ON KAON-HYPERON RESONANCES

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On the basis of data on the behavior of the cross section for the reactions $\pi^- + p \rightarrow \Sigma + K$ and $\pi^- + p \rightarrow \Lambda + K$ near the $\pi^- + K$ threshold, it is concluded that the $\Sigma + K$ system should have a $T = 1/2$ level with a binding energy $E \sim 30$ MeV. This should lead to a resonance in the $\pi^- + p \rightarrow \Lambda + K$ reaction below the $\Sigma + K$ threshold. The possibility of the existence of such levels in $\Lambda + K$ and other systems and their connection with the resonances near the corresponding thresholds are discussed.

1. INTRODUCTION

RESONANCES in the cross sections for the production and scattering of particles signify that a comparatively long-lived system, i.e., a compound nucleus, is produced in the intermediate state. This system can frequently be considered as a bound state of two particles. An interaction leading to the formation of this state can be roughly divided into two parts: "peripheral" interactions with a radius of the order m_π^{-1} due to the exchange of pions, and "inner" interactions with a much smaller radius connected with the exchange of heavier particles. Inner and peripheral interactions are quite different in their effects. If a bound state is produced as a result of an inner interaction, the binding energy, roughly speaking, is of the order of the particle masses (for example, in the Fermi-Yang model of the pion), and the concept of a bound state loses meaning. If, on the other hand, the peripheral interaction is not very small and is attractive, then the existence of a bound state, i.e., a resonance close to the production threshold of the particles, is quite probable.

In fact, the depth of the peripheral interaction is of the order $U \sim m_\pi$ and its radius is of the order $a \sim m_\pi^{-1}$. For a particle of mass M, the condition for the existence of a level in the well has, as we know, the form

$$MUa^2 \gtrsim 1,$$

and, in our case, is satisfied for particles with $M > m_\pi$. The binding energy of the state is, of course, smaller than the depth of the well and, consequently, the resonance should be situated comparatively close to the threshold. Hence the peripheral interaction can lead to a correlation between the positions of the resonances and the thresholds. The possibility of such a correlation has been considered by one of the authors.[1] If it turns out that the binding energy is small in comparison with the well depth, then the position and level width can be found from the energy dependence of the cross section for the production of the particles close to threshold.

In the present paper, a phenomenological analysis is made of experimental data on the cross sections for the reactions $\pi + N \rightarrow \Sigma + K$ and $\pi + N \rightarrow \Lambda + K$. From this analysis it follows that the $\Sigma + K$ system apparently has a $T = 1/2$ level with a binding energy of ~ 30 MeV. This level should be manifest in the form of a resonance in the $\pi + N \rightarrow \Lambda + K$ reaction cross section. We shall discuss the possibility that levels exist in other systems: $\Lambda + K$, $N + \rho$, $N + \omega$, and $N + K^*$. The analysis was carried out by the method of Dalitz and Tuan[2-5] based on the properties of unitarity, time reversal, and analyticity of the scattering matrix for the analysis of the $\bar{K}N$ interaction at low energies.

2. RELATION BETWEEN AMPLITUDES OF DIFFERENT CHANNELS

For the description of multichannel reactions, it is particularly convenient to use the K-matrix formalism.[3,4,6] The K-matrix is related to the scattering matrix S by the formulas

$$S = 1 + 2\pi i\delta (E - E_{in}) T, \qquad K = T - i\pi K\rho T, \qquad (1)$$

where the elements of the diagonal matrix ρ represent the statistical weight in the respective channels. For a fermion-boson system, the elements ρ_i have the form

$$\rho_i = k_i M_i/\pi E_i, \qquad (2)$$

where M_i is the fermion mass, E_i is the total energy, and k_i is the relative momentum. It is convenient to introduce the matrices $K' = \pi\rho^{1/2}K\rho^{1/2}$ and $T' = \pi\rho^{1/2}T\rho^{1/2}$. The cross section for a state of momentum J is then given by the expression

$$\sigma(i \to j) = 4\pi k_i^{-2}(I + {}^1/_2)\,|\langle i|T'|j\rangle|^2. \qquad (3)$$

The advantage of introducing the K-matrix for the case of several channels is connected with the fact that the conditions of unitarity and time reversal are formulated very simply for it: the K-matrix should be real and symmetric. When there are no transitions between channels, the K-matrix is diagonal and its matrix elements are equal to the tangents of the scattering phase shifts in the respective channels. The matrix T is simply expressed in terms of the K-matrix if only two particles are present in each channel. In our case, along with $N\pi$ and YK, there are also channels for the production of several pions. For simplicity, we will not take these channels into account at first; a qualitative discussion of their influence is given below. Hence, in the region above the ΛK threshold, the K′-matrix has the form

$$\begin{array}{c c} & \begin{array}{cc} \pi N & \Lambda K \end{array} \\ \begin{array}{c} \pi N \\ \Lambda K \end{array} & \begin{pmatrix} \alpha & b \\ b & \gamma \end{pmatrix} \end{array}, \qquad (5)$$

and above the ΣK threshold it has the form

$$K' = \begin{pmatrix} \alpha_N & \alpha_{N\Lambda} & b_N \\ \alpha_{N\Lambda} & \alpha_\Lambda & b_\Lambda \\ b_N & b_\Lambda & \gamma \end{pmatrix}. \qquad (4)$$

Close to the threshold, the statistical weight of the produced particles (under the assumption that they are produced in the S state) is a rapidly changing function of the momentum. In explicit form, we write this dependence as follows:

$$\gamma = k/\varkappa, \qquad b = \beta\,(k/|\varkappa|)^{1/2}, \qquad (6)$$

where κ and β are some new quantities.

Close to the ΛK threshold, we obtain for the T-matrix, by virtue of Eq. (1),

$$T' = \frac{1}{(1 - i\alpha)(1 - ik/\varkappa) + \beta^2 k/|\varkappa|} \times \begin{pmatrix} \alpha(1 - ik/\varkappa) + i\beta^2 k/|\varkappa| & \beta\,(k/|\varkappa|)^{1/2} \\ \beta\,(k/|\varkappa|)^{1/2} & k(1 - i\alpha)/\varkappa + i\beta^2 k/|\varkappa| \end{pmatrix}, \qquad (7)$$

and in the region close to the ΣK threshold, we obtain

$$T_{N\Sigma} = \frac{\beta_N(1 - i\alpha_\Lambda) + i\alpha_{N\Lambda}\beta_\Lambda}{\Delta(x)}\,\frac{(k/|\varkappa|)^{1/2}}{1 - ik/\varkappa}, \qquad (8a)$$

$$T_{N\Lambda} = (\alpha_{N\Lambda} - \beta_N\beta_\Lambda x)/\Delta(x), \qquad (8b)$$

$$T_{NN} = i\,[1 - (1 - i\alpha_\Lambda + i\beta_\Lambda^2 x)/\Delta(x)] \qquad (8c)$$

where

$$x = -ik\,(1 - ik/\varkappa)^{-1}\,|\varkappa|^{-1},$$

$$\Delta(x) = (1 - i\alpha_N)(1 - i\alpha_\Lambda) + \alpha_{N\Lambda}^2 + x\,[-2\alpha_{N\Lambda}\beta_N\beta_\Lambda + i\beta_N^2(1 - i\alpha_\Lambda) + i\beta_\Lambda^2(1 - i\alpha_N)].$$

Formulas (7) and (8) have been written for the region above the thresholds. The analytic extension of the amplitudes of T to the region below threshold is carried out by the substitution $k \to i\,|k|$, so that $x \to |k|\,(|\kappa| + |k|\,\mathrm{sign}\,\kappa)^{-1}$.

The quantity $1/\kappa$ has the meaning of a scattering length for the scattering of a K-meson on a hyperon and is roughly of the order of the interaction range a. The quantity a is determined by the smallest mass transfer in the interaction, so that in our case $a \sim 1/2m_\pi$. When there is a real or virtual level close to zero, κ is less than 1/a. In this case, for a change in k in the interval $\sim\kappa$, the quantities β and κ, which depend on $k^2/4m_\pi$, can be considered as constants. This holds even more so for the remaining elements of the K-matrix, which depend on the momenta of nonthreshold particles, since, for example, a change in momentum of the Σ and the K by the quantity k corresponds to a change $\Delta p_{\pi\,\mathrm{cms}} = k^2/500$ MeV/c.

Hence, in the region $|k| \lesssim |\kappa|$, the dependence of the T-matrix on the energy is given by formulas (7) and (8), where the quantities α, β, and κ are constants. Since the kinematical factors lead to a considerable narrowing of the corresponding regions of variation of p_π, a sharp "resonance-like" change in the cross sections appears in narrow intervals of p_π.

3. ΣK THRESHOLD

We first consider the vicinity of the ΣK threshold. In this case, the formulas are more complex than in the vicinity of the ΛK threshold, but this region has been better studied experimentally. The existing data permit the determination only of the signs and orders of magnitude. They are, however, sufficient for qualitative conclusions on the behavior of the ΛK production cross section.

We shall estimate the quantity κ from the energy dependence of the ΣK production cross section close to threshold. We are interested in the influence of the ΣK threshold on the ΛK channel, so that we should find the cross section for the reaction $\pi + N \to \Sigma + K$ in a state with isotopic spin $T = 1/2$. In order to determine it, we use the results of Wolf et al[7] on the cross sections for the reactions $\pi^- + p \to \Sigma^- + K^+$ and $\pi^- + p \to \Sigma^0 + K^0$. The experimental data of Baltay et al[8] then give cross sections for the $\pi^+ + p \to \Sigma^+ + K^+$ reaction close to threshold a value smaller than is allowed by charge independence.

We now assume that the interaction between the K meson and hyperon is charge independent. Taking for the cross section $\sigma_+ = \sigma(\pi^+ p \to \Sigma^+ K^+)$ at

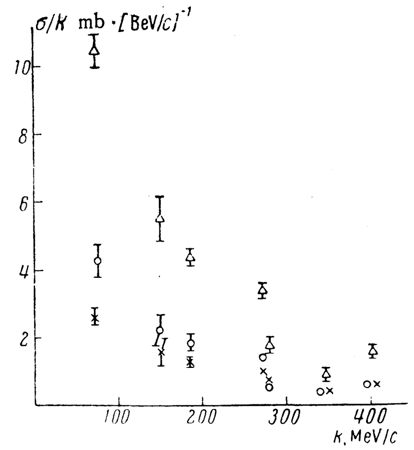

FIG. 1. Experimental values of normalized cross sections as functions of the c.m.s. K-meson momentum: x – for $\pi^- + p \to \Sigma^- + K^+$; o – for $\pi^- + p \to \Sigma^0 + K^0$; Δ – for $\pi + N \to \Sigma + K$ in a state with isotopic spin $T = 1/2$.[7,9]

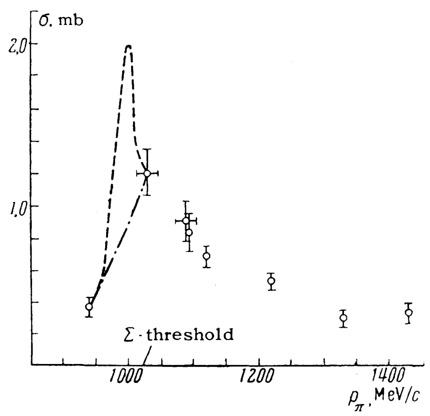

FIG. 2. Cross section for the $\pi^- + p \to \Lambda + K$ reaction as a function of the pion laboratory momentum.[7,9] The dashed curve represents the case of a real ΣK level and the dashed dotted curve represents the case of a virtual level.

low energies the smallest value allowed by charge independence, i.e., $\sigma_+ = \sigma_{3/2} = (\sqrt{2}\,\sigma_0 - \sqrt{\sigma_-}\,)^2$, we obtain for $\sigma_{1/2}$ the energy dependence shown in Fig. 1.

The dependence of the amplitude $T_{N\Sigma}$ on the energy is given by formula (8a). The quantity $\Delta(x)$ in the denominator of (8) can be written in the form

$$\Delta(x) = \Delta(0)\,[1 + (\varepsilon_1 + i\varepsilon_2)\,x].$$

From analysis of the data obtained in the threshold experiments of Wolf et al.,[7] it follows that $\epsilon_2 \sim -0.3$ and $|\epsilon_1| \sim 0.2$. Hence, above and in the direct vicinity of the threshold, the dependence of Δ on k can be neglected. We then obtain for the cross section $\sigma_{1/2}(\Sigma K)$

$$\sigma_{1/2} = \frac{\beta_N^2 + (\alpha_\Lambda\beta_N - \alpha_{N\Lambda}\beta_\Lambda)^2}{|\Delta(0)|^2}\,\frac{k\,|\varkappa|}{k^2 + \varkappa^2}\,. \tag{9}$$

Comparing (9) with Fig. 1, we obtain $|\kappa| \sim 140$ MeV. This means that the Σ + K system has a real or virtual level of energy $E \sim 30$ MeV.

To determine the sign of κ, we use the data on the energy dependence of the cross section for the $\pi N \to \Lambda K$ reaction in this region[7,9] (see Fig. 2). This cross section drops sharply as we go above and below the ΣK threshold energy. Such a drop can be understood with the aid of formula (8b). For large $|k| \gg |\kappa|$, we have $x = \operatorname{sign}\kappa$. Neglecting, as before, the dependence of Δ on k, we obtain for the partial cross section

$$\sigma(k \gg \varkappa)/\sigma(k = 0) = [1 - (\beta_N\beta_\Lambda/\alpha_{N\Lambda})\operatorname{sign}\varkappa]^2. \tag{10}$$

Hence

$$(\beta_N\beta_\Lambda/\alpha_{N\Lambda})\operatorname{sign}\varkappa \sim 1, \tag{11}$$

i.e., the sign of κ is the same as the sign of $\beta_N\beta_\Lambda/\alpha_{N\Lambda}$. A change of k by the quantity κ corresponds to $\Delta p_{\pi\,\mathrm{lab}} \sim 50$ MeV/c, so that the drop in the cross section $\sigma_{K\Lambda}$ in formula (8b) actually occurs in a narrow region of variation of p_π.

In order to find now the sign of $\beta_N\beta_\Lambda\alpha_{N\Lambda}^{-1}$, we use data on the character of the singularity in the cross section for the $\pi^- + p \to \Lambda + K$ reaction at the ΣK threshold.[10] Below threshold, $x \approx |k/\kappa|$, and the slope of the $\sigma_{\Lambda K}(E)$ curve is determined primarily by the numerator of formula (8b):

$$\sigma_{\Lambda K} = \sigma_{\mathrm{thr}} - 2\sigma_{\mathrm{thr}}\beta_N\beta_\Lambda\alpha_{N\Lambda}^{-1}\,|k|/|\varkappa|. \tag{12}$$

The experimental results (see Fig. 11 in [7]) apparently indicate an increase in $\sigma_{\Lambda K}$ below threshold. If this is so, then $\beta_N\beta_\Lambda\alpha_{N\Lambda}^{-1} < 0$, which, together with (11), leads to the conclusion that $\kappa < 0$, i.e., the level in the Σ + K system is real.

Below threshold, the formula for $T_{N\Lambda}$ takes the form

$$T_{N\Lambda} = \frac{\alpha_{N\Lambda}}{\Delta(0)}\,\frac{\varkappa + |k| - |k|\,\beta_N\beta_\Lambda\alpha_{N\Lambda}^{-1}\operatorname{sign}\varkappa}{\varkappa + |k| + |k|\,(\varepsilon_1 + i\varepsilon_2)\operatorname{sign}\varkappa}\,. \tag{13}$$

Since $\kappa < 0$ and ϵ_2 is small, the cross section for the $\pi + N \to \Lambda + K$ reaction close to $|k| = -\kappa$ will have a sharp peak of width $\sim|\kappa\epsilon_2|$. After a kinematical recalculation of the position and width of this resonance in the laboratory system, we obtain

$$p_\pi^{\mathrm{res}} \approx p_\pi^{\mathrm{thr}} - \frac{\varkappa^2}{(1+\varepsilon_1)^2}\,\frac{(M_\Sigma + m_K)^2}{2M_\Sigma M_N m_K} \sim 990\ \mathrm{MeV}/c,$$

$$\Delta p_\pi \approx 2\,|\varepsilon_2|\,(p_\pi^{\mathrm{res}} - p_\pi^{\mathrm{thr}}) \sim 25\ \mathrm{MeV}/c.$$

The height of the peak is more sensitive to the values of the parameters α and β and cannot be predicted reliably. The approximate behavior of the ΛK production cross section in this region, for the case $\kappa < 0$, is plotted in Fig. 2 as a dotted line. This ΛK resonance corresponds to the production of a "(Σ + K)-deuteron" in the intermediate state of the reaction $\pi + N \rightarrow \Lambda + K$.

Along with the absolute value of the cross section in this region, the angular distribution and the polarization of the Λ particle will also change sharply. The $S_{1/2}$ state should predominate in the resonance if Σ and Λ have the same parity, and the $P_{1/2}$ state should predominate if the parity is different. In both cases, the asymmetry of the angular distribution and the polarization in the resonance should be less than in the region close to the ΣK threshold.

A similar resonance determined by formula (8c) should also be observed in πN scattering with $T = 1/2$. However, the observation of this resonance is much more difficult than in the case of ΛK production, since it should be observed on a background of the large third πN resonance. The third resonance itself cannot be explained by means of the "(Σ + K)-deuteron." Its large value, $\sigma_{res} = 30 - 45$ mb $= (2 - 3) \times 4\pi\lambda_\pi^2$, signifies that the resonance goes to higher harmonics.[11] Moreover, the resonance is much broader than follows from (8c).

Above, we have used the three-channel formulas (8). If the channels for the production of several pions are taken into account, then the general form of the dependence of the matrix elements of T on the energy close to threshold has the same form as (8), but the quantities $\alpha_{N\Lambda}$ and β_N in formulas (8) become complex. The experimentally observed sharp drop in the cross section for the $p(\pi^-, K)\Lambda$ reaction away from the threshold indicates that the phase shift of the quantity $\beta_\Lambda \beta_N \alpha_{N\Lambda}^{-1}$ is small. In this case, allowance for the inelastic channels leads only to a redetermination of the constant, so that the foregoing qualitative conclusion still holds.

A resonance should also be observed in the inelastic channels $N + \pi \rightarrow N + n\pi$. Evidently, the value of the resonance in this reaction, as in the elastic scattering reaction, is smaller than in $\pi + N \rightarrow \Lambda + K$. The value of the resonance is proportional to the relative probability of the decay of the Σ + K system via the respective channels. It should be expected that the probability for the decay into ΛK is the largest, since the $\Sigma K \rightarrow \Lambda K$ transition proceeds through the exchange of pions and occurs in a region whose dimensions are larger than the dimensions of the region for the transition $\Sigma + K \rightarrow N + n\pi$. In a strong interaction not too close to the threshold, the probability for the transition is apparently proportional to the size of the interaction region. The measurements of the heights of the resonance peaks in the reactions $N + \pi \rightarrow N + \pi$ and $N + \pi \rightarrow N + n\pi$ are thus of physical interest, since they can serve as a measure of the corresponding interaction.

4. ΛK THRESHOLD

The connection between πN scattering and ΣK production close to threshold is described by formula (7). Although this formula contains only three parameters, the experimental data are insufficient for their determination and only estimates can be made. The estimate below gives $\beta^2 \ll 1$. Hence, above threshold, we can neglect the term $\beta^2 k/|\kappa|$ in the denominator of formula (7), and we then obtain for the production cross section

$$\sigma = 4\pi\lambda_\pi^2 \beta^2 k |\varkappa| / (1 + \alpha^2)(\varkappa^2 + k^2). \tag{14}$$

The energy dependence of the cross section is determined by the ΛK interaction in the final state and is the same in all reactions in which the Λ and K are produced close to threshold. Hence, to determine $|\kappa|$, we also use the data on photoproduction.[12] Despite the large statistical errors, it is seen from Fig. 3 that σ/k actually decreases with energy and $|\kappa| \sim 200$ MeV/c. In order to determine β, we use the value of the cross section for the reaction $\pi + N \rightarrow \Lambda + K$ at $p_\pi = 940$ MeV/c.[9] With the aid of formula (14), we obtain $\beta^2 \sim 0.06$.

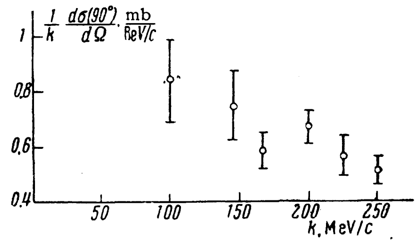

FIG. 3. Normalized differential cross section for the $\gamma + p \rightarrow \Lambda + K$ reaction at 90° as a function of the K-meson momentum in the rest system of the K meson. The data were obtained by the Cornell University group.[12]

The cross section for πN elastic scattering below the ΛK threshold is determined by formula (7), in which the substitution $k \rightarrow i|k|$ should be made:

$$T_{NN} = \frac{\alpha(\varkappa + |k|) - \beta^2 |k| \operatorname{sign} \varkappa}{(1 - i\alpha)(\varkappa + |k|) + i\beta^2 |k| \operatorname{sign} \varkappa}. \tag{15}$$

If $\kappa > 0$, then, owing to the smallness of β^2, there should be practically no effect on the elastic channel. If $\kappa < 0$, then, for $|k| = -\kappa$, the cross section will have a sharp peak. From formula (15), it is seen that, depending on the sign of α, the cross section has a minimum on the right or the left of the peak.

After kinematical recalculation, formula (15) can be brought to the form of the ordinary Breit-Wigner formula

$$S = 1 + 2iT = \frac{1 + i\alpha}{1 - i\alpha}\left(1 - \frac{i\Gamma_e}{E - E_m + i\Gamma_e/2}\right), \quad (16)$$

where $E_m^{lab} \approx 600$ MeV and $\Gamma_e \sim 24$ MeV.

Formulas (7) do not take into account the inelastic channels $N + n\pi$. Their influence can be taken into account in the usual way through the replacement of Γ_e in the denominator of formula (16) by the total width Γ, which makes the resonance somewhat broader. There should also be a resonance at this point in the inelastic cross section. Generally speaking, the resonance should be observed on a large background of nonresonance scattering. However, if the πN and ΛK parities are the same[5] and the reaction $N + \pi \rightarrow N + 2\pi$ proceeds primarily through the production of the 33 isobar:[13] $N + \pi \rightarrow N^* + \pi \rightarrow N + 2\pi$, then the nonresonance cross section in the $S_{1/2}$ channel is small, since the "isobaric" mechanism is forbidden by the momentum and parity selection rules. If it is established experimentally that $N + 2\pi$ is produced in the S state, then the increase in the cross section in the resonance region will be appreciable.

Since the quantities β and κ depend on $k^2/4m_\pi^2$, then the procedure above by which formula (15) was extrapolated with constant β and κ to $|k| = -\kappa$ for $|\kappa| \sim 200$ MeV/c $\sim 1.5\, m_\pi$, is not strictly justified. However, as in Sec. 3, only the sign of κ is important. If $\kappa(0) < 0$, then the ΛK "bound state" exists, and the described resonance phenomena in πN scattering occur, although the accurate position of the resonance for large $|\kappa / 2m_\pi| (\sim 1)$ cannot be obtained directly from experiments on ΛK production. The presence or absence of these types of peaks in πN scattering for the indicated region will also be a direct check on the existence of the "$(\Lambda + K)$-deuteron", since here the influence of the ΛK channel on the πN channel is small everywhere outside the resonance region, and there is no other way of determining the sign of $\kappa(0)$.

Experiment indicates that a second pion-nucleon resonance with $T = 1/2$ is located in this region. Its width, however, is 3 — 4 times as great as the estimates from (16). Nevertheless, it is not excluded that the second πN resonance is a consequence of the existence of a ΛK bound state, since the dependence of β on k and the influence of the inelastic channels can appreciably increase the resonance width.

5. GLOBAL SYMMETRY. OTHER RESONANCES

The behavior of the cross sections close to threshold can lead to some conclusions about the form of the interaction between K mesons and hyperons. If the K-hyperon bound state exists, this would directly indicate the predominance of attractive forces in the interaction. If such a bound state does not exist, then it is possible to draw some conclusions about the sign of the peripheral interaction from the behavior of the production cross section for these particles at low energies, since, after the separation of the statistical weight $\sim k$, this behavior is determined basically by the interaction in the final state. The K mesons are produced in the inner region of size $r \lesssim (m_K + m_\pi)^{-1}$ and then interact with the hyperon through exchange of pions. If this interaction is repulsive, then the penetrability of the potential barrier and the quantity σ/k should increase with the energy. If, as a simple example, we consider the Schrödinger equation with a rectangular potential well, it can be seen that in formula (7) the quantity $\beta(k)$ then increases while $\kappa(k)$ drops, so that the quantity $\beta^2 |\kappa| (\kappa^2 + k^2)^{-1}$ increases. Since the experimental data indicate that, for the production of ΣK with $T = 1/2$ and ΛK, the value of σ/k decreases as we go away from the threshold, the peripheral interaction between these particles is apparently attractive.

The results obtained contradict the hypothesis on global symmetry. According to this hypothesis, the interaction between hyperons and K mesons can be expressed through the interaction of nucleons with K mesons. The exact same situation also holds for the $Y + N$ and $N + N$ interactions.[4] The Schrödinger equations for the systems $K + \Lambda$ and $K + \Sigma$ with $T = 1/2$ have the form

$$-(2\mu_\Sigma)^{-1}\nabla^2\psi_\Sigma + V_{\Sigma\Sigma}\psi_\Sigma + V_{\Sigma\Lambda}\psi_\Lambda = (E - m_\Sigma - m_K)\psi_\Sigma,$$
$$-(2\mu_\Lambda)^{-1}\nabla^2\psi_\Lambda + V_{\Lambda\Lambda}\psi_\Lambda + V_{\Lambda\Sigma}\psi_\Sigma = (E - m_\Lambda - m_K)\psi_\Lambda. \quad (17)$$

Here μ_Λ and μ_Σ are the ΛK and ΣK reduced masses; the matrix elements of V are expressed in terms of the V_0 and V_1 interactions of K mesons with nucleons in states with isotopic spins 0 and 1, respectively:

$$V_{\Lambda\Lambda} = \tfrac{1}{4}(V_0 + 3V_1), \qquad V_{\Sigma\Lambda} = V_{\Lambda\Sigma} = \tfrac{1}{4}\sqrt{3}\,(V_0 - V_1),$$
$$V_{\Sigma\Sigma}(T = 1/2) = \tfrac{1}{4}(3V_0 + V_1). \quad (18)$$

Experiments on the scattering of K mesons on protons and nuclei[14] indicate that V_0 and V_1 are greater than zero, so that the potentials $V_{\Lambda\Lambda}$,

$V_{\Lambda\Sigma}$, and $V_{\Sigma\Sigma}$ obtained from the global symmetry formulas (18) cannot lead to the indicated attractive effect in the ΣK and ΛK systems.

The interaction between K mesons rather resembles the interaction of the $\overline{K}$ with a nucleon. Dalitz and Tuan[2-5] showed that the Y_1 and the recently discovered Y_0 resonance[16] can be interpreted as weakly bound states of the $\overline{K} + N$ system. The $\overline{K} + N$ binding energy is then 46 MeV for Y_1 and 27 MeV for Y_0, which is close to the values ~ 30 MeV and ~ 60 MeV obtained above for the $\Sigma + K$ and $\Lambda + K$ systems, respectively (if the ΛK level is real).

We shall now consider which other known resonances in the high-energy region can be associated with threshold effects. We can, for example, expect a sharp change in the cross sections close to the production thresholds for the unstable mesons ρ, ω, and K^*.[17] As has been shown above, these effects will be particularly appreciable if the attraction proves sufficient for the production of a bound state. The production thresholds for ρ and ω mesons in the reactions $N + \pi \rightarrow N + \rho$ and $N + \pi \rightarrow N + \omega$ are $T_\pi = 900$ and $T_\pi = 965$ MeV, respectively. That is why the third πN resonance at $T_\pi = 890$ MeV can be connected with the existence of levels in the $N + \rho$ and (or) $N + \omega$ systems with binding energies $E_\rho \approx 5$ MeV and $E_\omega = 42$ MeV, respectively. If this interpretation is correct and if the $N + \rho$ and $N + \omega$ systems are in the S state, then the angular momentum of this resonance cannot be greater than $3/2$.

Similarly, the K^-p resonance Λ^* with mass[18] 1812 MeV can be associated with the threshold for the $K^- + p \rightarrow K^* + N$ reaction situated at $E_{cms} = 1823$ MeV. The possibility that the third πN and K^-p resonances are connected with the thresholds for the $N + \rho$ and $N + K^*$ reactions has been considered by Ball and Frazer[19] with the aid of the dispersion relations.

The threshold for the $K^- + p \rightarrow \Xi + K$ reaction[1)] is also located in the region of the K^-p resonance. If it is assumed that the Ξ and K form a bound state, then the K^-p total cross section will have a resonance. But since the cross section for the $K^- + p \rightarrow \Xi + K$ reaction is very small,[20] the width of the resonance should be small ~ 5 MeV.

[1)]We note the remarkable closeness of the thresholds for the reactions $\pi + N \rightarrow \Sigma + K (M_\Sigma + m_K = 1690$ MeV) and $\pi + N \rightarrow N + \rho (M + m_\rho \approx 1690$ MeV) and also the reactions $K^- + p \rightarrow \Xi + K (M_{\Xi^-} + m_{K^+} = 1812$ MeV) and $K^- + p \rightarrow K^* + N$ ($M_p + m_{K^*_-} = 1823$ MeV).

6. CONCLUSIONS

If the considerations on the threshold origin of the resonances discussed above are correct, then these resonances are secondary phenomena due to peripheral interactions of the particles in the threshold vicinity. In this case, the study of the resonances, together with the behavior of the production cross section close to threshold, can be a convenient means of obtaining information on the interaction between strange and unstable particles at low energies.

In order to explain the character of the interaction between the Σ and K in the $T = 1/2$ state, it would be of interest to investigate the behavior of the cross section for the $\pi + N \rightarrow \Lambda + K$ reaction in the interval $T_\pi = 810 - 900$ MeV (Fig. 2) and the $\pi^- + p \rightarrow \Sigma + K$ reaction close to threshold. The study of the $\pi^+ + p \rightarrow \Sigma^+ + K^+$ reaction close to threshold would be of interest to obtain information on the ΣK interaction in the $T = 3/2$ state as well as for the removal of the contradictions with charge independence (see Sec. 3). In order to study the ΛK interaction, we should seek the fine structure of the second πN resonance (Sec. 4) and also find the behavior of the cross sections for the $\pi + N \rightarrow \Lambda + K$ and $\gamma + p \rightarrow \Lambda + K$ reactions close to threshold. Finally, it would be very interesting to study the behavior of the production cross sections for unstable particles $K^- + N \rightarrow K^* + N$ and $\pi + N \rightarrow \rho + N$ as well as $\Xi + K$ close to their thresholds.

Note added in proof: According to recent reports on experimental work, a ΛK resonance was found in [21] in the region under consideration, while no resonance was found in [22]. Further experimental investigations are desirable.

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