SCATTERING OF HIGH ENERGY ELECTRONS ON ELECTRONS AND THE DIPOLE STRUCTURE OF THE ELECTRON

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The differential cross section for scattering of high-energy electrons on electrons is calculated with the anomalous magnetic and electric dipole moments taken into account.

THE possibility that the electron may possess an electric dipole moment, and the associated violations of space and time parity conservation in electromagnetic interactions at small distances, have been discussed in recent years by a number of authors.[1-5] On the basis of an analysis of the experimental results on the Lamb shift and other atomic effects [1-2] it has been established that $\lambda < 10^{-3}$ (λ stands for the electric dipole moment of the electron in units* of $e/\sqrt{4\pi}\,m$; e and m are respectively the charge and mass of the electron).

Experiments on the determination of the g-factor of the free electron[6] made it possible to lower this limit to 2×10^{-5}. The electron energy in all of the above mentioned experiments was comparatively low—it did not exceed 100 keV.[6]

For the determination of an upper limit on λ an experiment with ultrarelativistic electrons has been proposed by Avakov and Ter-Martirosyan.[7] The calculation of the differential scattering cross section for an electron with an electric dipole moment in the Coulomb field of a spinless nucleus has shown that for large momentum transfers from the electron to the nucleus ($q \sim 500$ MeV) the correction, due to the electric dipole moment, becomes noticeable. At these values of q it becomes necessary to take into account the magnetic dipole structure of the electron.[7] It turns out to be impossible to determine separately the corrections due to the electric and anomalous magnetic moments because these moments enter the cross section only in the form of the sum

$$\alpha^2(q^2) = \mu^2 F_2^2(q^2) + \lambda^2 F_3^2(q^2),$$

where μ is the anomalous magnetic moment of the electron in units of $e/\sqrt{4\pi}\,m$, and $F_2(q^2)$ and $F_3(q^2)$ are the corresponding form factors. Experiments on scattering of 300 MeV electrons with $q = 250$—450 MeV on He^4 nuclei gave $\alpha < 2 \times 10^{-4}$.[8]

It has been repeatedly remarked in the literature that experiments on electron-electron scattering at high energies (over 20 BeV in the laboratory frame) would yield valuable information on the structure of the electron and on the validity of quantum electrodynamics at small distances. Avakov[9] and Baĭer[10] have calculated the electron-electron scattering cross section taking into account the smearing out of the electron charge and magnetic moment (Dirac and anomalous).

In the present work the differential cross section is calculated for the scattering of fast electrons on electrons in which also the smearing out of the dipole structure (magnetic and electric) is taken into account.

The matrix element for the process is of the form

$$M = (2\pi)^4 e^2 \delta(p_1 + p_1' - p_2 - p_2') \times \left\{ \frac{[\bar{u}_2(p_2)\Gamma_\nu(q)u_1(p_1)][\bar{u}_2'(p_2')\Gamma_\nu(-q)u_1'(p_1')]}{(p_1 - p_2)^2} - \frac{[\bar{u}_2'(p_2')\Gamma_\nu(f)u_1(p_1)][\bar{u}_2(p_2)\Gamma_\nu(-f)u_1'(p_1')]}{(p_1 - p_2')^2} \right\}, \qquad (1)$$

where p_1 and p_1' are the 4-momenta of the electrons in the initial state, p_2 and p_2' are the 4-momenta in the final state,

$$q_\nu = (p_2 - p_1)_\nu = (p_1' - p_2')_\nu,$$

$$f_\nu = (p_2' - p_1)_\nu = (p_1' - p_2)_\nu.$$

The vertex operator

$$\Gamma_\nu(q) = F_1(q^2)\gamma_\nu + \frac{i\mu}{4m} F_2(q^2)[\gamma_\nu, \hat{q}] + \frac{i\lambda}{4m} F_3(q^2)[\gamma_\nu, \hat{q}]\gamma_5 \qquad (2)$$

takes into account the presence of structure (i.e., the smearing out) in the electron charge and electric and magnetic dipole moments.

*We make use of a system of units in which $\hbar = c = 1$, $e^2/4\pi = \alpha = 1/137$.

For small values of q the function F_1 describes the distribution of the charge and Dirac magnetic moment of the electron, F_2 —the distribution of the anomalous (including the contributions due to radiative corrections) magnetic moment, and F_3 —the distribution of the electric dipole moment. When, however, q becomes larger than, or of the order of, the inverse length of the distribution of charge and dipole moments, then all three functions F_1, F_2, and F_3 describe the distribution of charge, anomalous magnetic and electric dipole moments.

By substituting Eq. (2) into Eq. (1) we calculate in the usual manner the differential scattering cross section. For $q \gg m$ we have in the barycentric frame the relations

$$(p_1p_2) = (p_1'p_2') = -q^2/2 = -2\varepsilon^2 \sin^2(\vartheta/2),$$
$$(p_1p_2') = (p_1'p_2) = -f^2/2 = -2\varepsilon^2 \cos^2(\vartheta/2),$$
$$(p_1p_1') = (p_2p_2') = -(q^2+f^2)/2 = -2\varepsilon^2,$$

where ϵ and ϑ are respectively the energy and the scattering angle of the electron.

The scattering cross section in this frame is given by*

$$\begin{aligned}
d\sigma = \frac{\pi r_0^2}{\gamma^2}\Big\{ & F_1^4(q^2)\Big(1+\cos^4\frac{\vartheta}{2}\Big)\Big/ 4\sin^4\frac{\vartheta}{2} \\
& + F_1^2(q^2)F_1^2(f^2)\Big/ 2\sin^2\frac{\vartheta}{2}\cos^2\frac{\vartheta}{2} \\
& + F_1^4(f^2)\Big(1+\sin^4\frac{\vartheta}{2}\Big)\Big/ 4\cos^4\frac{\vartheta}{2} \\
& + F_1^2(q^2)\left[\mu^2F_2^2(q^2)+\lambda^2F_3^2(q^2)\right]\gamma^2\mathrm{ctg}^2\frac{\vartheta}{2} \\
& + F_1^2(f^2)\left[\mu^2F_2^2(f^2)+\lambda^2F_3^2(f^2)\right]\gamma^2\mathrm{tg}^2\frac{\vartheta}{2} \\
& + F_1^2(q^2)\left[\mu^2F_2^2(f^2)-\lambda^2F_3^2(f^2)\right] \\
& \times\gamma^2\cos^2\frac{\vartheta}{2}\Big(1+\sin^2\frac{\vartheta}{2}\Big)\Big/ 4\sin^2\frac{\vartheta}{2} \\
& + F_1^2(f^2)\left[\mu^2F_2^2(q^2)-\lambda^2F_3^2(q^2)\right] \\
& \times\gamma^2\sin^2\frac{\vartheta}{2}\Big(1+\cos^2\frac{\vartheta}{2}\Big)\Big/ 4\cos^2\frac{\vartheta}{2} + \frac{1}{8}\left[\mu^2F_2^2(q^2)\right. \\
& \left. + \lambda^2F_3^2(q^2)\right]^2\gamma^4\Big(1+\cos^2\frac{\vartheta}{2}\Big)^2 \\
& + \frac{1}{8}\left[\mu^2F_2^2(f^2)+\lambda^2F_3^2(f^2)\right]^2\gamma^4\Big(1+\sin^2\frac{\vartheta}{2}\Big)^2 \\
& + \frac{1}{8}\left[(\mu^2F_2(q^2)F_2(f^2)+\lambda^2F_3(q^2)F_3(f^2))^2\right. \\
& \left. +\mu^2\lambda^2(F_2(q^2)F_3(f^2)+F_2(f^2)F_3(q^2))^2\right]\gamma^4 \\
& \times\Big(2+\sin^2\frac{\vartheta}{2}\cos^2\frac{\vartheta}{2}\Big)\Big\}\sin\vartheta\, d\vartheta,
\end{aligned} \tag{3}$$

*ctg = cot, tg = tan.

where $r_0 = \alpha/m$ is the classical radius of the electron, and $\gamma = \epsilon/m$.

If we set $\lambda = 0$ in Eq. (3) we obtain an expression which coincides with the result of Baĭer [10] (provided we take in the latter the limit $m \to 0$). We note that in the formula for the cross section given by Avakov [9] the last two terms are incorrect.

Setting in Eq. (3) $F_1 = 1$, $F_2 = F_3 = 0$, we obtain the well-known formula for Møller scattering.

To go over to the laboratory frame of reference it is sufficient to replace in Eq. (3) γ and ϑ by respectively γ_1 and ϑ_1, given by

$$\gamma = \sqrt{\gamma_1/2}, \qquad \cos\vartheta = (2-\gamma_1\sin^2\vartheta_1)/(2+\gamma_1\sin^2\vartheta_1).$$

The relative kinetic energy loss Δ of the incident electron in the laboratory frame is given by $\Delta = \sin^2(\vartheta/2)$. Upon substitution of that expression into Eq. (3) one obtains the distribution in energy of the secondary electrons produced when an electron traverses matter.

The resultant formulas allow in principle the determination of the values of the form factors F_1, μF_2, and λF_3 for various values of the arguments.

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