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HYDRODYNAMIC DESCRIPTION OF THE MOTION OF A CHARGED PARTICLE IN A WEAKLY IONIZED PLASMA

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A statistical derivation is given for the hydrodynamic equations that describe the motion of a charged particle in a weakly ionized plasma. Among the hydrodynamic functions required for the description of the motion of the charged particle in a weakly ionized plasma are the space correlation functions. The interaction of charged particles with neutral particles is described by the introduction of an effective collision number. Several solutions of the equations are given.

In deriving the hydrodynamic equations for a highly ionized plasma one can use the kinetic equations for the charged-particle distribution functions.[1] The kinetic equations can be used for a highly ionized plasma because when $\mu = e^2/r_d kT \ll 1$ the time required for establishing the momentum equilibrium state is given by $\tau \sim 1/\omega_L\mu \gg 1/\omega_L$. This time is appreciably greater than the correlation time for distribution functions $g_{ab}(\mathbf{q}, \mathbf{q}', \mathbf{p}, \mathbf{p}', t)$ that describe initial states of the plasma that do not deviate greatly from equilibrium states i.e., $\tau \gg \tau_{cor}$. For this reason the plasma correlation functions g_{ab} (here a and b are subscripts denoting the particle species) can be expressed in terms of slowly varying distribution functions. By slowly varying we mean $\partial f_a/\partial t \ll f_a/\tau_{cor}$ and $\partial f_a/\partial q \ll f_a/r_{cor}$.

In turn, the use of the kinetic equations leads to a closed system of equations for the hydrodynamic functions, which do not change greatly even in a time $\tau \gg \tau_{cor}$.

The situation is different in a weakly ionized plasma. Here, the plasma frequency ω_L is smaller than, or of the order of, the collision frequency for collisions between charged particles and neutrals, i.e., $\omega_L \lesssim \nu$. But $\omega_L\mu \ll \nu$. This condition is then the criterion for a weakly ionized plasma. When it is satisfied the time required to establish a local Maxwellian distribution over the charged particles ($\sim 1/\nu$) is found to be smaller than, or of the order of, the time required for establishing the equilibrium correlation function, the Debye function. For this reason, in this case the correlation functions $g_{ab}(\mathbf{q}, \mathbf{q}', \mathbf{p}, \mathbf{p}', t)$ can not be expressed in general form in terms of the initial distribution functions; thus one cannot obtain closed equations for the initial distribution functions (the kinetic equations for the charged particles).

In a weakly ionized plasma it is sometimes possible to use a method for obtaining the hydrodynamic equations that was developed by Bogolyubov, Gurov, and Born and Green[2], in which one starts out with a chain of equations for the distribution functions. In the present paper we present another method for deriving these equations; our method is based on the equations that describe the random phase densities.[3]

The equations obtained below differ from the usual hydrodynamic equations:[4] in our case, in addition to the equations describing the density and velocity of charged particles there is also an equation for the space correlation functions. By using this system of equations one can also provide a hydrodynamic description of non-equilibrium processes in cases in which the basic effect is the temporal change of a spatial spectrum while the average velocity and density remain essentially constant. Appropriate examples will be considered below. We first derive the equations, using the notation

$$N_a(\mathbf{q}, \mathbf{p}, t) = \sum_{1 \leqslant i \leqslant N_a} \delta(\mathbf{q} - \mathbf{q}_i(t))\, \delta(\mathbf{p} - \mathbf{p}_i(t)),$$

$$N_n(\mathbf{q}, \mathbf{p}, t) = \sum_{1 \leqslant i \leqslant N_n} \delta(\mathbf{q} - \mathbf{q}_i(t))\, \delta(\mathbf{p} - \mathbf{p}_i(t))$$

to denote the phase-space density of the charged particles and neutrals respectively. Here, the subscript a refers only to the charged particles; N_a and N_n are the total numbers of particles of the corresponding species.

In the presence of an external electric field

$\mathbf{E}_0$ the equation for the functions N_a is of the form

$$\frac{\partial N_a}{\partial t} + \mathbf{v}\frac{\partial N_a}{\partial \mathbf{q}} + e_a\mathbf{E}_0\frac{\partial N_a}{\partial \mathbf{p}} - \frac{\partial}{\partial \mathbf{q}}\sum_b \int \frac{e_a e_b}{|\mathbf{q}-\mathbf{q}'|} N_b(\mathbf{q}', \mathbf{p}', t)\, d\mathbf{q}'\, d\mathbf{p}' \frac{\partial N_a}{\partial \mathbf{p}} = S_a. \quad (1)$$

The right side of this equation

$$S_a = \frac{\partial}{\partial \mathbf{q}}\sum_n \int U_{an}(\mathbf{q}, \mathbf{q}') N_n(\mathbf{q}', \mathbf{p}', t)\, d\mathbf{q}' d\mathbf{p}' \frac{\partial N_a}{\partial \mathbf{p}} \quad (2)$$

describes the effect of neutrals on the charged particles while U_{an} is the potential energy of the interaction between particles of species a and n. Below we shall also introduce equations for the functions $\delta N_a = N_a - \overline{N}_a$. The bar denotes an average over an ensemble.

When $\mu = e^2/r_d kT \ll 1$ we can neglect the deviation $\delta N_a \delta N_b - \overline{\delta N_a \delta N_b}$. We also neglect the term containing the mean internal field in the plasma since this term is usually small compared with others; we then obtain the equation

$$\frac{\partial \delta N_a}{\partial t} + \mathbf{v}\frac{\partial \delta N_a}{\partial q} + e_a\mathbf{E}_0\frac{\partial \delta N_a}{\partial \mathbf{p}} - \frac{\partial}{\partial \mathbf{q}}\sum_b \int \frac{e_a e_b}{|\mathbf{q}-\mathbf{q}'|}\delta N_b d\mathbf{q}' d\mathbf{p}' \frac{\partial \overline{N}_a}{\partial \mathbf{p}} = \delta S_a. \quad (3)$$

Equations for the functions f_a and g_{ab} can be obtained from (1) and (3). Account must be taken of the fact that

$$\overline{N}_a(\mathbf{q}, \mathbf{p}, t) = n_a f_a(\mathbf{q}, \mathbf{p}, t), \qquad n_a = N_a/V, \quad (4)$$

$$\overline{\delta N_a(\mathbf{q}, \mathbf{p}, t)\, \delta N_b(\mathbf{q}', \mathbf{p}', t)} = \delta_{ab}\delta(\mathbf{q}-\mathbf{q}')\,\delta(\mathbf{p}-\mathbf{p}')\, n_a f_a + n_a n_b g_{ab}. \quad (5)$$

Using (1) and (3) we can write an infinite system of equations for the hydrodynamic random functions (defined as the moments $\int v_i v_j \ldots N_a \times (\mathbf{q}, \mathbf{p}, t)\, d\mathbf{p}$) and their deviations from the mean values. We have

$$\frac{\partial}{\partial t}\int N_a d\mathbf{p} + \frac{\partial}{\partial q_i}\int v_i N_a d\mathbf{p} = 0, \quad (6)$$

$$\frac{\partial}{\partial t}\int v_i N_a d\mathbf{p} + \frac{\partial}{\partial q_j}\int v_i v_j N_a d\mathbf{p} - \frac{e_a}{m_a} E_{0i}\int N_a d\mathbf{p} + \frac{1}{m_a}\frac{\partial}{\partial q_i}\sum_b \int \frac{e_a e_b}{|\mathbf{q}-\mathbf{q}'|} N_b(\mathbf{q}', \mathbf{p}', t)\, d\mathbf{q}' d\mathbf{p}' \int N\, dp = \int v_i S_a d\mathbf{p} \quad (7)$$

etc. Correspondingly, the deviations are given by

$$\frac{\partial}{\partial t}\int \delta N_a d\mathbf{p} + \frac{\partial}{\partial q_i}\int v_i \delta N_a d\mathbf{p} = 0, \quad (8)$$

$$\frac{\partial}{\partial t}\int v_i \delta N_a d\mathbf{p} + \frac{\partial}{\partial q_i}\int v_i v_j \delta N_a\, d\mathbf{p} - \frac{e_a}{m_a} E_{0i}\int \delta N_a d\mathbf{p} + \frac{1}{m_a}\frac{\partial}{\partial q_i}\sum_b \int \frac{e_a e_b}{|\mathbf{q}-\mathbf{q}'|}\delta N_b d\mathbf{q}' d\mathbf{p}' \int \overline{N}_a d\mathbf{p} = \int v_i \delta S_a d\mathbf{p} \quad (9)$$

etc. These equations are used for the derivation of the hydrodynamic equations.

It has been noted above that in a weakly ionized plasma one can not express the correlation functions for the charged particles in the plasma in terms of the initial distribution functions to obtain the kinetic equations. As a consequence, in the hydrodynamic approximation the system of equations that describes the motion of the charged particles not only contains equations for the hydrodynamic functions ρ_a and u^a, but also contains equations for the space correlation functions $\gamma_{ab}(q, q', t)$.

In order to see explicitly the difference in the hydrodynamic approximation for a weakly ionized plasma we shall treat the case in which the density and temperature of the neutral particles have fixed assigned values while the mean velocity is zero. Since the collision frequency $\nu \gg \omega_L \mu = 1/\tau$ it may be assumed that the momentum distribution of the particles is approximately Maxwellian. Thus, in the first approximation we have

$$n_a f_a(q, p, t) = \rho_a(q, t)\left[\frac{1}{2\pi m_a\Theta}\right]^{3/2}\exp\left[-\frac{(p - m_a u^a(q, t))^2}{2m_a\Theta}\right],$$

$$g_{ab}(q, q', p, p', t) = \gamma_{ab}\left[\frac{1}{4\pi^2 m_a m_b \Theta^2}\right]^{3/2}\exp\left[-\frac{(p - m_a w_a^b)^2}{2m_a\Theta} - \frac{(p' - m_b w_b^a)^2}{2m_b\Theta}\right], \quad (10)$$

Here

$$\gamma_{ab}(q, q', t) = \int g_{ab}\, dp dp', \quad W_a^b(q, q', t) = \frac{1}{\gamma_{ab}}\int v g_{ab} dp dp', \quad w_b^a = \frac{1}{\gamma_{ab}}\int v' g_{ab} dp dp'. \quad (11)$$

To obtain equations for the functions ρ_a and u^a we multiply (6) and (7) by n_a and average. The result is

$$\frac{\partial \rho_a}{\partial t} + \frac{\partial}{\partial \mathbf{q}}(\rho_a \mathbf{u}^a) = 0, \quad (12)$$

$$\frac{\partial(\rho_a u_i^a)}{\partial t} + \frac{\partial}{\partial q_j}(\rho_a u_i^a u_j^a) = \frac{e_a}{m_a}E_i\rho_a - \frac{1}{m_a}\frac{\partial}{\partial q_i}(\rho_a\Theta) - \frac{n_a}{m_a}\sum_b n_b \int \frac{\partial}{\partial q_i}\frac{e_a e_b}{\varepsilon_0|\mathbf{q}-\mathbf{q}'|}\gamma_{ab}(\mathbf{q}, \mathbf{q}')\, d\mathbf{q}' - \nu_a \rho_a u_i^a. \quad (13)$$

Here ε_0 is the dielectric constant of the neutral component. The system of equations is not closed since the function γ_{ab} appears everywhere. Multiplying (8) and (9) by $\delta N_b(\mathbf{q}, \mathbf{q}', t)$, integrating over $\mathbf{p}'$, and averaging, we obtain the corresponding equations for the b component, the functions γ_{ab}, and w_a^b:

$$\frac{\partial \gamma_{ab}}{\partial t} + \frac{\partial}{\partial \mathbf{q}} (\gamma_{ab} \mathbf{w}_a^b) + \frac{\partial}{\partial \mathbf{q}'} (\gamma_{ab} \mathbf{w}_b^a) = 0, \tag{14}$$

$$\frac{\partial}{\partial t} \Big(\gamma_{ab} w_{ai}^a + \frac{\partial}{\partial q_j} (\gamma_{ab} w_{ai}^b w_{aj}^b) = \frac{e_a}{m_a} E_i \gamma_{ab}$$

$$- \frac{1}{m_a} \frac{\partial}{\partial q_i} (\gamma_{ab} \Theta) - \frac{\rho_a \rho_b}{n_a n_b m_a} \frac{\partial}{\partial q_i} \frac{e_a e_b}{\varepsilon_0 |\mathbf{q} - \mathbf{q}'|}$$

$$- \frac{1}{m_a} \sum_c n_c \frac{\partial}{\partial q_i} \int \frac{e_a e_b}{\varepsilon_0 |\mathbf{q} - \mathbf{q}'|} \gamma_{bc} (\mathbf{q}', \mathbf{q}'') \, d\mathbf{q}'' - \nu_a \gamma_{ab} w_{ai}^b \tag{15}$$

and an analogous equation for the function w_b^a.

The relations that have been obtained then give a closed system of equations for the functions ρ_a, u^a, γ_{ab}, w_a^b and w_b^a. If

$$\partial (\rho_a \mathbf{u}^a)/\partial t \ll \nu_a \rho_a \mathbf{u}^a, \qquad \partial (\gamma_{ab} \mathbf{w}_b^a)/\partial t \ll \nu_a \gamma_{ab} \mathbf{w}_a^b$$

in (13) and (15) and the terms containing $\rho_a u_i^a u_j^a$ and $\gamma_{ab} w_{ai}^b w_{aj}^b$ are small, we can eliminate the functions u^a, w_a^b and w_b^a and obtain a closed system of equations for the functions ρ_a and γ_{ab}; these coincide with the corresponding equations of the classical theory of electrolytes.[5]

In the derivation of (13) and (15) it has been assumed that the microscopic force exerted on a charged particle of species a by the neutral particles is proportional to the velocity of the charged particle, that is to say,

$$S_a = \frac{\partial}{\partial q} \sum_n \int U_{an} (\mathbf{q}, \mathbf{q}') \, N_n (\mathbf{q}', \mathbf{p}', t) \, d\mathbf{q}' d\mathbf{p}' \frac{\partial N_a}{\partial \mathbf{p}}$$

$$= \frac{\partial}{\partial \mathbf{p}} (m_a \nu_a \mathbf{v} N_a (\mathbf{q}, \mathbf{p}, t)).$$

Here ν_a is the corresponding collision frequency. This approximation is a good one if, as assumed above, the neutral particles are characterized by a local Maxwellian distribution and their mean velocity is zero.

Using this formula we obtain the expression appearing on the right side of (13)

$$\int \mathbf{v} \overline{S}_a dp = - \sum_n \frac{\partial}{\partial \mathbf{q}} \int U_{an} (\mathbf{q}, \mathbf{q}') \, \overline{N_n (\mathbf{q}', \mathbf{p}') \, N_a (\mathbf{q}, \mathbf{p})} \, d\mathbf{q}' d\mathbf{p}' dp$$

$$= - \nu_a \mathbf{u}^a \rho_a.$$

In the derivation of (15) we have utilized the fact that

$$\int \mathbf{v} \delta S_a \delta N_a d\mathbf{p} d\mathbf{p}' = - \int \nu_a \mathbf{v} \, [\overline{N_a (\mathbf{q}, \mathbf{p}) \, N_b (\mathbf{q}', \mathbf{p}')}$$

$$- \overline{N_a (\mathbf{q}\ \mathbf{p})} \; \overline{N_b (\mathbf{q}', \mathbf{p}')}] \, d\mathbf{p} d\mathbf{p}'$$

$$= - \nu_a \int v \, \overline{\delta N_a \delta N_b} d\mathbf{p} d\mathbf{p}'.$$

Using (5) in (15) we obtain the term $-\nu_a \gamma_{ab} w_a^b$.

As an example we shall treat a uniform electron plasma with a positive background $\rho_e = n_e$, $\mathbf{E}_0 = 0$. Using (14) and (15) and neglecting the $\gamma_{ab} w_{ai}^b N_{aj}^b$ terms we obtain the equation

$$\nu \frac{\partial \gamma (\mathbf{r}, t)}{\partial t} + \frac{\partial^2 \gamma}{\partial t^2} = s_e^2 (\Delta \gamma - \varkappa^2 \gamma) - \frac{2\omega_L^2}{n_e} \delta (\mathbf{r}). \tag{16}$$

Here

$$\varkappa^2 = 4\pi e^2 n_e / \varepsilon_0 \Theta, \qquad s_e^2 = 2\Theta / m_e, \qquad \omega_L^2 = 4\pi e^2 n_e / \varepsilon_0 m_e.$$

For any initial deviation from the Debye distribution the spatial Fourier components of the correlation function $\gamma(\mathbf{r}, t)$ can be written in the form

$$\gamma (k, t) = - \frac{1}{n_e} \frac{\varkappa^2}{k^2 + \varkappa^2} + e^{-\nu t/2} \left[A_k \frac{\cos \eta t}{\cosh \eta t} + B_k \frac{\sin \eta t}{\sinh \eta t} \right],$$

$$\eta^2 = | s_e^2 (k^2 + \varkappa^2) - \nu^2/4 |. \tag{17}$$

It is evident from the solution that has been obtained that the establishment of the equilibrium correlation Debye function $\gamma^{(0)}(\mathbf{k}) = -\kappa^2/n_e(k^2 + \kappa^2)$ depends on the ratio of the frequency ω_L to the frequency ν and the initial distribution over wave number. For the particular spatial spectrum in which the condition $\omega_L^2 + \Theta k^2/m_e > \nu^2/8$ is satisfied equilibrium is established in a time of order $1/\nu$ and is accompanied by oscillations. The oscillatory mode is sharply expressed if the coefficients A_k and B_k differ from zero when $k \gg \kappa$. If $\omega_L^2 + \Theta k^2/m_e < \nu^2/8$ equilibrium is established aperiodically and slower than for a Maxwellian. If the initial functions A_k and B_k differ from zero only when $k \lesssim \kappa$ and if $\omega_L \ll \nu$, then the equilibrium correlation function is established in a time $\tau_{rel}^1 \sim \nu/\omega_L^2$. This time corresponds to the relaxation time in the theory of strong electrolytes of Debye and Falkenhagen.[5]

We now consider the behavior of a weakly ionized uniform plasma in a weak alternating electric field $E = E_0 e^{i\omega t}$. We introduce the function $\gamma_{ab}^{(1)}(\mathbf{r}, \omega) = \gamma_{ab}(\mathbf{r}, \omega) - \gamma_{ab}^{(0)}$ (here $\gamma_{ab}^{(0)}(\mathbf{r})$ is the equilibrium correlation function, $\gamma_{ab}^{(1)} \ll \gamma_{ab}^{(0)}$). We obtain an equation for $\gamma_{ab}^{(1)}$ from (14) and (15):

$$i\omega \gamma_{ab}^{(1)} = \Theta \left[\frac{1}{m_a (\nu_a + i\omega)} + \frac{1}{m_b (\nu_b + i\omega)} \right] \frac{\partial^2}{\partial r^2} \gamma_{ab}^{(1)}$$

$$+ \left[\frac{e_a}{m_a (\nu_a + i\omega)} - \frac{e_b}{m_b (\nu_b + i\omega)} \right] E \frac{\partial \gamma_{ab}^{(0)}}{\partial r}$$

$$- \frac{4\pi}{\varepsilon_0} \sum_c n_c \left[\frac{e_a e_c}{m_a (\nu_a + i\omega)} \gamma_{cb}^{(1)} (r) + \frac{e_b e_c}{m_b (\nu_b + i\omega)} \gamma_{ac}^{(1)} (r) \right]. \tag{18}$$

When $e_2 = -e_1 = e$ the solution of (18) is

$$\gamma_{11}^{(1)} = \gamma_{22}^{(1)} = 0, \qquad \gamma_{12}^{(1)} = -\gamma_{21}^{(1)},$$

$$\gamma_{12}^{(1)} = \frac{e}{4\pi n\Theta} \frac{1}{1+\omega^2/\omega_L^2 - i\omega\nu_1/\omega_L^2} E \frac{\partial}{\partial r}\left[\frac{e^{-\varkappa r} - e^{-\alpha\varkappa r}}{r}\right]. \tag{19}$$

Here

$$\alpha^2 = \frac{1}{2}\left[1 - \frac{\omega^2}{\omega_L^2} + i\omega\frac{\nu_1}{\omega_L^2}\right], \qquad \omega_L^2 = \frac{4\pi e^2 n}{\varepsilon_0 m_1}$$

and it is assumed that $m_2 \gg m_1$.

The intensity of the electric field acting on the charged particles in the plasma is given by

$$E_g = E_0 e^{i\omega t}\left[1 - \frac{e^2}{6\varepsilon_0\Theta}\frac{\varkappa}{1+\alpha}\right]. \tag{20}$$

The second term in this expression arises since we have taken account of the correlation between charged particles in the plasma. Using (20) we can easily obtain an $\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$ in a weakly ionized plasma taking account of the correlation of charged particles. In view of the uniformity of the plasma and (20) we can write (13) in the form

$$\frac{\partial u^a}{\partial t} = \frac{e_a}{m_a} E_0 e^{i\omega t}\left(1 - \frac{e^2}{6\varepsilon_0\Theta}\frac{\varkappa}{1+\alpha}\right) - \nu u^a.$$

Eliminating the function u^a by means of the equation for the total current we obtain the following expressions for $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$:

$$\varepsilon'(\omega) = \varepsilon_0\left[1 - \frac{\omega_L^2}{\nu_1^2+\omega^2} + \frac{\omega_L^2}{\nu_1^2+\omega^2}\frac{e^2\varkappa}{6\varepsilon_0\Theta} \times\left(\mathrm{Re}\frac{1}{1+\alpha} - \frac{\nu_1}{\omega}\mathrm{Im}\frac{1}{1+\alpha}\right)\right], \tag{21}$$

$$\varepsilon''(\omega) = -\varepsilon_0\frac{\omega_L^2}{\nu_1^2+\omega^2}\frac{\nu_1}{\omega} \times\left[1 - \frac{e^2\varkappa}{6\varepsilon_0\Theta}\mathrm{Re}\frac{1}{1+\alpha} - \frac{\omega}{\nu_1}\frac{e^2\varkappa}{6\varepsilon_0\Theta}\mathrm{Im}\frac{1}{1+\alpha}\right]. \tag{22}$$

If the correlation effects are neglected (21) and (22) become the usual hydrodynamic expressions.[4]

At low frequencies ($\omega \ll \nu_1$) the conductivity $\sigma = \omega\varepsilon''/4\pi$ coincides with the corresponding formula from the theory of electrolytes.[5] When $\omega \ll \nu_1$ (21) differs from the corresponding expression for ε' in the theory of electrolytes by virtue of the term $-\omega_L^2/\nu_1^2$.

The terms due to the interaction in (21) and (22) are important when $\omega_L^2(\nu_1^2 + \omega^2) \sim 1$. It should be kept in mind, however, that the equations given here are valid under the condition that the collision frequency is not small compared with ω_L. When $\nu_1 \ll \omega_L$ additional dissipation processes associated with Landau damping can be important.

In the example considered here the use of the equation in (12)—(15) leads to the appearance of additional terms in (20)—(22) that arise because correlation is taken into account. These additional terms are of order $\varepsilon \sim e^2/r_d\Theta$ and are not important in most cases. For instance, we could have used the usual hydrodynamic equations (12) and (13) with the term containing the correlation function omitted. The additional equations (14) and (15) for the correlation functions in (12)—(15) are important when the effect on the plasma is such that it is important to take account of the temporal change in the space correlation function. An example is the problem of establishment of the stationary state when an external fixed electric field is switched on. The same problem in electrolytes has been treated by Khalatnikov.[6]

Here we shall treat another example: assume that the plasma is subject to a uniform (this is possible at low frequencies) random electric field $\mathbf{E}_0$ whose average value is zero; however, $\overline{E_0^2} \neq 0$. Thus, $\delta\mathbf{E}_0 = \mathbf{E}_0(t)$. In this case it is convenient to start with the equations for the random deviations of the functions $\int v\delta N_a dp$ and $\delta\mathbf{E}$. We write these equations taking account of the uniformity of the plasma and using the total-current equation in place of Poisson's equation. In the linear approximation we obtain the following expressions for δj and $\delta\mathbf{E}$:

$$\frac{\partial\delta j}{\partial t} = \sum_a \frac{e_a^2 n_a}{m_a}\delta E - \nu\delta j; \quad -\frac{\partial E}{\partial t} = 4\pi\delta j + 4\pi\delta j_0. \tag{23}$$

Here, $\delta j_0 = -(4\pi)^{-1}\,\partial\delta E_0/\partial t$ is the fluctuation of the external "transverse" current. Using (23) we find

$$(\delta E\delta E)_\omega = (\delta E_0\delta E_0)_\omega/(\varepsilon'^2 + \varepsilon''^2). \tag{24}$$

At low frequencies ($\omega \ll \nu$) and when $\omega_L \ll \nu$

$$\varepsilon(\omega) = \varepsilon' + i\varepsilon'' = 1 + i\omega_L^2/\nu\omega = 1 + i/\tau_{rel}^1\omega. \tag{25}$$

In (24) $(\delta\mathbf{E}\delta\mathbf{E})_\omega$ is the time spectral function of the electric field in the plasma.

If $(\delta\mathbf{E}_0\delta\mathbf{E}_0)_\omega$ is taken to mean the spectral function of the transverse field, or the equivalent effect of the thermal motion, then

$$(\delta E_0\delta E_0)_\omega = \frac{8\pi}{\omega}\varepsilon''(\omega)\, kT$$

and (24) becomes the Nyquist formula.

If $(\delta\mathbf{E}_0\delta\mathbf{E}_0)_\omega$ is the spectral function of the external field, whose strength is greater than the thermal effect, using (25) we have

$$(\delta E\delta E)_\omega = \frac{(\omega\tau_{rel}^1)^2}{1+(\omega\tau_{rel}^1)^2}(\delta E_0\delta E_0)_\omega, \qquad \tau_{rel}^1 = \frac{\nu}{\omega_L^2}, \tag{26}$$

whence it follows that the reduction in the energy density of the external field due to plasma polarization is

$$\frac{1}{2\pi}\int\frac{(\delta E_0\delta E_0)_\omega - (\delta E\delta E)_\omega}{8\pi}d\omega = \frac{1}{16\pi^2}\int_{-\infty}^{\infty}\frac{(\delta E_0\delta E_0)_\omega}{1+(\omega\tau_{rel}^1)^2}d\omega. \tag{27}$$

If the function $(\delta \mathbf{E}_0 \delta \mathbf{E}_0)_\omega$ is independent of frequency (white noise) the right side of (27) becomes

$$(\delta E_0 \delta E_0)/16\pi\tau^{l}_{rel}.$$

Thus, in a weakly ionized uniform plasma the energy of the external field can be converted into heat by relaxation processes; the corresponding relaxation time is τ^{1}_{rel}.

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