SCATTERING OF NEUTRINOS ON NUCLEONS IN THE MODEL OF THE ANOMALOUS MUON INTERACTION

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The cross sections for the scattering of neutrinos on nucleons due to the exchange of a neutral vector χ meson, i.e., in the model of the anomalous muon interaction of Kobzarev and Okun', are calculated. The values obtained for these cross sections are of the order of $\sim 10^{-31}$ cm^2, implying that experiments for testing the theory should be feasible.

KOBZAREV and Okun' [1] have proposed the model of the anomalous muon interaction. This interaction involves hypothetical neutral vector particles, χ mesons, which interact with muons as well as with the muonic neutrino and the baryons. This model has the attractive features of being γ_5 invariant and renormalizable. The latter is particularly important at the present stage.

One of the most feasible experiments to verify this model is, at present, the measurement of the cross section for the scattering of the muonic neutrino on nucleons due to the exchange of a virtual χ meson. The cross sections for this process are calculated in this paper.

Within the framework of the Sakata model, the χ meson can interact with the baryon current in two ways:

1) the χ meson interacts with the nucleon current $\sqrt{4\pi}\, f(\bar{p}\gamma_\alpha p + \bar{n}\gamma_\alpha n)$,

2) the χ meson interacts with the Λ current $\sqrt{4\pi}\, \bar{\Lambda}\gamma_\alpha\Lambda$.

The graph describing the scattering of the neutrino on the nucleon is, in case 1), analogous to the graph describing the usual electromagnetic scattering of an electron by a proton (see Figs. 1, a and b).

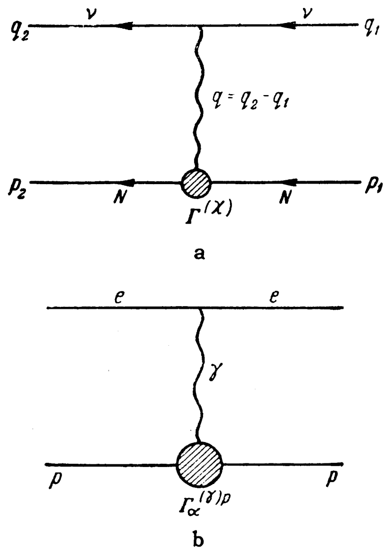

FIG. 1

As is known, the proton vertex $\Gamma_\alpha^{(\gamma)p}(q^2)$ in Fig. 1, b representing the effect of strongly interacting virtual particles can be written in the form

$$\Gamma_\alpha^{(\gamma)p}(q^2) = \gamma_\alpha F_{ep}(q^2) + \sigma_{\alpha\beta}q^\beta F_{mp}(q^2),$$
$$\sigma_{\alpha\beta} = \tfrac{1}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha),$$

where $F_{ep}(q^2)$ and $F_{mp}(q^2)$ are the charge and magnetic form factors of the proton.

Making use of the analogy between the graphs of Figs. 1, a and b, we can determine the quantity $\Gamma_\alpha^{(\chi)}(q^2)$. Indeed, let us rewrite the electromagnetic current of the proton $\sqrt{4\pi}\, e\bar{p}\gamma_\alpha p$ in such a way as to separate out the nucleon current $\bar{p}\gamma_\alpha p + \bar{n}\gamma_\alpha n \equiv \bar{N}\gamma_\alpha N$, in which we are interested:

$$\sqrt{4\pi}e\bar{p}\gamma_\alpha p = \tfrac{1}{2}\sqrt{4\pi}e(\bar{N}\gamma_\alpha N + \bar{N}\gamma_\alpha\tau_3 N),$$

where $N = \begin{pmatrix} p \\ n \end{pmatrix}$ is the wave function of the nucleon and $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is a matrix acting in the isotopic space of the nucleon. The "accumulation" of virtual particles in the proton vertex induces the replacement

$$\bar{N}\gamma_\alpha N \to \bar{N}\Gamma_\alpha^{(\gamma)S}(q^2)N, \qquad \bar{N}\gamma_\alpha\tau_3 N \to \bar{N}\Gamma_\alpha^{(\gamma)V}(q^2)\tau_3 N,$$

where the indices S and V distinguish between the isoscalar and isovector components of the nucleon form factor. Since the nucleon vertex cannot depend on which quantum (χ or γ) is emitted (neither participates in the strong interactions), and since the electromagnetic and f currents of the nucleon ($\sqrt{4\pi}\, e\bar{N}\gamma_\alpha N$ and $\sqrt{4\pi}\, f\bar{N}\gamma_\alpha N$) differ only by a factor, it follows that the vertex $\Gamma_\alpha^{(\chi)}(q^2)$

in Fig. 1, a is equal to the isoscalar component of the nucleon form factor: $\Gamma_\alpha^{(\chi)}(q^2) = \Gamma_\alpha^{(\gamma)S}(q^2)$.

Let us now consider the case where it is the Λ particle which has the f charge. The scattering of a neutrino by a nucleon is in this case due, for example, to the process described by the graph of Fig. 2. Including the remaining possible graphs and representing their sum by a graph with a blob in the nucleon vertex, we again obtain the graph of Fig. 1, a with the only difference that the nucleon no longer has an f charge. The electromagnetic analog of this process is the scattering of an electron by a neutron, which does not have an e charge.

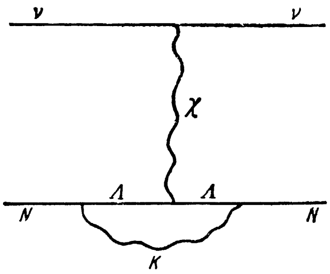

FIG. 2

We note, however, that in the second case we cannot determine the form of the nucleon vertex in the way we did in the case of the χN interaction. Therefore, we know neither $\Gamma_\alpha^{(\chi)}(q^2)$ nor $\Gamma_\alpha^{(\chi)}(0)$ with certainty. In view of the singlet nature of the Λ particle, it would be reasonable to choose the scalar form factors, but the scalar charge form factor of the nucleon does not vanish in the static limit. The calculations have therefore been carried out with the scalar magnetic and the neutron electric form factors.

The matrix element for the scattering process has in both cases the form

$$M = \frac{4\pi f^2}{m_\chi^2 - q^2} (\bar{u}_\nu \gamma_\alpha u_\nu)(\bar{u}_N \Gamma_\alpha^{(\chi)}(q^2) u_N)$$

(the term $q_\alpha q_\beta / m_\chi^2$ in the meson propagator gives a vanishing contribution on account of the transversality of the vertex).

Since $m_\chi \approx 10m$ (m is the nucleon mass),* it is natural to assume $q^2 \ll m_\chi^2$, and the matrix element is then written down in the form of an effective four-fermion interaction

$$M = C(\bar{u}_\nu \gamma_\alpha u_\nu)(\bar{u}_N \Gamma_\alpha^{(\chi)}(q^2) u_N)$$
$$= C(\bar{u}_\nu \gamma_\alpha u_\nu)\{\bar{u}_N [\chi_e F_e(q^2)\gamma_\alpha + \chi_m F_m(q^2)\sigma_{\alpha\beta} q^\beta] u_N\}.$$

Here we have used the following notation: $C = 4\pi f^2/m_\chi^2 \approx 1/10\,m^2$, χ_e and χ_m are the charge and the anomalous f moment of the nucleon, and $F_e(q^2)$ and $F_m(q^2)$ are the charge and magnetic form factors of the nucleon.

As usual,

$$dW = \frac{(2\pi)^4}{(2\pi)^6} \frac{|M|^2}{2\omega_1 2E_1} \frac{d\mathbf{p}_2}{2E_2} \frac{d\mathbf{q}_2}{2\omega_2} \delta^{(4)}(p_2 + q_2 - p_1 - q_1),$$

$$p = (E, \mathbf{p}), \quad q = (\omega, \mathbf{q}).$$

For the cross section we have, in terms of the invariants s and t,

$$d\sigma = dt\,|M(s,t)|^2/16\pi(s - m^2)^2;$$
$$s = (p_1 + q_1)^2 = (p_2 + q_2)^2,$$
$$t = q^2, \quad -t_0 \leqslant t \leqslant 0, \quad t_0 = (s - m^2)^2/s$$

[in the laboratory system $s = m^2(1 + 2\omega_1/m)$, where ω_1 is the energy of the incident neutrino].

The quantity $|M|^2$ is easily computed with the help of the formula

$$\bar{u}_N \sigma_{\alpha\beta} q^\beta u_N = 2m\bar{u}_N \gamma_\alpha u_N - p_\alpha \bar{u}_N u_N \quad (p = p_1 + p_2),$$

which simplifies the evaluation of the traces.

With the notations

$$r = -\frac{t}{t_0}, \quad Z_e = \frac{t_0}{s} = \left(\frac{s - m^2}{s}\right)^2, \quad Z_m = \frac{s - 2m^2}{s},$$

$$a = 0.214\,F^2, \quad X = at_0,$$

we obtain for the final formula for $\sigma = \sigma_e + \sigma_{em} + \sigma_m$:

$$\sigma_e = \frac{C^2\chi_e^2}{4\pi a} X \int_0^1 F_e^2(r)\left[1 - r + \frac{r^2}{2} Z_e\right] dr,$$

$$\sigma_{em} = \frac{C^2(2m\chi_e\chi_m)}{4\pi a} XZ_e \int_0^1 F_e(r) F_m(r) r^2 dr,$$

$$\sigma_m = \frac{C^2\chi_m^2}{4\pi a^2} X^2 \int_0^1 F_m^2(r)[1 - rZ_m] r\,dr.$$

To obtain the cross sections for the first and second cases, one must substitute the corresponding form factors.

For example, in case 1)*

$$F_{e1}(q^2) = F_{eS}(q^2) = 0.44\left(1 + \frac{1.27}{1 - 0.214\,q^2}\right), \qquad \chi_{e1} = 1,$$

$$F_{m1}(q^2) = F_{mS}(q^2) = 4.0 - \frac{3.0}{1 - 0.214\,q^2},$$

$$\chi_{m1} = \frac{1.91}{2m} - \frac{1.79}{2m} = \frac{0.12}{2m}$$

(the index S refers to the scalar nature of the form factor).

In case 2), neither $\Gamma_\alpha^{(\chi)}(q^2)$ nor $\Gamma_\alpha^{(\chi)}(0)$ is known with certainty, and the calculations have

*The values of m_χ and C are taken from the estimates of Kozbarev and Okun'.[1].

*The expressions for the form factors are based on the data of Hofstadter and Herman.[2]

been carried out assuming that

$$F_{e2}(q^2) = F_{en}(q^2) = 0.32 + \frac{0,28}{1-0.214\,q^2} - \frac{0,60}{1-0.10\,q^2},$$

$$\chi_{e2} = \alpha\chi_{e1},$$

$$F_{m2}(q^2) = F_{mS}(q^2), \qquad \chi_{m2} = \beta\chi_{m1}$$

where the unknown coefficients α and β have been arbitrarily set equal to unity.* The index n refers to the neutron.

In the first case we can find the total cross section ($\sigma_1 = \sigma_{1e} + \sigma_{1em} + \sigma_{1m}$). This is impossible in the second case, since we do not know the coefficients α and β. The dependence of the cross sections on the energy of the incident neutrino ω_1 is shown in Fig. 3.

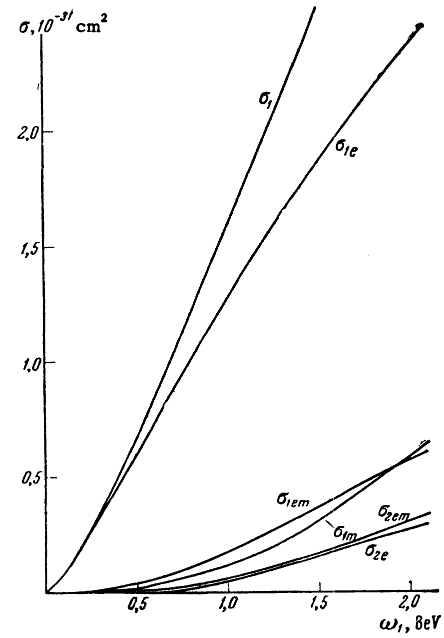

FIG. 3. Dependence of the cross sections on the energy of the neutrino (the total cross section for the process is $\sigma = \sigma_e + \sigma_{em} + \sigma_m$). The index 1 refers to the first case (χN interaction), the index 2 to the second case ($\chi\Lambda$ interaction).

Moreover, for $\omega_1 = 1$ BeV, we have calculated the distributions of the recoil protons over the angles θ (Fig. 4) and the spectra in the laboratory system (Fig. 5).

We have also calculated the average momentum transfer $\sqrt{|\bar{t}|}$ and the average scattering angle $\bar{\theta}$ for the same energy $\omega_1 = 1$ BeV (θ is the angle of emission of the recoil proton):

Cross sections	σ_{1e}	σ_{1em}	σ_{1m}	σ_1	σ_{2e}	σ_{2em}
$\sqrt{\overline{\lvert t\rvert}}$, BeV/c	0.53	1.0	0.90	0.60	0.86	0.98
$\overline{\cos\theta}$	0.53	0,91	0.84	0.59	0.81	0.91

*The constants f, m_χ, α, and β are parameters of the theory and must be determined from experiment. However, the shape of the curves representing the total cross section and the angular and energy distributions cannot depend on them. A change in these constants is equivalent to the multiplication of these curves by some number, i.e., to a change in the scale of the ordinate.

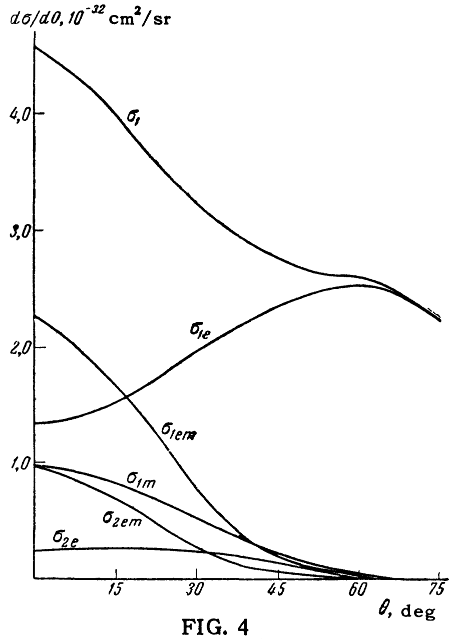

FIG. 4

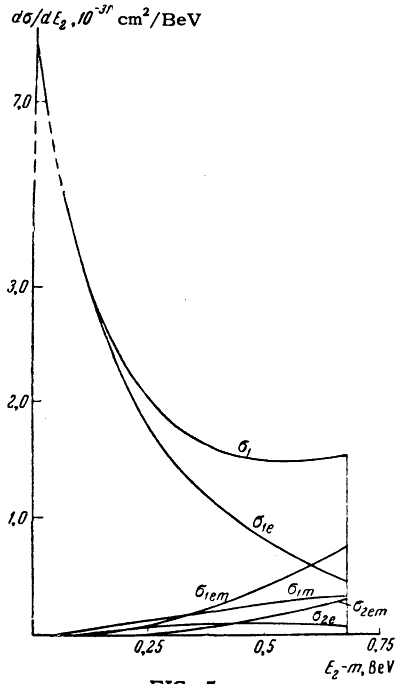

FIG. 5

It follows from the results of this paper that the cross sections amount to $\sim 10^{-31}$ cm^2 in optimal cases, which makes the corresponding experiments entirely feasible.

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[1] I. Yu. Kobzarev and L. B. Okun', JETP **41**, 1205 (1961), Soviet Phys. JETP **14**, 859 (1962).
[2] R. Hofstadter and R. Herman, Phys. Rev. Lett. **6**, 293 (1961).

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