THEORY OF NEUTRON TRANSFER IN NUCLEAR COLLISIONS

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The energy dependence of the cross section for transfer of a neutron in nuclear collisions is treated for arbitrary Q-value of the reaction.

At present there is great interest in reactions resulting from the collision of heavy ions. In particular there have been studies [1-6] of the transfer from N^{14} to various nuclei at energies where the Coulomb field plays a decisive role.

The theory of neutron transfer has been treated in the quasiclassical approximation ($\eta_i = Z_1Z_2e^2/\hbar v_i \gg 1$) in numerous papers.[7-12] In an earlier paper,[12] to explain the angular distribution from the reaction $N^{14}(N^{14}N^{13})N^{15}$, the assumption was made (in analogy to the elastic scattering of ions [13-15]) that in neutron transfer reactions, at energies above the Coulomb barrier the nucleus can be treated as an absorbing body. Consequently only the case where the Q-value of the reaction was small compared to the kinetic energy of the colliding nuclei was treated. The present paper gives formulas for the energy dependence of the cross section for arbitrary Q-values.

At energies below the Coulomb barrier, the cross section for transfer of a neutron in a nuclear collision is given by the expression [9]

$$\sigma(E_i) \sim \frac{k_f}{k_i}\int |I(\theta)|^2 d\Omega, \tag{1}$$

$$\begin{aligned} I(\theta) &= \int \psi_{\mathbf{k}_f}^{(-)*} \frac{e^{-\alpha r}}{r} \psi_{\mathbf{k}_i}^{(+)}(\mathbf{r})\, d\mathbf{r} \\ &= 8\pi^2 \left[\frac{\eta_i\eta_f}{(e^{2\pi\eta_f}-1)(e^{2\pi\eta_i}-1)}\right]^{1/2} \left[\frac{(\alpha - ik_f)^2 + k_i^2}{(\mathbf{k}_i - \mathbf{k}_f)^2 + \alpha^2}\right]^{i\eta_f} \\ &\times \left[\frac{(\alpha - ik_i)^2 + ik_f^2}{(\mathbf{k}_i - \mathbf{k}_f)^2 + \alpha^2}\right]^{i\eta_i} \\ &\times \frac{1}{(k_i - k_f)^2 + \alpha^2} \frac{F(-i\eta_f, -i\eta_i, 1; -\zeta)}{1+\zeta}, \\ \zeta &= \frac{4k_ik_f}{(k_i - k_f)^2 + \alpha^2} \sin^2\frac{\theta}{2}. \end{aligned} \tag{2}$$

The subscripts i and f refer to the initial and final states, $\psi_{\mathbf{k}}^{(\pm)}(\mathbf{r})$ are the Coulomb functions, $F(-i\eta_f, -i\eta_i, 1; -\zeta)$ is the hypergeometric function, $\alpha = \sqrt{2M\epsilon}/\hbar$, ϵ' is the binding energy of the captured neutron, θ is the angle of scattering and M is the neutron mass. The cross section, which depends on $|I(\theta)|^2$, increases exponentially with increasing angle θ.

For energies above the top of the Coulomb barrier, effects related to compound nucleus formation are important. According to classical mechanics, the distance of closest approach of the colliding nuclei, $Z_1Z_2e^2(2E)^{-1}[1 + \sin^{-1}(\theta/2)]$, is equal to the sum of their radii $R \approx r_0(A_1^{1/3} + A_2^{1/3})$ at the scattering angle $\theta = \theta_0$, i.e.,

$$R = Z_1Z_2e^2(2E)^{-1}[1 + \sin^{-1}(\theta_0/2)]. \tag{3}$$

At angles θ greater than θ_0, one should observe a rapid fall-off in the angular distribution for the neutron transfer process, since under these conditions the probability of compound nucleus formation is large, while the probability of its decay into a particular channel is small (since the number of decay channels is very large).

The experimental data [5,6] indicate that such a diffraction picture in qualitatively correct. According to the data of McIntyre et al,[6] for the angular distribution from the reaction $Au^{197}(N^{14}N^{13})Au^{198}$ at energies above the Coulomb barrier, in accordance with formula (3), the greatest contribution to the cross section comes from N^{14} nuclei whose distance of closest approach to the Au^{197} nucleus is equal to $(12.7 \pm 0.5) \times 10^{-13}$ cm. Setting this quantity equal to $R = r_0(A_1^{1/3} + A_2^{1/3})$, the authors found a value $(1.55 \pm 0.06) \times 10^{-13}$ cm for r_0.

Despite the fact that such a classical diffraction picture is not sufficient for a quantitative explanation of the sharp drop in the angular distribution of the N^{13} ions, which in all probability can be explained on the optical model, we can estimate the dependence of the neutron transfer cross section on energy by using the asymptotic expression for the hypergeometric function $F(-i\eta_f, -i\eta_i, 1; -\zeta)$ when $\eta_i \gg 1$ and $\eta_f \gg 1$, and integrating (1) from $\theta = 0$ to $\theta = \theta_0$, where θ_0 is defined by (3).

The result is

$$\sigma(E_i) \sim [4E_i - (\sqrt{E_i(E_i+Q)} - E_i)\zeta']^{-1} \exp\{-2\Phi(E_i, Q)\}, \tag{4}$$

where

$$\Phi(E_i, Q) = \eta_i\{(\varphi_i - \psi_i) - \rho(\varphi_f - \psi_f)\},$$

$$\varphi_i = \operatorname{arctg}\frac{2\sqrt{E_i\varepsilon' A}}{-AQ-\varepsilon'} \qquad (0 \leqslant \varphi_i \leqslant \pi);$$

$$\varphi_f = \operatorname{arc\,tg}\frac{2\sqrt{(E_i+Q)\varepsilon' A}}{-AQ+\varepsilon'} \qquad (0 \leqslant \varphi_f \leqslant \pi),$$

$$\psi_i = \arccos\frac{(1-\rho)\zeta'+2}{2\sqrt{1+\zeta'}} \qquad (0 \leqslant \psi_i \leqslant \pi);$$

$$\psi_f = \arccos\frac{(1-\rho)\zeta'-2\rho}{2\rho\sqrt{1+\zeta'}} \qquad (0 \leqslant \psi_f \leqslant \pi),$$

$$\rho = \frac{\eta_f}{\eta_i} = \sqrt{\frac{E_i}{E_i+Q}},$$

$$\zeta' = \frac{4\sqrt{E_i(E_i+Q)}}{(\sqrt{E_i}-\sqrt{E_i+Q})^2+\varepsilon'/A}\sin^2\frac{\theta_0}{2},$$

$$A = A_1A_2/(A_1+A_2), \qquad Q = E_f - E_i = \varepsilon' - \varepsilon. \tag{4'}$$

Here E_i is the energy of the colliding nuclei in the center-of-mass system (CMS), and ϵ and ϵ' are the binding energies, in the incident and residual nuclei, of the neutron which is transferred in the collision. A graph of the function $L_\rho(\zeta') = 2(\psi_i - \rho\psi_f)$ is given in Fig. 1 of Ter-Martirosyan's paper.[9]

We list the limiting cases of formula (4).

(1) $\theta_0 = \pi$. Subbarrier transfer of the neutron:

$$\sigma(E_i) \sim [4E_i - (\sqrt{E_i(E_i+Q)} - E_i)\zeta']^{-1} \exp\Big\{-2Z_1Z_2e^2\hbar^{-1}\sqrt{2MA} \times \Big[\frac{1}{\sqrt{E_i}}\operatorname{arc\,tg}\frac{AQ+\varepsilon'}{2\sqrt{A\varepsilon' E_i}} - \frac{1}{\sqrt{E_i+Q}}\operatorname{arc\,tg}\frac{AQ-\varepsilon'}{2\sqrt{A\varepsilon'(E_i+Q)}}\Big]\Big\}. \tag{5}$$

2) $Q/E_i \ll 1$, $\epsilon'/AE_i \ll 1$, $\theta_0 = \pi$. Small energy transfer with subbarrier penetration of the neutron:

$$\sigma(E_i) \sim E_i^{-1}\exp\{-2\alpha Z_1Z_2e^2/E_i\}. \tag{6}$$

3) $Q/E_i \ll 1$, $\epsilon'/AE_i \ll 1$. Capture of the neutron with small energy transfer when the energy of the colliding ions is above the barrier:

$$\sigma(E_i) \sim \frac{1}{E_i}\exp\Big\{-\frac{\alpha Z_1Z_2e^2}{E_i}\Big(1+\sin^{-1}\frac{\theta_0}{2}\Big)\Big\} = \frac{\text{const}}{E_i}. \tag{7}$$

Here we have used (3). Except for the factor in front of the exponential, formula (6) was given in a paper of E. Lifshitz.[8]

*arc tg = $\tan^{-1}$; arc cos = $\cos^{-1}$.

In the papers of Breit and Ebel[10] and Gol'danskiĭ,[16] it was pointed out that the data on transfer of a neutron from N^{14} to B^{10} and N^{14} do not fit the dependence (6); the cross sections for these reactions rise too slowly. Breit and Ebel[11] pointed out the possibility of explaining the experiments on nucleon transfer in collisions of N^{14} ions at energies below the Coulomb barrier by including the effect of virtual excitation of the nuclei because of the long-range nature of the Coulomb forces.

It is characteristic of neutron transfer reactions with N^{14} that, for energies above the Coulomb barrier, the rise in cross section with energy gradually slows down. In the case of neutron transfer to a B^{10} nucleus (in the energy range 9—16 MeV in the c.m.s.), to N^{14} (10—13 MeV) and to Au^{197} (110—130 MeV), the cross section no longer depends on energy. As the energy increases further, the cross section for the $B^{10}(N^{14}N^{13})B^{11}$ reaction decreases (for the other reactions given above, the graph of the dependence of σ on E ends). For this reaction, the ratio $\sigma(9.95\text{ MeV})/\sigma(10.83\text{ MeV}) = 1.13$ (energies in the CMS), which is reproduced well by formula (7). We may expect that such a picture of the dependence of the neutron transfer cross section holds for all reactions where the Q-value is small compared to the kinetic energy of the colliding nuclei.

For the reaction of transfer of a neutron from N^{14} to C^{12} and O^{16}, the ratio of the Q-value to the height of the Coulomb barrier is approximately −0.6. The dependence of the cross section for these reactions on energy above the top of the Coulomb barrier, is given by (4). But to find this dependence explicitly requires the knowledge of the angular distribution of the N^{13} nuclei at these energies.

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