}/g_{V1} = 4.5$ and $E_e < 200$ MeV, then $\overline{|m|^2}$ is practically independent of $\cos\theta$. If $g_{V2}/g_{V1} = -0.5$, then $\overline{|m|^2}$ can be plotted against $\cos\theta$ as a straight line of slope

$$\frac{0.9\overline{\beta}_e}{1.1 + \overline{(m_e/E_e)}} \qquad (> 0.4\overline{\beta}_e).$$

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APPENDIX

CALCULATION OF THE STATISTICAL ERRORS

For each part of the histogram, we constructed the quantity $\Sigma \Delta n_i/f_i$ corresponding to the integral

$$\int \frac{dW}{fdE_e} dE_e \, d\cos\theta,$$

where the sum and integral were extended over the chosen interval of variation of $\cos\theta$ and E_e. Here, Δn_i is the experimentally measured number of events falling in a given small region $\Delta E_e \Delta \cos\theta$. Since this region can be chosen sufficiently small, $\Delta n_i = 0$ or 1. Hence $\Sigma \Delta\eta_i/f_i$ is equal to the sum $\Sigma 1/f$ taken from the experimental points. To estimate the errors, it is necessary to know the dispersion $D(\Sigma \Delta n/f)$ of the quantity $\Sigma \Delta n/f$. It can be obtained in the following way. We have the relation

$$\Delta n/f = \varphi(\cos\theta) \sim \overline{|m|^2}.$$

For simplicity, let $\varphi(\cos\theta)$ = const for a given interval of $\cos\theta$. Then the mean number of particles $\overline{\Delta n}$ falling in a small region $\Delta E_e \Delta \cos\theta$ is

$$Cf\bar{N}\Delta E_e \Delta\cos\theta, \qquad C = (\Sigma f \Delta E_e \Delta\cos\theta)^{-1},$$

where $\Sigma\overline{\Delta n} = \bar{N}$ is the mean number of particles falling in the entire interval of $\cos\theta$ under consideration. The quantity Δn has a Poisson distribution, i.e., it has a dispersion $\Delta n D(\Delta n) = \overline{\Delta n}$. Then the dispersion is

$$D\left(\sum \frac{\Delta n}{f}\right) = \sum \frac{\overline{\Delta n}}{f^2} = \bar{N}\frac{\sum f^{-1}\Delta E_e \Delta\cos\theta}{\sum f\Delta E_e \Delta\cos\theta} = \bar{N}\frac{\int f^{-1} dE_e d\cos\theta}{\int f dE_e d\cos\theta}.$$

From this formula, we also estimated the errors shown in the histograms. The quantity f in our case was normalized by the condition

$$\int f^{-1} dE_e \, d\cos\theta = 2.$$

It is clear that we can estimate the quantity $\bar{N}$ only approximately if we replace $\bar{N}$ by the experimentally measured number of events for a given interval of $\cos\theta$.

A more accurate formula can be readily obtained if the variable φ is taken into account:

$$D\left(\sum \frac{\Delta n}{f}\right) = \frac{\bar{N}\int f^{-1}\varphi dE_e \, d\cos\theta}{\int \varphi f dE_e d\cos\theta}.$$

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[1] L. B. Okun', UFN **68**, 449 (1959), Ann. Rev. of Nuclear Sci. **9**, 61 (1959).

[2] A. Pais and S. B. Treiman, Phys. Rev. **105**, 1616 (1957).

[3] Luers, Mittra, Willis, and Yamamoto, Phys. Rev. Lett. **7**, 255 (1961).

[4] Brene, Egardt, and Quist, Nuclear Phys. **22**, **553** (1961).

Translated by E. Marquit
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ERRATA

Vol	No	Author	page	Correction
15	6	Turov	1098	The article contains an erroneous statement that weak ferromagnetism cannot exist in any cubic crystal (with collinear or weakly noncollinear antiferromagnetic structure. This was found to be true only for crystal classes T and T_h, and for others weak ferromagnetism will appear in antiferromagnets with magnetic structure type $3^+ 4^-$, and only due to invariants of third and higher orders in the antiferromagnetism vector L. Consequently a line (14) should be added to the table on p. 1100: $14 \mid 207\text{-}230 \mid \text{Cubic} \mid 3^+, 4^- \mid M_x L_x (L_y^2 - L_z^2) + M_y L_y (L_z^2 - L_x^2) + M_z L_z (L_x^2 - L_y^2) \mid \text{VI}$ The Cartesian axes are directed here parallel to the fourfold symmetry axes. The tensors $g^{(1)}$ and $g^{(2)}$ for this (sixth) group of weakly ferromagnetic structures will be identically equal and isotropic: $g_{\alpha\beta}^{(1)} = g_{\alpha\beta}^{(2)} = g\delta_{\alpha\beta}$
16	1	Valuev	172	At the end of the article there are incorrect expressions pertaining to $K\mu_3$ decay. The correct formula can be easily obtained from the main formula of the article by putting $g_S = g_T = 0$. The tangent of the angle between the $\mid m \mid^2$ curve and the $\cos\theta$ axis will be $\approx \beta_e$ if $g_{V2}/g_{V1} = -0.5$ and ≈ 0 if $g_{V2}/g_{V1} = 4.5$ and $\beta_e \sim 1$, so that in fact the difference in the angle correlations between these cases is even somewhat stronger than indicated in the article.
16	1	Zhdanov et al	246	The horizontal parts of curves 2 and 3 in Fig. 2 should be drawn with solid lines (they correspond to the asymptotic calculated values of the ionization losses, i.e., to the region in which the theory describes the relation between g/g_0 and the particle energy exactly).
16	1	Deutsch	478 & 481	When account is taken of thermoelectric processes it is necessary to add in the first curly bracket of (24) the term $A = 3v_0^2 H_y c(\alpha_{xz} - \alpha_{zx})/2$ and in Eq. (31) the term $A/9$.
16	1	Nguyen	920 Eqs. (4), (6), (7), & (8)	The combinations $V^1 \pm V^2$, $A^1 \pm A^2$, and $I^1 \pm I^2$ should be divided by $\sqrt{2}$.
16	1	Gershtein et al	1097 Eq. (1)	Reads $G/\sqrt{2}$, should read $G/2$
16	5	Gurevich		An error has crept into Eq. (30). The right half of this formula is actually equal to $\varepsilon_E \frac{\delta_{kz}}{2\pi E}\left[F_0(\varepsilon) + 2\varepsilon \frac{d}{d\varepsilon} F_0(\varepsilon)\right]$.