AN EXPERIMENTAL INVESTIGATION OF THE EFFECT OF AN INTERMEDIATE MEDIUM ON GRAVITATIONAL INTERACTION

V. B. BRAGINSKIĬ, V. N. RUDENKO, and G. I. RUKMAN

Moscow State University

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The effect of an intermediate medium on gravitational interaction is investigated. The sensitive measuring device could respond to the change in the weight of a 10 kg mass when screened by a steel sheet 10 cm thick. A modulation technique was employed along with an electromechanical transducer and an electronic device for optimal separation of the signal from noise. Statistical methods were used for the analysis and the estimate of the reliability of the results. Under the conditions of the experiments no effect of the intermediate medium on gravitational interaction was detected. The probability that an effect of the order $\geq 1.3 \times 10^{-10}$ of the gravitational interaction was not noticed is ~ 0.04 (and correspondingly ~ 0.002 at a level $\geq 2 \times 10^{-10}$).

EXPERIMENTS on the effect of an intermediate medium on the gravitational interaction between two masses were made by various workers [1-4]. Majorana [1-3] stated that he observed an intermediate-medium effect amounting to $\sim 10^{-9}$ of the Newtonian interaction. These results were not confirmed in subsequent researches [4], in which, however, the experimental conditions differed appreciably from those of Majorana.

The effect of the medium on gravitational interaction was recently discussed by Hoffman [5] and Radzievskii and Kagal'nikova [6], who suggested experiments aimed at observing small effects of this type. One must apparently agree with Hoffman's opinion [5] that there are no grounds for negating a priori the effect of the medium on the gravitational interaction. From the nonlinear equation of general relativity it follows that gravitational fields are not additive, and consequently the intermediate medium can have an effect. However, the magnitudes of the possible effects should be appreciably smaller than could be observed under the experimental conditions in [1-4].

We describe in the present article the results of an experiment similar in formulation to those of Majorana.[1,3] We used in the experiment a modulation procedure, an optimal electronic system to separate weak signals from noise, and statistical methods to analyze and estimate the reliability of the results.

DESCRIPTION OF APPARATUS AND MEASUREMENT PROCEDURE

To investigate the effect of an intermediate medium on gravitational interaction, we assembled a setup with principal elements as shown in Fig. 1. Two identical brass cubes 1 and 3 (each of 10 kg mass) are connected by a duraluminum frame with one another and with a counterweight 4. This system of three masses can execute low-frequency oscillations about a knife edge 5 in a vertical plane. The natural frequency of the system oscillations can be varied by changing the stiffness of spring 6, which connects the upper part of the system with a rigid nonmagnetic structure to which the prism is secured. The role of the intermediate medium ("screen") is assumed by a steel two-blade rotor 2, 10 cm thick and 60×30 cm[2] in cross section. The rotor, fastened to a moving stand 7, turns about a vertical axis in such a way that when the stand is in the extreme left position (position A of Fig. 1) the blades of the rotor enter the gap between cubes 1 and 3. Figure 1 shows a rotor position with one of the blades between the cubes.

If the force of interaction F_0 between the upper cube 1 and the earth (the weight of the cube) changes under the influence of the intermediate body (the rotor blade) by a certain amount F_X, then rotation of the rotor will cause a periodic force $F_X \sin \omega_m \tau$ to act on the system of three

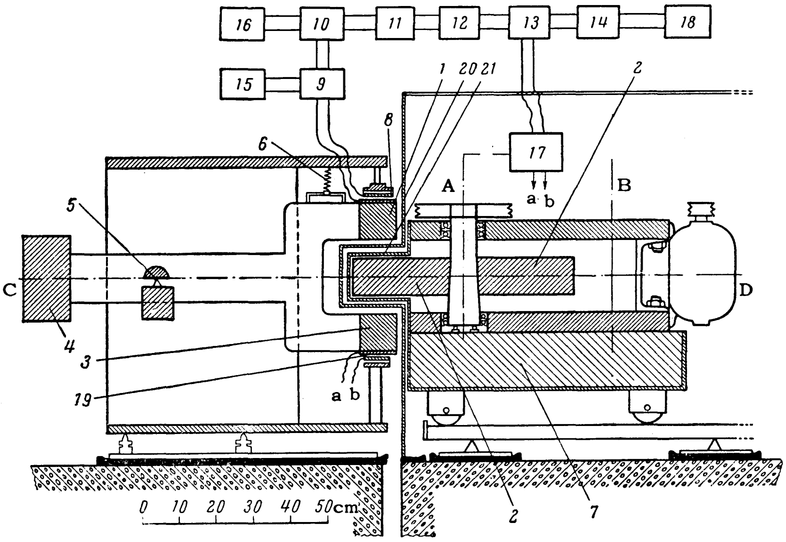

FIG. 1

masses 1, 3, and 4 in synchronism with the entry of the rotor blades between the cubes. The lower cube 3 is intended to offset the Newtonian attraction of the upper cube 1 to the rotor. To observe the small oscillations of the system, brought about by the expected force $F_x \sin \omega_m \tau$, a capacitive transducer-discriminator 9 is used, with a sensitivity of about 5000 V/cm, along with the electronic system 9 — 18. The "working" capacitance 8 of the transducer includes the surface of cube 1 and the surface of an insulated plate rigidly secured to the structure. A similar capacitance 19 is formed by a second analogous plate and the surface of the lower cube 3, and is intended for calibration of the electronic equipment. By applying to the capacitance an alternating voltage of known magnitude it is possible to simulate a force F_x (from $\sim 10^{-6}$ dyn to $\sim 10^{+2}$ dyn) and compare it with the signal received by the recording part of the setup. This device operates on a principle inverse to that of the absolute electrostatic voltmeter.

Seismic vibrations of the foundation and low-frequency acoustic jolts are the principal types of noise interfering with the measurement and registration of the small forces F_x with this setup. It was found by direct measurements that the resolving power of the setup increases with decreasing natural frequency ω_0 approximately as $1/\omega_0$. In this case the frequency ω_m should not differ from the natural frequency ω_0 of the three-mass system relative to the knife-edge by more than ω_m/Q, where Q is the figure of merit of this degree of freedom. A modulation frequency $f_m = \omega_m/2\pi = 0.3$ cps (a rotor revolution frequency of 0.15 cps) was chosen to permit the employ ordinary electronic circuitry for the separation of the signals from the noise.

The capacitive transducer 9, connected with the generator 15, the detector 10, and the indicator 16 (which has a time constant on the order of three seconds) make it possible to register a calibration force F_c on the order of 0.1 — 0.2 dyn against the noise background. With the upper cube 1 having a weight $F_0 = 10^{+7}$ dyn, this corresponds to the possibility of observing a relative change in cube weight $F_x/F_0 = (2 - 10) \times 10^{-8}$. The second half of the electronic circuitry extracts the weak signal due to the action of the small periodic force $F_c = 10^{-3} - 10^{-1}$ dyn. It comprises a low-frequency filter 11, amplifier 12, synchronous detector 13, integrating network 14, reference-voltage generator 17, and galvanometer 18. The reference generator 17 delivers a voltage synchronized at twice the rotor frequency, to synchronous detector 13. The same voltage (or a part of it) is used to obtain the calibration force F_c.

In separating a signal proportional to a small force, we made use of the fact that the frequency and phase of both forces F_x and F_c are known. The force $F_x \sin(\omega_m \tau + \varphi)$ should produce periodic oscillations of the mass system; these oscillations are converted by the transducer, detector, and amplifier into an electric signal $U_x \sin(\omega_m \tau + \varphi_1)$. Seismic and acoustic noise also produces oscillations in the system, and consequently the total signal at the input of amplifier 12 is $U_x \sin(\omega_m \tau + \varphi_1) + \xi(\tau)$. For optimum separation of the signal U_x against the background of the noise $\xi(\tau)$, for known ω_m and φ_1, it is necessary to carry out the following operation [7]:

$$\delta_x = C \int_{\tau}^{\tau+\tau_0} [U_x \sin(\omega_m \tau + \varphi_1) + \xi(\tau)] \sin(\omega_m \tau + \varphi_1)\, d\tau. \quad (1)$$

The mathematical expectation $M(\delta_X)$ is directly proportional to F_X. When the integration time τ_0 is increased, the confidence interval for δ_X decreases as $1/\sqrt{\tau_0}$, so that the greater τ_0 the smaller the values of F_X that can be reliably registered (see [8] for details). The synchronous detector 13 and the integrating network 14 carry out an operation that is nearly the same as (1). The galvanometer 18 records at time intervals τ_0 values of δ that differ from δ_X by a certain constant $k(\tau)$, namely $\delta = \delta_X + k(\tau)$.

The value of $k(\tau)$ "drifts" somewhat over time intervals on the order of several hours. This is a common shortcoming of any synchronous detector with a large linear dynamic range (on the order of $^1/_{500}$). To observe the small expected force F_X, which might have been created by the effect of the rotor blades on the gravitational interaction between the upper cube 1 and the earth, an operation of the type (1) was repeated many times, and the values obtained were statistically processed. The measurement was made in the following sequence: the stand 7 holding the rotating rotor was swung into position B (see Fig. 1), the voltage from reference generator 17 was applied to the calibrating capacitance 19 so that the system was swayed by an accurately known force F_C, less than 0.2 dyne (the upper limit of the dynamic range of the synchronous detector). The synchronous detector and the integrating network were used to measure the quantity $\delta^0 = \delta_X|_{F=F_C} + k(\tau)$.

The integration time τ_0 amounted to one minute. The operation was repeated five times, after which the reference generator was disconnected from the plates of calibrating capacitor 19 and the quantities $\delta^- = \delta_X|_{F=0} + k(\tau)$ were measured at the same position of the stand, B. The integration time was maintained the same, $\tau_0 = 1$ min. After measurement of ten values the stand 7 was shifted into the extreme left position A, in which the rotor blades 2 pass between masses 1 and 3, and ten values of $\delta^+ = \delta_X|_{F=F_X} + k(\tau)$ were measured. The stand was then moved to position B and readings of δ^- again taken, etc. At the end of one such measurement series, yielding from 30 to 70 values of δ^- and δ^+, the calibration was repeated. This was done to ensure that the electromechanical transfer function of the system 9 — 12 did not change in the time necessary for the measurement of one series, which amounted to several hours. Subsequent statistical processing was applied only to those series, in which the quantities $(\overline{\delta^0} - \overline{\delta^-})$ (proportional to the force F_C), measured at the start and the end of the series, did not differ by more than double the width of the confidence interval, calculated from five values of δ^0.

PROCESSING OF THE MEASUREMENT RESULTS

In order to exclude the value of $k(\tau)$ (the "null" of the synchronous detector), we plotted a regression line $y(\tau)$ for the average values of $\overline{\delta^-}$ for 10 values of δ^- in succession. The influence of the null drift $k(\tau)$ was excluded for the values of Δ^+ and Δ^-, reckoned from the regression line and calibrated with the aid of F_C, namely:

$$\Delta^- = \frac{F_C[\delta^- - y(\tau)]}{[\overline{\delta^0} - \overline{\delta^-}]}, \qquad \Delta^+ = \frac{F_C[\delta^+ - y(\tau)]}{[\overline{\delta^0} - \overline{\delta^-}]}. \tag{2}$$

The mathematical expectation $M(\overline{\Delta^+} - \overline{\Delta^-})$ is equal to F_X. Thus, by processing the obtained sampling of Δ^+ and Δ^- one can get a statistical estimate for F_X.

Figure 2 shows 30 values of δ^- and δ^+ from one of the series (these are denoted by dots and crosses, respectively). At the start and end of the series are shown the values of δ^0 corresponding to $F_C = 0.147$ dyn. The figure shows the regression line $y(\tau)$ for three mean values of $\overline{\delta^-}$. To the left in the same figure are shown 120 values of Δ^+ and Δ^-, obtained in three series of measurements. A zero value of Δ^+ or Δ^- denotes that the corresponding values of δ^+ or δ^- fall on the regression line. A positive sign of the difference $(\overline{\Delta^+} - \overline{\Delta^-})$ corresponds to "screening" of the gravitational potential, while a negative sign corresponds to its "intensification." The total number of statistically-processed series was 18, each with 850 values of Δ^+ and Δ^-. The total integration time $2n\tau_0$ thus amounts to approximately 30 hours, which is equivalent to a receiver bandwidth of about 10^{-5} cps.

To estimate the significance of $(\overline{\Delta^+} - \overline{\Delta^-})$ it is convenient to use the Student t-criterion[9]. It was first established with the aid of the Kolmogorov $K(\lambda)$ criterion that the distribution of Δ^+ and Δ^- obeys in each series a normal law, and this made it possible to employ the effective agreement criteria applicable to normally-distributed quantities. With the aid of Fisher's F-criterion we eliminated the swings of the measurements (possibly due to random relatively strong seismic shocks). There were 14 swings among the 1700 values of Δ^+ and Δ^-. We then determined for each series the values of the Student statistics t_β. In all series, the values of t_β turned out to be small (less than 0.6). It should be considered, in accordance with Student's t-criterion, that in each series the quantities Δ^+ and Δ^- can belong to one

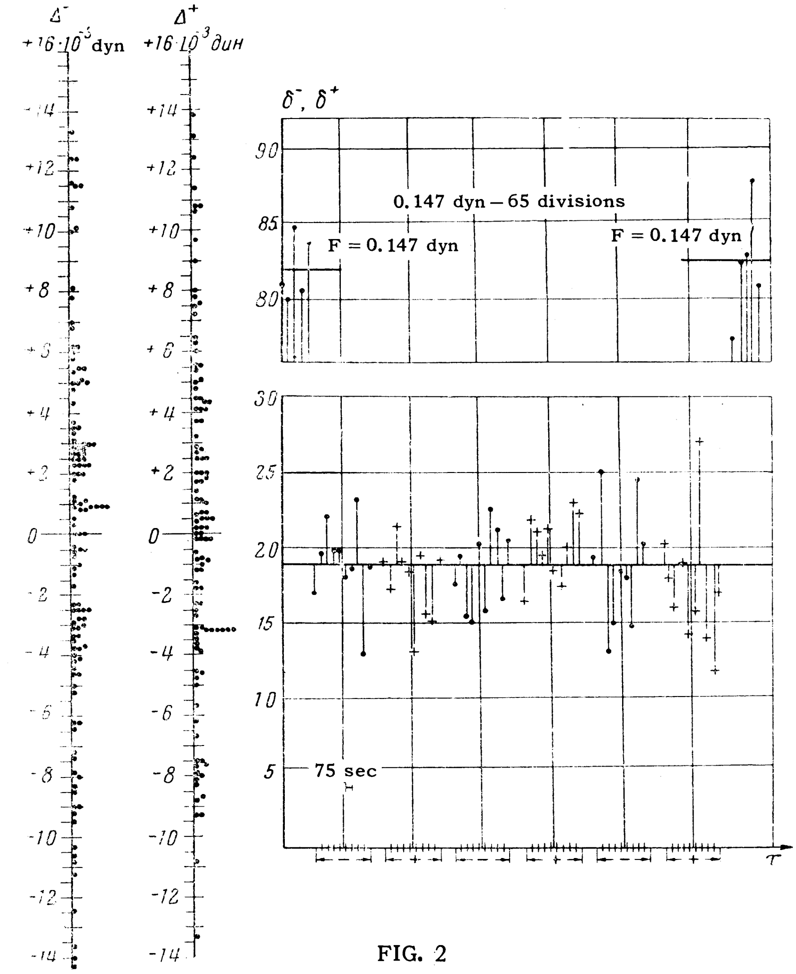

FIG. 2

general assembly and the resultant value of $(\overline{\Delta^+} - \overline{\Delta^-})$ for such a number of values of Δ^+ and Δ^- and for the determined dispersions cannot be regarded as significant. We note that the sign of $(\overline{\Delta^+} - \overline{\Delta^-})$ is positive in nine series, and negative in eight, while in one we have $(\overline{\Delta^+} - \overline{\Delta^-}) \simeq 0$. The unification of several series of measurements, which in principle increases the probability of resolving the difference $(\overline{\Delta^+} - \overline{\Delta^-})$, was made difficult by the fact that the dispersions $D(\Delta^+)$ and and $D(\Delta^-)$, which can be regarded as constant within each series, vary greatly from series to series. This is caused by the fact that the spectral density of the seismic noise is not constant in time. We therefore unified the sets of Δ^+ and Δ^- only for those series, in which the dispersions $D(\Delta^+)$ and $D(\Delta^-)$ did not go beyond the 5% probability level as measured by the Bartlett criterion[10]. There were 12 such series with a total number of 525 for Δ^+ and 524 for Δ^-. The statistical characteristics for these assemblies of Δ^+ and Δ^- are

$$(\overline{\Delta^+} - \overline{\Delta^-}) = -0.03 \cdot 10^{-3}\,\text{dyn}, \quad \sqrt{D(\overline{\Delta^+})} = 0.44 \cdot 10^{-3}\,\text{dyn},$$
$$\sqrt{D(\overline{\Delta^-})} = 0.42 \cdot 10^{-3}\,\text{dyn}.$$

Using again the Student t-criterion, we find that in this case the quantity $(\overline{\Delta^+} - \overline{\Delta^-})$ must not be regarded as significant, i.e., the sets Δ^+ and Δ^- can belong to one general assembly.

We note that discarding any of the 12 unified series makes the value of $(\overline{\Delta^+} - \overline{\Delta^-})$ not larger than 0.2×10^{-3} dyn, which is considerably below that limiting value of this difference, $B_{lim} = 0.7 \times 10^{-3}$ dyn, for which one could call the difference significant in the one-sided Student criterion $(\beta = 0.10)$.[1)]

From the determined values of $D(\overline{\Delta^+})$ and $D(\overline{\Delta^-})$ and the limiting value it is easy to calculate the error of the second kind[9]. According to Student's t-criterion, we assume in accord with the determined values of $(\overline{\Delta^+} - \overline{\Delta^-})$, $D(\overline{\Delta^+})$, and $D(\overline{\Delta^-})$ a zero hypothesis (H_0), namely $F_X = 0$. Putting $F_X \neq 0$ (the H_F hypothesis) we can easily calculate, with the aid of the theory of confidence intervals, the probability $P\{H_0/H_F\}$ that the H_0 hypothesis will be erroneously assumed (whereas

1)The remaining six series have a larger average dispersion, corresponding to the greater limiting values, and therefore yield less information on $M(\overline{\Delta^+} - \overline{\Delta^-})$.

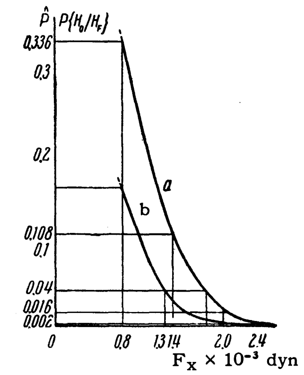

FIG. 3

H_F is valid) for all possible values $(\overline{\Delta^+} - \overline{\Delta^-}) \leq B_{lim} = 0.7 \times 10^{-3}$ dyn, which could be obtained in this experiment. We recall that B_{lim} is determined uniquely by the values of $D(\overline{\Delta^+})$ and $D(\overline{\Delta^-})$ and the assumed values of the significance level β. Figure 3 (curve a) shows the probability of the error of the second kind $P\{H_0/H_F\}$, plotted from our data as a function of the assumed force F_X.

COHERENT NOISE. DISCUSSION OF RESULTS

As can be seen from Fig. 3, the probability of not observing a force of 1.4×10^{-3} dyn (the relative change in the weight $F_X/F_0 = 1.4 \times 10^{-10}$) for all possible values $(\overline{\Delta^+} - \overline{\Delta^-}) \leq B_{lim}$ amounts to approximately 10%, while the probability of not observing a force at a level 2×10^{-3} dyne $(F_X/F_0 = 2 \times 10^{-10})$ is approximately 1.6%. However, the value obtained for $(\overline{\Delta^+} - \overline{\Delta^-})$ is appreciably smaller than B_{lim}. Therefore the probability of missing a change $F_X/F_0 \geq 1.4 \times 10^{-10}$ is even smaller if we take into account the obtained realization of $(\overline{\Delta^+} - \overline{\Delta^-})$ (see below).

Let us dwell briefly on the coherent noise capable of neutralizing the expected force F_X.

1. The knife edge 5 and the center of mass of cubes 1 and 3 were not more than 0.1 cm away from the symmetry plane C — D of the rotor 2. Calculation shows that the difference in the Newtonian attraction of the masses 1 and 3 to the blades of rotor could not exceed 5×10^{-5} dyn as a result of such inaccuracy.

2. In order to eliminate air bumps due to the rotation of rotor 2, steel cases 20 and 21 were used (see Fig. 1). It is easy to calculate that vibration of case 20, coherent with rotation of the rotor and with amplitude $a_0 = 10^{-3}$, with a gap of 1 cm between cube 1 and case 20, lower-face area of 120 cm^2, and a vibration frequency $f_m = 0.3$ cps could cause a force of about 6×10^{-8} dyn to act on the system of three masses. Under the experimental conditions the value of a_0 did not exceed 10^{-3} cm.

3. The intensity of the constant magnetic field (~ 1 Oe) and the amplitude (~ 0.01 Oe) and amplitude gradient (~ 0.02 Oe/cm) of the alternating magnetic field connected with the rotation of the rotor were measured near the masses 1 and 2. From these data and from the known permeability and specific conductivity of the brass from which the cubes 1 and 3 were made we could calculate accurately the coherent force acting on a sphere with a volume equal to that of the cube[11]. Under our conditions this force did not exceed 5×10^{-6} dyn.

4. Under the experimental conditions the structure with the system of three masses and the stand 7 with the rotor were mounted on separate foundations. To attenuate the shocks that might be transmitted through the mount, we used liners of sponge rubber (see Fig. 1). Assuming, on the other hand, both halves of the system to rest on a common concrete foundation occupying the entire half space, we can calculate the amplitude of the displacement under the structure with the three masses from the value of the centrifugal force (due to the inaccuracy in centering the rotor on the shaft)[12]. We can then determine from the stiffness of the spring 6 the force tending to swing the three-mass system. This force does not exceed 6×10^{-4} dyn (under the experimental conditions, the eccentricity of the rotor did not exceed 0.01 cm, the centrifugal force was not larger than $5 \times 10^{+3}$ dyn, and the stiffness of the spring 6 was $K_S = 2 \times 10^{-5}$ dyn/cm). We note that this is a highly exaggerated estimate of the noise, for in fact separate foundations were used. In addition, this force had half the frequency of the expected force. Thus, at the attained resolution level, the possible types of coherent noise could not neutralize the force that could have been observed.

A comparison of the results obtained with those given by Majorana[1,3] can apparently be made only within the framework of the hypothesis developed in [1,3], since the experimental conditions are not fully comparable. In accordance with this hypothesis, and recognizing that the "screening" of cube 1 was incomplete, we can calculate the expected force F_X^*. For the smallest value of the "screening constant" obtained in [1,3] we get $F_X^* \gtrsim 1.3 \times 10^{-3}$ dyn.[2)] From the values obtained for $D(\overline{\Delta^+})$ and $D(\overline{\Delta^-})$ we can calculate the probability $\hat{P}$ that the result of our measurement, $(\overline{\Delta^+} - \overline{\Delta^-}) = +0.2 \times 10^{-3}$ dyne, (the largest positive value with an enforced discard of one series of measurements) could be realized when $M(\overline{\Delta^+} - \overline{\Delta^-}) \geq F_X^*$. Figure

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2)The size of the 'screen' was smaller than in [1,3].

3 shows the dependence of $\hat{P}$ on F_X (curve b). As can be seen from Fig. 3, the probability that the result of our measurements $[(\overline{\Delta^+} - \overline{\Delta^-}) = +0.2 \times 10^{-3}$ dyn] could be obtained for $M(\overline{\Delta^+} - \overline{\Delta^-}) \geq F_X^* = +1.3 \times 10^{-3}$ dyn does not exceed 0.04. This casts doubts on the results of the Majorana experiments[1,3]. We note that in the Radzievskiĭ and Kagal'nikova[6] interpretation of the effect observed in [1,3] the value of F_X^* should be twice as large (the mass of the cube 3 should also be "screened" by the rotor).

Unfortunately, the reliability of the results obtained in [1-3] cannot be estimated for lack of statistical processing of the measurement results. For example, it is possible to calculate the width of the confidence interval in [2], which approximately corresponds to the resolving power, from the experimentally obtained three values of F_X/F_0 ($+2.8 \times 10^{-11}$, -1.2×10^{-11}, and $+2.0 \times 10^{-11}$). According to these three figures, the width of the confidence interval or the resolving power at a confidence level 0.95 is 1.0×10^{-10} (and not 2×10^{-11}, as suggested by the authors of [2]). However, even the conclusion that the confidence interval is 1×10^{-10} and that there is no effect for $F_X/F_0 \leq 1 \times 10^{-10}$ cannot be regarded as reliable, since it is impossible to judge whether these three values belong to a normal assembly.

The resolving power obtained in the present work from the determination of F_X/F_0 does not differ greatly from the value 4.7×10^{-10} stated by Majorana[1,3], because the main quantity which determines the accuracy of the measurements is the time consumed in the measurement, which is sufficiently long in the experiments of [1-3]. Other experiments on the measurement of weak gravitational effects are in a similar situation. For example, the accuracy with which Eotvos found the ratio of inertial mass to gravitational mass to be constant for different bodies amounted to 10^{-8}, while in the latest analogous experiments by Dicke[13] the accuracy was increased to 3×10^{-10}.[13]

In conclusion let us point out that under the conditions of our experiment the order of magnitude of the force F_X^0, connected with nonlinear effects resulting from the general relativity theory, can be estimated with the aid of known expressions for the energy of a system of point masses[14]. After simple derivations we can obtain an approximate expression for such a force F_X^0, which has the same sign as the "screening",

$$F_x^0 \cong \gamma^2 m_1 m_2 M / 2c^2 r_{12} R^2. \quad (3)$$

Here γ is the gravitational constant, m_1 the mass of cube 1, m_2 the "screening" mass of the blade of rotor 2, M the mass of the earth, R the radius of the earth, r_{12} the distance between m_1 and m_2, and c the velocity of light. After substituting the numerical values we obtain $F_X^0 \approx 10^{-18}$ dyn ($F_X^0/F_0 \approx 10^{-25}$). It must be noted that this estimate is an upper limit, since the expression (3) has been derived under the assumption that the masses m_1, m_2, and M are point-like. Improvement of the procedure described in the paper, within the framework of the measurement scheme described, can increase the measurement accuracy by one or two orders of magnitude, which is insufficient to observe an effect on the order of F_X^0/F_0. This does not lead, however, to a principal impossibility of detecting the force F_X^0, since the nature of the noise did not correspond under our experimental conditions to the measured effect.

In conclusion the authors consider it their pleasant duty to thank Professor V. V. Migulin, Prof. A. Z. Petrov, V. G. Zubov, and L. D. Meshalkin for valuable discussions and for help with the work.

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