REMARK ON THE RANGE-ENERGY RELATIONSHIP

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The range of heavy charged particles in matter can be expressed with sufficient accuracy by a universal function. The range-energy relation can be written out with the aid of this function in the form $E = aR^m$, where m is independent of the composition of the substance.

1\. As is well known, the energy loss of a heavy particle of charge ze passing through matter is given by

$$\frac{dE}{dR} = \frac{4\pi z^2 e^4}{mv^2} \frac{N_0 Z}{A} B, \qquad (1)$$

where R is the range in g/cm^2, N_0 is Avogadro's number, A and Z the atomic weight and the atomic number of the substance, and v is the particle velocity. We shall call the coefficient B the braking ability of the substance.

Lindhard and Scharff [1] have shown, using Bloch's law [2] $I = KZ$ (where I is the average ionization energy of the atom) that in the approximation of the Thomas-Fermi model the braking ability B depends only on a quantity x, given by

$$x = v^2/Zv_0^2, \qquad v_0 = e^2/\hbar. \qquad (2)$$

Using this fact, it is easy to show that the range of the particle in the substance will have the form $R = AZz^{-2}F(x)$, i.e., it will be expressed in terms of a universal function of x.

However, experiment shows that Bloch's law is inaccurate. Figure 1 shows the Z-dependence of the quantity $K = I/Z$, obtained as a result of several investigations. The full circles denote data for protons with energy higher than 340 MeV [3-5]. In reducing these data, only corrections for the relativistic effect were taken into account in the formula for the losses. The average ionization energy of aluminum was taken to be 163 eV. The light circles denote the results of investigations made with low-energy protons [6,7]. The curve in the figure is a plot of $I = KZ^n$, ($K = 17$, $n = 0.86$). The experimental points fit the curve quite well, whereas according to Bloch they should form a line parallel to the Z axis.

The experimental data presented show apparently that Bloch's law $I = KZ$ does not hold, i.e., the braking ability cannot be a function of x only. Nonetheless, the range can be expressed in terms

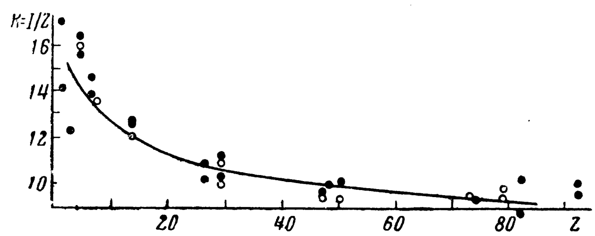

FIG. 1

of a universal function, provided x is replaced by the different parameter[1)]

$$y = E/Ia_0, \qquad (3)$$

where E and a_0 are the kinetic energy and the mass number of the particle (E should be expressed here in MeV and I in keV). Although the expression for y does contain the average ionization energy I, the exact value of I is immaterial for what follows.

We assume that the braking ability depends only on y not only in the approximation of the Thomas-Fermi model, but in the general case. Then the range R is expressed in terms of a function F which depends only on y:

$$R = (a_0 AI^2/z^2 Z)\, F(y). \qquad (4)$$

In the Bethe approximation [9], the function F(y) can be obtained theoretically. In the general case it is difficult to calculate F(y), for in spite of numerous papers [10-16], the exact form of the expression for the braking ability is unknown. We shall determine the function F(y) from the experimental data. Since the form of F(y) does not depend on the composition of the substance, i.e., F(y) is a universal function, we use for the construction of F(y) the most exact and complete

1)As usual, it is assumed that I is independent of the energy. The parameter y was previously investigated by Maksimov in a calculation of the universal range-energy relation.[8]

data obtained in experiments with nuclear emulsions.

2. Figure 2 shows the experimental data obtained by Barkas et al [17], who made their measurements over a wide range of energies, from 1 to 700 MeV. The results are given for a proton in a standard emulsion. We see that in the energy interval 7—200 MeV the experimental points fit the straight line quite well. If the energy is expressed in MeV and the range in microns, then the range-energy relation in this interval can be expressed by

$$E = aR^m, \tag{5}$$

where the constants[2)] a and m are equal to

$$a = 0.265 \pm 0.002, \qquad m = 0.574 \pm 0.001. \tag{6}$$

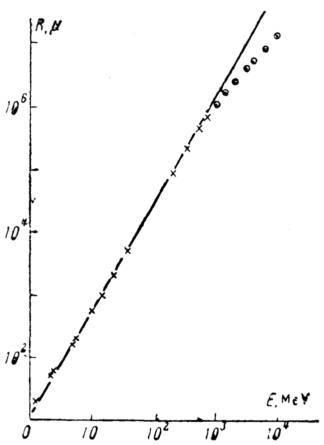

FIG. 2. Range-energy relations for protons in standard emulsion: × – experimental data[17], ○ – results of calculation.[16]

In the 200—1000 MeV interval the range-energy ratio can also be represented in the form (5). If the energy is expressed in MeV and the range in cm, then the values of the constants will be:

$$a = 41.41 \pm 0.40, \qquad m = 0.656 \pm 0.003. \tag{7}$$

Comparison with the experimental data has shown that formula (5) holds true with high accuracy in the energy interval 7—1000 MeV.

3. Let us construct the function F(y) for a proton ($a_0 = 1$, $Z = 1$). Using (5), (4), and (3) we obtain

$$F(y) = ky^{1/m}, \tag{8}$$

$$k = ZI^{1/m-2}/a^{1/m} A. \tag{9}$$

Inasmuch as F(y) is a universal function, the constants k and m entering into the range-energy relations (5) are also universal. Since the average ionization energy of a standard nuclear emulsion is I = 0.331 keV,[3,21] it follows that the energy interval 7—200 MeV corresponds to a y interval from 20 to 600. The range-energy relation for all substances is expressed in this y interval, in accordance with (5) and (6), by

$$E = aR^{0.574}. \tag{10}$$

Only the quantity a depends here on the composition of the substance and on the parameters of the particle. In the interval $600 \le y \le 2700$ the range-energy ratio is expressed, in accordance with (5) and (7), by

$$E = aR^{0.656}. \tag{11}$$

4. We used the derived relations in connection with a water-emulsion chamber [22,23]. To determine the constant a in (10) we measured the average range of the muons from $\pi - \mu - e$ decay and found it to be $R_\mu = 1010 \pm 16\,\mu$. Taking the muon energy to be 4.12 MeV [24], we obtained a = 0.197. The range-energy relation for a water-emulsion chamber thus has the form

$$E = 0.197R^{0.574}. \tag{12}$$

Using a = 0.197 and relation (9), we obtain for the average ionization energy in the water-emulsion chamber:

$$I_{\text{w.e.}} = 0.206\,\text{keV}. \tag{13}$$

5. Several experimental facts can be cited to corroborate the universality of the constant m, and consequently the hypothesis that the retarding ability depends only on y.

A. The Interval $20 \le y \le 600$

1) Ranges were measured for 69 protons of known energy in a water-emulsion chamber [23] with a composition greatly different from that of ordinary emulsion. The results were reduced by least squares. The relation obtained

$$E = (0.201 \pm 0.008)\, R^{0.573 \pm 0.005}$$

agrees well with (12).

2) Figure 3 shows the deviations of the experimental ranges R_e from the values R_t calculated by formula (10) for aluminum, copper, lead, and nuclear emulsion. The relative deviations are given as functions of y.

[2)]Other values were obtained in some investigations for the constants a and m: a = 0.262, m = 0.575[18]; a = 0.251, m = 0.581[19]; a = 0.286, m = 0.568[20]. The differences in the constant m can be attributed to inaccuracies in the measurement and to the fact that the measurements were carried out in different energy intervals.

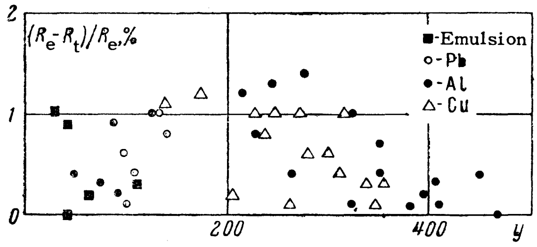

FIG. 3

As can be seen from the figure, the deviations do not exceed about one per cent [17,25-27].

B. The Interval $600 \le y \le 2700$

1) Differentiating (5) and taking (9) into account, we obtain an expression for the energy loss in the form

$$dE/dR = m\,(Z/Ak)\,I^{1/m-2}\,E^{1-1/m}. \tag{14}$$

Using this expression and the universality of the constants k and m, we obtain for the ratio of the braking abilities

$$B/B_0 = (A_0Z/AZ_0)(I/I_0)^{1/m-2}. \tag{15}$$

We see that the energy does not enter into the expression for the ratio B/B_0; this is in good agreement with experiment [3,4,28-30].

2) From experiments by Mather and Segre (340-MeV protons) and of Zrelov and Stoletov (660-MeV) in copper [28,5], we obtained $m = 0.65 \pm 0.02$, which agrees with (11) within the limits of experimental error.

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