STORAGE OF COLD NEUTRONS

A. G. DOROSHKEVICH

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We consider the possibility of storing cold neutrons in a 1 m^3 cubic cavity with beryllium walls. The lifetime of neutrons against capture and heating is estimated. The probability of heating of the neutrons by surface vibrations of the walls is also estimated.

IT is known that slow neutrons undergo total internal reflection when they are incident at grazing angles on most materials. At sufficiently low speeds, the neutrons do not enter the material even for normal incidence. This opens the possibility of storage of cold neutrons in cavities of sufficiently large volume, even though the wall temperature is much higher than the temperature of the neutrons.

A paper of Zel'dovich[1] gives estimates of the lifetime of neutrons in a graphite cavity against their capture by the wall material. It is also of interest to estimate the losses of neutrons due to heating when scattered by the wall material and in collisions with the vibrating walls of the cavity.

For beryllium, the critical neutron velocity is $v_m \approx 7$ m/sec. At such energies the processes of scattering and capture of neutrons occur with absorption of one phonon. The total cross section has a $1/v$ dependence on the neutron velocity and is strongly dependent on the lattice temperature. According to Hughes' data for a neutron energy $E_n = 3 \times 10^{-4}$ eV and a lattice temperature $T_e = 300$°K, the total cross section is $\sigma_u = 1.25$ barns; $E_n = 9 \times 10^{-4}$ eV and $T_e = 100$°K, the cross section is $\sigma_u = 0.095$ barns.[2] The cross section for absorption of neutrons by beryllium is independent of the lattice temperature and has a $1/v$ dependence on the neutron energy. For $E_n = 9 \times 10^{-4}$ eV, we have $\sigma_a = 0.055$ barns.[3]

The lifetime of a neutron in the wall material until it is captured or heated is independent of E_n and is equal to 2.7×10^{-4} sec at $T_e = 300$°K and 2.3×10^{-3} sec at $T_e = 100$°K. The lifetime of the neutron before capture is independent of E_n and T_e and is equal to 4.2×10^{-3} sec. Assuming that the neutron energy distribution is uniform for $E < E_{cr}$, we find that the average depth of penetration of the neutrons into the wall material is $\sim 10^{-6}$ cm. The lifetime of the neutrons in a cubic cavity of volume 1 m^3 until capture of the neutrons by the wall material (beryllium) is 6.2×10^4 sec ≈ 17 h. The total lifetime of neutrons in a 1 m^3 cubic cavity is 4.3×10^3 sec ≈ 1.2 h for $T_e = 300$°K and 3.6×10^4 sec ≈ 10 h for $T_e = 100$°K.

Heating of the neutrons can also occur on reflection from the vibrating walls of the cavity. The vibrations of the wall are caused by the emergence at the surface of the Debye volume waves as well as by Rayleigh waves which are damped exponentially as they go into the crystal.[4]

The probability of such heating can be estimated by solving the one-dimensional problem of scattering by a potential of the form

$$V = V_1 + V_2, \qquad V_1 = \begin{cases} U_0, & x > 0 \\ 0, & x < 0 \end{cases},$$

$$V_2 = \begin{cases} 0, & x > x_0 \sin\omega' t > 0 \\ -U_0 & x_0 \sin\omega' t > x > 0 \\ U_0, & x_0 \sin\omega' t < x < 0 \\ 0, & x < x_0 \sin\omega' t < 0 \end{cases}, \tag{1}$$

where U_0 is the potential energy of the neutron in the crystal, ω' and x_0 are the frequency and amplitude of the wall vibrations. For beryllium, $U_0 \approx 4 \times 10^{-19}$ erg. We have to find the probability of transition from states with $E < U_0$ to a state with $E > U_0$; this is conveniently done using a method described by Zel'dovich.[6]

Let us assume that heating occurs only when $\lambda' \geq \lambda_n$, where λ' is the wavelength of the wall vibrations and λ_n is the neutron wavelength. This condition limits the frequency of vibration of the wall:

$$\omega' \leqslant \frac{c}{v_0}\omega_0 \leqslant 4 \cdot 10^{11}\ \mathrm{sec}^{-1} \tag{2}$$

where c is the sound velocity in the wall material, and ω_0 and v_0 are the frequency and the velocity of the neutron. Thus we include only the long-wave vibrations of the wall, which enables us to treat the wall vibrations macroscopically and to describe them by a potential V.

The probability of heating must be averaged over the neutron energies and over the frequency distribution of the wall vibrations. The energy distribution of the neutrons can be assumed to be uniform and the frequency distribution of the volume waves can be assumed to be the Debye spectrum. The distribution of the Rayleigh waves can be found in the same way as that of the Debye waves.

Let us estimate the amplitude of oscillation of the wall. For Debye waves, the energy density in the low frequency part of the spectrum $\omega' \leq \omega = 4 \times 10^{11}\ \text{sec}^{-1}$ is

$$\varepsilon \sim kT \frac{4\pi}{c^3} \omega^3. \tag{3}$$

On the other hand, according to (1), $\epsilon \sim \rho x_0^2 \omega_1^2/2$. For $T = 300°K$, we have $x_0^2 \sim 10^{-18}\ \text{cm}^2$. The amplitude of the Rayleigh waves is of this same order of magnitude. Carrying out all the necessary computations, we find that for a cube of volume $1\ \text{m}^3$, the probability of heating by the wall vibrations is $W \approx 10^{-7}\ \text{sec}^{-1}$.

The estimates given here are very rough. For example, we have not included the change in the tangential component of the neutron velocity, and the method of describing the wall vibrations does not include certain types of waves. The results give only an order of magnitude estimate of the probability of heating of the neutrons by the surface vibrations. But it is clear that the vibrations of the surface of the cavity walls do not cause any essential change in the neutron lifetime so long as the cavity is sufficiently large, which confirms the qualitative analysis given by Zel'dovich.[1]

Thus the total lifetime of neutrons against capture and heating in a cubic cavity of volume $1\ \text{m}^3$ with beryllium walls is equal to $\sim$1.2 h for $T_e = 300°K$ and $\sim$10 h for $T_e = 100°K$. In addition to the method of magnetic containment of neutrons, described by Vladimirskiĭ,[7] one can also store cold neutrons in cavities of sufficiently large volume.

In conclusion I express my sincere gratitude to Ya. B. Zel'dovich for his continued interest in the work.

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[4] L. D. Landau and E. M. Lifshitz, Theory of Elasticity, Pergamon, 1959.

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[6] Ya. B. Zel'dovich, JETP **33**, 1488 (1957), Soviet Phys. JETP **6**, 1148 (1958).

[7] V. Vladimirskiĭ, JETP **39**, 1062 (1960), Soviet Phys. JETP **12**, 740 (1960).

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