CALCULATION OF THE POLARIZATION OF LOW-ENERGY COSMIC-RAY MUONS

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The polarization of vertical and inclined muon beams at sea level is calculated as a function of energy in the range from 0 to 2 BeV under the assumption that all muons are produced in pion decays. It is shown that the polarization increases with the muon energy and also that the polarization of the inclined beam is greater than that of the vertical beam at energies $\lesssim$200 MeV.

At the present time almost all the experimental data on the polarization of muons have been obtained for low energies, 0—2 BeV.[1-9] It is of interest to compare these results with those expected theoretically and calculated with account of the character of the pion spectrum at energies lower than several BeV.

The variation of the slope of the pion decay spectrum with pion energy causes differences in the polarization of the muons that are generated at different heights and have an energy E_0 at sea level. Actually, owing to ionization losses, the production energy of such muons increases with height, and at the same time the slope of the corresponding portion of the muon decay spectrum, which determines the degree of polarization, also increases. We calculate below the polarization at sea level with account of muon production over the entire depth of the atmosphere.

1. POLARIZATION OF VERTICAL BEAM

The in-flight polarization of a muon in $\pi \rightarrow \mu$ decay was calculated by Hayakawa[10] and Johnson:[11]

$$\cos\theta = EE^*/pp^* - \varepsilon m_\mu^2/pp^* m_\pi, \qquad (1)$$

where θ —angle between the muon spin and its momentum, E and p —energy and momentum of the muon at the instant of its production, in the laboratory system of coordinates, E^* and p^* —energy and momentum of the muon in the pion rest system, ϵ —energy of the pion whose decay gave rise to the muon under consideration, m_μ and m_π —rest masses of the muon and pion. We use a system of units with $c = 1$ throughout.

We first find the average polarization $\eta_0(E_0, x)$ of a vertical beam of muons with energy E_0 at sea level, produced at a depth x in the atmosphere. For this purpose we determine from the magnitude of the ionization losses the energy $E(x)$ of the muon at the instant of production (muon scattering is disregarded), and formula (1) is then averaged over the function representing the dependence of the number of muons of energy E, produced at a given height, on the energy of the pions whose decay gave rise to the muons under consideration. This function is

$$N(\varepsilon)/\varepsilon^2 \sqrt{1 - m_\pi^2/\varepsilon^2},$$

where $N(\epsilon)$ —air spectrum of the pions, ϵ^{-1} —a quantity proportional to the pion decay probability, $1/\epsilon\sqrt{1 - m_\pi^2/\epsilon^2}$ —a quantity proportional to the probability of production of a muon with energy in the interval under consideration. Using the pion-production spectrum calculated by Olbert[12] we determined by numerical integration the polarization $\eta_0(E_0, x)$ as a function of the depth of muon production in the atmosphere. To determine the average polarization of the vertical muon flux, $\eta_0(E_0, x)$ was averaged over a function (calculated by Sands[13]) representing the probability that a muon with sea-level energy E_0 is produced at a depth x of the atmosphere.

The results of the calculations are represented in Fig. 1 by the portion of the curve for the energy region $E_0 \leq 2$ BeV. The curve was then continued in accord with the data of [14]. The increase in polarization in the low-energy region is due to the change in the effective height of the muon production and to the increase in the slope of the pion spectrum.

2. POLARIZATION OF INCLINED BEAM

The polarization of an inclined muon beam is greater than the polarization of the vertical beam

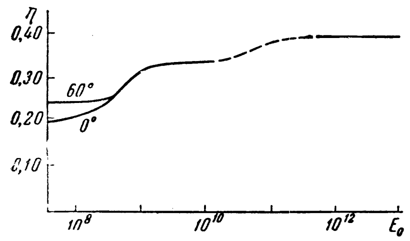

FIG. 1. Muon polarization as a function of the energy at sea level (the figures on the curve indicate the inclination of the beam).

of the same energy, since the effective energy for the production of the former is greater than that for the latter.

We introduce the quantity

$$\Phi_\varphi(E_0, x) = \mu_\varphi(E, x)\, W_\varphi(E_0, x), \tag{2}$$

where $\mu_\varphi(E, x)$ is the number of muons generated at a height x with energy E and traveling at an angle φ to the vertical, $W_\varphi(E_0, x)$ is the probability of a muon created at a height x reaching sea level without decaying and with energy E_0, along a line making an angle θ with the vertical. This probability is given by the formula

$$W_\varphi(E_0, x) = (xp_0/x_0p)^{m_\mu H/\tau_0 p_0' \cos\varphi}, \tag{3}$$

where $x_0 = 1000$ g/cm^2 —depth of the atmosphere at sea level, p_0 —momentum of the muon under consideration at sea level, p′ —momentum of the muon with sea-level momentum p_0 at the instant of its production on the top of the atmosphere (x = 0), τ_0 —lifetime of the muon in its rest frame, and H = 8000 m —height of the homogeneous atmosphere. In the derivation of (3) we took into account ionization losses and the consequent reduction in the muon lifetime.

In order to find the function $\mu_\varphi(E, x)$ let us consider the diffusion equation for a vertical low-energy pion beam

$$\frac{\partial \pi_0(\varepsilon, x)}{\partial x} = -\frac{1}{L_\pi}\pi_0(\varepsilon, x) + f(\varepsilon)\exp\left(-k(\varepsilon)\frac{x}{L_p}\right) - \frac{E_\pi}{x\varepsilon}\pi_0(\varepsilon, x). \tag{4}$$

Here $\pi_0(\epsilon, x)$ —number of muons of energy ϵ at a depth x (all the quantities pertaining to the vertical flux are denoted by a zero subscript), L_π —effective range for inelastic collisions between pions and air nuclei, $E_\pi = 10^{11}$ eV, $f(\epsilon)$ —pion production spectrum, and L = 125 g/cm^2 —effective range of the primary-component protons. The second term in the right part of the equation describes the production of pions of the primary component. Pions of energy ϵ are generated in the vertical beam by nucleons traveling in a cone with aperture angle determined by the pion energy. Therefore the total effective range of the generating component is somewhat greater than the depth of the atmosphere x at which the generation is considered. The factor $k(\epsilon)$, which takes this effect into account, is the reciprocal of the cosine of the average angle between the vertical and the momentum of the nucleon from the cone under consideration.

It is shown in [14] that the solution of the equation (4) for $\epsilon < 10^{10}$ eV is described with good accuracy by the function

$$\pi_0(\varepsilon, x) = BN(\varepsilon)\, x \exp(-k(\varepsilon)\, x/L_p), \tag{5}$$

where B is a constant, $N(\epsilon) = \epsilon f(\epsilon)$ —the air spectrum of the pions. From Olbert's calculation[12] and also from the calculations of Garibyan and Gol'dman[15] it follows that the pion air spectrum changes little with the depth of the atmosphere. This means that $k(\epsilon)$ is very weakly dependent on the energy. When $\epsilon \geq 10^9$ eV, the problem can be regarded as one-dimensional, i.e., $k(\epsilon) = 1$. At lower energies, $k(\epsilon)$ should exceed unity only slightly.

Let us find now the number of muons generated at a height x with energy E and traveling vertically downward

$$\mu_0(E, x) = Bx\exp\left(-\frac{\langle k\rangle}{L_p}x\right)\int_{\varepsilon_-}^{\varepsilon_+}\frac{N(\varepsilon)}{\varepsilon^2\sqrt{1 - m_\pi^2/\varepsilon^2}}\,d\varepsilon. \tag{6}$$

Here ϵ_- and ϵ_+ are the smallest and the largest energy of pions, in the decay of which muons of energy E are produced, $\langle k\rangle$ —average value of $k(\epsilon)$ in the energy interval $\epsilon_- \leq \epsilon \leq \epsilon_+$. Approximating the function $N(\epsilon)\epsilon^{-2}(1 - m_\pi^2/\epsilon^2)^{-1/2}$ in the interval $\epsilon_- \leq \epsilon \leq \epsilon_+$ by the power function $\epsilon^{-\gamma(E)}$ we obtain from (4)

$$\mu_0(E, x) = B\frac{\varepsilon_-^{1-\gamma(E)}}{\gamma(E) - 1}\left[1 - \left(\frac{\varepsilon_-}{\varepsilon_+}\right)^{\gamma(E)-1}\right]x\exp\left(-\frac{\langle k\rangle}{L_p}x\right). \tag{7}$$

We assume that the angular distribution of the muons generated at a height x has the form

$$\mu_\varphi(E, x) = \mu_0(E, x)(\cos\varphi)^{n(E,x)}, \tag{8}$$

where $n(E, x)$ is the exponent in the angular distribution of the muons generated at a height x, and find the average polarization of the muons at sea level, traveling at an angle φ to the vertical

$$\bar{\eta}_\varphi(E_0) = \frac{\int_0^{x_0}\eta_\varphi(E_0, x)\,\Phi_\varphi(E_0, x)\,dx}{\int_0^{x_0}\Phi_\varphi(E_0, x)\,dx}, \tag{9}$$

where $\eta_\varphi(E_0, x)$ —average polarization of the muons at the instant of production at a height x, for an inclined beam; this polarization is calculated by the method indicated in Sec. 1 for $\eta_0(E_0, x)$.

We have actually obtained the increase in the polarization of an inclined beam of muons, compared with a vertical beam of the same energy at sea level:

$$\Delta\bar{\eta} = \bar{\eta}_\varphi(E_0) - \bar{\eta}_0(E_0),$$

where $\eta_0(E_0)$ is the polarization of the vertical muon beam, calculated by formula (9). All the calculations are made by numerical integration; $\langle k \rangle$ is determined from the condition that $\bar{\eta}_0(E_0)$ coincide with the polarization calculated in Sec. 1. Satisfactory agreement is obtained even when $\langle k \rangle = 1$. A change in $\langle k \rangle$ influences $\Delta\bar{\eta}$ very little. We have assumed that the exponent in the muon angular distribution has the form $n(E_0, x) = 2.8x/x_0$, since $n = 2.8$ when $x = x_0$ and $n = 0$ when $x = 0$ for low energies E_0. The assumed x-dependence of $n(E_0, x)$ also influences the value $\Delta\bar{\eta}$ very little.

The result is that a noticeable increase in polarization with the angle φ takes place only for energies $E_0 < 200$ MeV. By way of illustration Fig. 1 shows the energy dependence of the longitudinal polarization of a muon beam making an angle 60° with the vertical.

3. COMPARISON WITH EXPERIMENT

Figure 2 shows the polarization of a vertical muon beam at sea level, as calculated in Sec. 1.

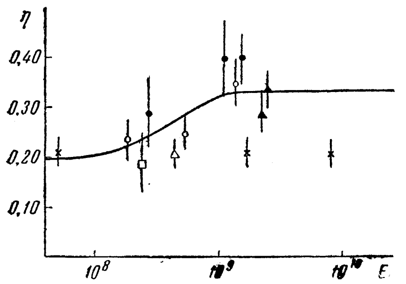

FIG. 2. Comparison with experiment: □ — [1], △ — [4], × — [7], ●, ○ — [9], ▲ — [8]

The same figure shows the experimentally measured polarizations.[1,4,7,9] As can be seen from the figure, all the experimental data except those of Clark and Van der Bradt[7] are in satisfactory agreement with the theoretical calculations. The results of many other investigations[2,3,5,6] are also in agreement with calculation, but owing to the insufficient statistical accuracy they are not shown in the figure.

As regards the aforementioned discrepancy, we note that[7] reports only preliminary experimental results. The authors indicate that the absolute values of the polarization, obtained in their investigation, are not final. The polarization does not increase with energy in this experiment probably because of the overestimated value of the polarization at the minimum muon energy (60 MeV). This overestimate is due to the considerable contribution of the inclined muons, since the experimental apparatus subtended a very large solid angle ($\sim 2\pi$).

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