ASYMPTOTIC SMOOTHING OUT OF A DISCONTINUITY IN A MONATOMIC GAS

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The structure of a shock wave in a monatomic gas is studied with the help of the integral kinetic equations derived by Wallander.[2] Asymptotic laws for smoothing out of the density, velocity and temperature profiles can be determined in a first approximation to the gas-dynamic solution. The form of the asymptotic formulas depends appreciably on the behavior of the collision cross section at large values of the relative velocity of the colliding particles.

1. INTRODUCTION

LYUBARSKIĬ,[1] in an investigation of the structure of a low-intensity shock wave in a monatomic gas, using the simplified kinetic equation for the distribution function f

$$v_x \partial f / \partial x = (f_0 - f)/\tau, \qquad \tau = \text{const}, \tag{1.1}$$

found that at large distances from the shock-wave front the hydrodynamic quantities tend to their limiting values as

$$c_1 \exp\{-c_2 |x|^{2/3}\}; \qquad c_1, c_2 = \text{const}. \tag{1.2}$$

Comparing this result with the exponential smoothing-out law obtained by Becker, Zoller, and Grad, who used the equations for the moments, Lyubarskiĭ arrived at the conclusion that when the kinetic equation is replaced by a finite system of ordinary differential equations it is impossible in principle to obtain the correct asymptotic expression for the solutions of the kinetic equation. In the present paper we attempt to determine the structure of the shock wave at large distances from the front, using likewise the kinetic approach, but without simplifying the kinetic equation. It is found that the asymptotic behavior of the hydrodynamic quantities depend essentially on properties of the collision integral, the general form of which is

$$J = \Phi - fQ, \tag{1.3}$$

where Φ — "production function" and Q — "collision function."[2] In (1.1) we have

$$\Phi = f_0/\tau, \qquad Q = 1/\tau = \text{const}.$$

The asymptotic behavior of the shock equalization is determined by the behavior of the collision function at large particle velocity, which in turn is a direct consequence of the behavior of the collision cross section $\sigma(v)$ at large values of the relative velocity v. If $\sigma(v) \to \sigma_0 \neq 0$ as $v \to \infty$ (see Sec. 5 below), then the equalization is exponential. If, on the other hand, $\sigma(v) = O(v^{-\gamma})$ for large v, with $\gamma > 0$ (Sec. 4), then the discontinuity is smoothed out like

$$c_1 \exp\{-c_2 |x|^{2/(\gamma+2)}\}. \tag{1.4}$$

In particular, when $\gamma = 1$ we get $Q = O(1)$ and (1.4) coincides with (1.2).

These results are derived below without assuming the discontinuity to be of low intensity and in a simpler manner than in [1].

In Sec. 6 we consider separately the case of a discontinuity of maximum intensity ($M_1 = \infty$).

2. FORMULATION OF THE PROBLEM

One-dimensional stationary motion of a monatomic gas in the direction of the x axis is described by the following system of integral equations[2]:

$$f(x, \mathbf{u}) = \int_0^\infty \Phi(x - \tau u_x, \mathbf{u}) \Pi(x, \mathbf{u}, \tau) d\tau, \tag{2.1}$$

$$\Phi(x, \mathbf{u}) = \iint_{-\infty}^{\infty} |\mathbf{u}_1 - \mathbf{u}_2| \sigma(|\mathbf{u}_1 - \mathbf{u}_2|) f(x, \mathbf{u}_1) f(x, \mathbf{u}_2) \times T(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}) d\mathbf{u}_1 d\mathbf{u}_2; \tag{2.2}$$

$$\Pi(x, \mathbf{u}, \tau) = \exp\left\{-\int_0^\tau Q(x - q u_x, \mathbf{u}) dq\right\}, \tag{2.3}$$

$$Q(x, \mathbf{u}) = \int_{-\infty}^{\infty} |\mathbf{u} - \mathbf{u}_1| \sigma(|\mathbf{u} - \mathbf{u}_1|) f(x, \mathbf{u}_1) d\mathbf{u}_1, \tag{2.4}$$

where f — distribution function, u — velocity of the particle, σ — collision cross section, and T — transformant of the collisions[2].

To describe the structure of the shock wave we must find the solution of this system of equations, satisfying the boundary conditions

$$f(x,\mathbf{u}) \xrightarrow[x\to-\infty]{} f_1(\mathbf{u}) \equiv n_1 (h_1/\pi)^{3/2} \exp\{-h_1(\mathbf{u}-\mathbf{U}_1)^2\},$$

$$f(x,\mathbf{u}) \xrightarrow[x\to+\infty]{} f_2(\mathbf{u}) \equiv n_2 (h_2/\pi)^{3/2} \exp\{-h_2(\mathbf{u}-\mathbf{U}_2)^2\}, \quad (2.5)$$

and the quantities n_1, U_1, h_1, and n_2, U_2, h_2 should be connected by the conservation laws[3]

$$n_1U_1 = n_2U_2, \qquad n_1(U_1^2 + RT_1) = n_2(U_2^2 + RT_2),$$
$$U_1^2 + 5RT_1 = U_2^2 + 5RT_2;$$
$$h_i = 1/2RT_i, \quad \mathbf{U}_i = \{U_i, 0, 0\}, \qquad i = 1, 2. \quad (2.6)$$

Putting

$$h = \tfrac{5}{6} M_1^2 \quad (2.7)$$

(M_1 is the Mach number ahead of the discontinuity), we obtain from (2.6)

$$\frac{n_2}{n_1} = \frac{U_1}{U_2} = \frac{8h}{2h+5}, \qquad \frac{h_2}{h_1} = \frac{32h}{(2h+5)(6h-1)},$$
$$s_1 \equiv U_1\sqrt{h_1} = \sqrt{h}, \qquad s_2 \equiv U_2\sqrt{h_2} = \sqrt{\frac{2h+5}{2(6h-1)}}. \quad (2.8)$$

3. FIRST APPROXIMATION SOLUTION

Substitution of (2.2) — (2.4) in (2.1) yields an integral equation for f in the form f = Vf, which is best solved by successive approximations: $f^{(n)} = Vf^{(n-1)}$. If we take as the zeroth approximation the gas-dynamic solution

$$f^{(0)} = \begin{cases} f_1(\mathbf{u}), & x<0 \\ f_2(\mathbf{u}), & x>0 \end{cases}, \quad (3.1)$$

Then we obtain in the first approximation (as shown by A. V. Belova)

$$f^{(1)} = \begin{cases} f_1, & x<0, \quad u_x>0 \\ f_1 + (f_2 - f_1)\exp\{-xu_x^{-1}Q_1(|\mathbf{u}-\mathbf{U}_1|)\}, & x<0, \quad u_x<0 \\ f_2 + (f_1 - f_2)\exp\{-xu_x^{-1}Q_2(|\mathbf{u}-\mathbf{U}_2|)\}, & x>0, \quad u_x>0 \\ f_2, & x>0, \quad u_x<0 \end{cases}, \quad (3.2)$$

where

$$Q_i(v) = n_i\left(\frac{h_i}{\pi}\right)^{3/2} \int_{-\infty}^{\infty} |\mathbf{v}-\mathbf{v}_1|\,\sigma(|\mathbf{v}-\mathbf{v}_1|)\exp\{-h_i v_1^2\}\,d\mathbf{v}_1. \quad (3.3)$$

The corresponding density, velocity, and temperature are determined by the formulas

$$n = \int_{-\infty}^{\infty} f^{(1)}\,d\mathbf{u}, \qquad \mathbf{U} = \frac{1}{n}\int_{-\infty}^{\infty} \mathbf{u}f^{(1)}\,d\mathbf{u},$$

$$T = \frac{1}{3Rn}\int_{-\infty}^{\infty} (\mathbf{u}-\mathbf{U})^2 f^{(1)}\,d\mathbf{u}. \quad (3.4)$$

Since it is our purpose to establish the order of the principal term in the asymptotic approximation as $|x| \to \infty$, we do not seek the next approximation, and confine ourselves to the first. This step is natural if we start from the physical meaning of the operator V.[2] An additional justification is the fact that the results obtained for $\gamma = 1$ agree with those of Lyubarskiĭ[1].

Writing

$$n = \begin{cases} n_1(1 - n_{11} + n_{12}), & x<0 \\ n_2(1 - n_{22} + n_{21}), & x>0 \end{cases},$$

$$U = \begin{cases} U_1 \dfrac{n_1}{n}(1 - U_{11} + U_{12}), & x<0 \\ U_2 \dfrac{n_2}{n}(1 - U_{22} + U_{21}), & x>0 \end{cases},$$

$$T = \begin{cases} T_1 \dfrac{n_1}{n}(T_{1,0} - T_{11} + T_{12}), & x<0 \\ T_2 \dfrac{n_2}{n}(T_{2,0} - T_{22} + T_{21}), & x>0 \end{cases}, \qquad T_{i0} = 1 + \frac{|\mathbf{U}_i - \mathbf{U}|^2}{3RT_i}, \quad (3.5)$$

we obtain from (3.4), (3.2) and (2.5), after making the substitution $\mathbf{u} - \mathbf{U}_i = \mathbf{v}$,

$$n_{11} = \left(\frac{h_1}{\pi}\right)^{3/2} \int_{v_x<-U_1} \exp\left\{-h_1v^2 - \frac{xQ_1(v)}{v_x+U_1}\right\}d\mathbf{v},$$

$$n_{12} = \frac{n_2}{n_1}\left(\frac{h_2}{\pi}\right)^{3/2} \int_{v_x<-U_1} \exp\left\{-h_2(\mathbf{v}+\mathbf{U}_1-\mathbf{U}_2)^2 - \frac{xQ_1(v)}{v_x+U_1}\right\}d\mathbf{v},$$

$$n_{21} = \frac{n_1}{n_2}\left(\frac{h_1}{\pi}\right)^{3/2} \int_{v_x>-U_2} \exp\left\{-h_1(\mathbf{v}+\mathbf{U}_2-\mathbf{U}_1)^2 - \frac{xQ_2(v)}{v_x+U_2}\right\}d\mathbf{v},$$

$$n_{22} = \left(\frac{h_2}{\pi}\right)^{3/2} \int_{v_x>-U_2} \exp\left\{-h_2v^2 - \frac{xQ_2(v)}{v_x+U_2}\right\}d\mathbf{v}. \quad (3.6)$$

U_{11}, and U_{12} differ from n_{11} and n_{12} respectively by the factor $(v_x + U_1)/U_1$ under the integral sign, and from T_{11} and T_{12} respectively by the factor $(\mathbf{v} + \mathbf{U}_1 - \mathbf{U})^2/3RT_1$. Analogously, U_{21} and U_{22}, unlike n_{21} and n_{22}, contain the factor $(v_x + U_2)/U_2$, while T_{21} and T_{22} contain the factor $(\mathbf{v} + \mathbf{U}_2 - \mathbf{U})^2/3RT_2$.

To investigate the asymptotic behavior of the functions n_{ij}, U_{ij}, T_{ij}, $(i, j = 1, 2)$ as $|x| \to \infty$, we must know the behavior of the functions $Q_i(v)$ as $v \to \infty$. If we assume that at infinity

$$\sigma(v) = O(v^{-\gamma}), \quad \gamma \geqslant 0, \quad (3.7)$$

then, as follows from (3.3),

$$Q_i(v) = O(v^{1-\gamma}). \quad (3.8)$$

4. ASYMPTOTIC EXPRESSION FOR $\gamma > 0$

Let us examine n_{11} as $x \to -\infty$. Introducing the spherical coordinates (v, ϑ, φ) defined through $v_x = v\cos\vartheta$, $v_y = v\sin\vartheta\cos\varphi$, and $v_z = v\sin\vartheta\sin\varphi$, we obtain after integrating with respect to φ and putting $\cos\vartheta = t$

$$n_{11} = \frac{2}{\sqrt{\pi}}\, h_1^{3/2} \int_{U_1}^{\infty}\int_{-1}^{-U_1/v} \exp\left\{-h_1v^2 - \frac{xQ_1(v)}{vt+U_1}\right\} v^2\,dt\,dv. \quad (4.1)$$

The lowest value of the coefficient of x in the exponent is obtained when $t = -1$. Expanding this coefficient in powers of $(t + 1)$, we obtain for the principal term of the asymptotic expansion

$$n_{11} \approx \frac{2}{\sqrt{\pi}} h_1^{3/2} \int_{U_1}^{\infty} \exp\left\{-h_1 v^2 - \frac{|x| Q_1(v)}{v - U_1}\right\} \frac{v(v-U_1)^2}{|x| Q_1(v)} dv. \quad (4.2)$$

At large v the two terms in the exponent compete with each other, and the lowest order of magnitude is attained when they are equivalent, i.e., by virtue of (3.8), when $v = O(|x|^{1/(\gamma+2)})$. This fact dictates the subsequent steps in the procedure. First, it is convenient to divide the integration interval in two parts separated by the point

$$|x|^{1/(\gamma+2)-\varepsilon}, \qquad 0 < \varepsilon < 1/(\gamma+2).$$

We can then replace in the second integral (which contains the principal term of the asymptotic expression) the function $Q_1(v)/(v - U_1)$ by its asymptotic expression $c_1 v^{-\gamma}$. Changing over now to the integration variable $u = v|x|^{-1/(\gamma+2)}$, we obtain

$$n_{11} \approx \frac{2}{c_1\sqrt{\pi}} h_1^{3/2} |x|^{1/(\gamma+2)} \times \int_{|x|^{-\varepsilon}}^{\infty} \exp\{-|x|^{2/(\gamma+2)}(h_1 u^2 + c_1 u^{-\gamma})\} u^{\gamma+2} du. \quad (4.3)$$

The function $F(u) = h_1 u^2 + c_1 u^{-\gamma}$ has in the interval $(0, \infty)$ a single minimum at the point

$$u_0 = (\gamma c_1/2h_1)^{1/(\gamma+2)}, \quad (4.4)$$

with $F''(u_0) = 2h_1(\gamma + 2)$.

Following now the usual Laplace method[4] we get

$$n_{11} \approx \frac{2h_1^{3/2}}{c_1\sqrt{\pi}} |x|^{1/(\gamma+2)} \exp\{-|x|^{2/(\gamma+2)} F(u_0)\} \times \int_{|x|^{-\varepsilon}}^{\infty} \exp\left\{-|x|^{2/(\gamma+2)} \frac{F''(u_0)}{2}(u-u_0)^2\right\} u^{\gamma+2} du$$

$$\approx \frac{\gamma}{\sqrt{\gamma+2}} \exp\left\{-|x|^{2/(\gamma+2)} h_1 \left(\frac{\gamma+2}{\gamma}\right)\left(\frac{\gamma c_1}{2h_1}\right)^{2/(\gamma+2)}\right\}. \quad (4.5)$$

The remaining integrals are estimated in similar fashion.

The final result is written in dimensionless form. For this purpose we put (for large v)

$$\sigma \approx \sigma_0 (v\sqrt{h_2})^{-\gamma}.$$

Then

$$Q_i \approx n_i \sigma_0 v (v\sqrt{h_2})^{-\gamma},$$

And in place of $|x|$ we can naturally introduce the dimensionless distance

$$\xi = |x| n_1 \sigma_0. \quad (4.6)$$

As a result we obtain

$$n_{11} \approx \frac{\gamma}{\sqrt{\gamma+2}} \exp\left\{-\frac{\gamma+2}{\gamma}\left(\frac{h_1}{h_2}\right)^{\gamma/(\gamma+2)}\left(\frac{\gamma}{2}\xi\right)^{2/(\gamma+2)}\right\},$$

$$n_{12} \approx \frac{n_2}{n_1}\frac{\gamma}{\sqrt{\gamma+2}} \exp\left\{-\frac{\gamma+2}{\gamma}\left(\frac{\gamma}{2}\xi\right)^{2/(\gamma+2)}\right\},$$

$$n_{21} \approx \frac{n_1}{n_2}\frac{\gamma}{\sqrt{\gamma+2}} \exp\left\{-\frac{\gamma+2}{\gamma}\left(\frac{h_1}{h_2}\right)^{\gamma/(\gamma+2)}\left(\frac{n_2}{n_1}\frac{\gamma}{2}\xi\right)^{2/(\gamma+2)}\right\},$$

$$n_{22} \approx \frac{\gamma}{\sqrt{\gamma+2}} \exp\left\{-\frac{\gamma+2}{\gamma}\left(\frac{n_2}{n_1}\frac{\gamma}{2}\xi\right)^{2/(\gamma+2)}\right\}; \quad (4.7)$$

$$U_{11} \approx -\left(\frac{h_2}{h_1}\right)^{1/(\gamma+2)}\left(\frac{\gamma}{2}\xi\right)^{1/(\gamma+2)}\frac{1}{U_1\sqrt{h_2}} n_{11},$$

$$U_{12} \approx -\left(\frac{\gamma}{2}\xi\right)^{1/(\gamma+2)}\frac{1}{U_1\sqrt{h_2}} n_{12},$$

$$U_{21} \approx \left(\frac{h_2}{h_1}\right)^{1/(\gamma+2)}\left(\frac{n_2}{n_1}\frac{\gamma}{2}\xi\right)^{1/(\gamma+2)}\frac{1}{s_2} n_{21},$$

$$U_{22} \approx \left(\frac{n_2}{n_1}\frac{\gamma}{2}\xi\right)^{1/(\gamma+2)}\frac{1}{s_2} n_{22}; \quad (4.8)$$

$$T_{11} \approx \frac{2}{3}\left(\frac{h_1}{h_2}\right)^{\gamma/(\gamma+2)}\left(\frac{\gamma}{2}\xi\right)^{2/(\gamma+2)} n_{11},$$

$$T_{12} \approx \frac{2}{3}\frac{h_1}{h_2}\left(\frac{\gamma}{2}\xi\right)^{2/(\gamma+2)} n_{12},$$

$$T_{21} \approx \frac{2}{3}\left(\frac{h_2}{h_1}\right)^{2/(\gamma+2)}\left(\frac{n_2}{n_1}\frac{\gamma}{2}\xi\right)^{2/(\gamma+2)} n_{21},$$

$$T_{22} \approx \frac{2}{3}\left(\frac{n_2}{n_1}\frac{\gamma}{2}\xi\right)^{2/(\gamma+2)} n_{22}. \quad (4.9)$$

One of the interesting qualitative consequences of these formulas is that the velocity profile is not monotonic when $x > 0$.

5. ASYMPTOTIC EXPRESSION FOR $\gamma = 0$

When $Q_1(v)/(v \pm U_i) \to c_i > 0$ as $v \to +\infty$, the principal term of the asymptotic expression will always contain a factor of the form $e^{-c\xi}$. The coefficient c is determined by the position of the minimum of the function $Q_i(v)/(v \pm U_i)$, which depends on the specific properties of the function $Q_i(v)$. We put

$$Q_i(v) = n_i \sigma_0 h_i^{-1/2} R(v\sqrt{h_i}), \quad (5.1)$$

where $R(u)$ is a dimensionless function such that $\lim R(u)/u = 1$. In particular, when $\sigma = \text{const}$, we have

$$R(u) = \frac{1}{\sqrt{\pi}} \exp\{-u^2\} + (1 + 2u^2)\frac{\operatorname{erf} u}{2u}. \quad (5.2)$$

We consider first n_{22}. Going over from (3.6) to a form similar to (4.2) we obtain after making the substitution $v\sqrt{h_2} = u$

$$n_{22} \approx \frac{n_1}{n_2}\frac{2}{\sqrt{\pi}} \int_0^{\infty} \exp\left\{-u^2 - \frac{n_2}{n_1}\frac{\xi R(u)}{u+s_2}\right\} \frac{u(u+s_2)}{\xi R(u)} du. \quad (5.3)$$

Let the function $H(u, s_2) = R(u)/(u + s_2)$ assume its lowest value at a finite positive point $u_0 = u_0(s_2)$, and let $H''(u_0, s_2) = R''(u_0)/(u_0 + s_2) > 0$. We then obtain by the Laplace method

$$n_{22} \approx \left(\frac{n_1}{n_2}\frac{2}{\xi}\right)^{3/2} \exp\left\{-u_0^2 - \frac{n_2}{n_1}\frac{\xi R(u_0)}{u_0 + s_2}\right\} \frac{u_0(u_0+s_2)^{3/2}}{R(u_0)\sqrt{R''(u_0)}}. \quad (5.4)$$

The general result for $x > 0$ can be written in the form

$$\frac{n_{22}}{n_{22}^*} = \frac{n_{21}}{n_{21}^*} = \frac{U_{22}}{U_{22}^*} = \frac{U_{21}}{U_{21}^*} = \frac{T_{22}}{T_{22}^*} = \frac{T_{21}}{T_{21}^*}$$

$$\approx \left(\frac{n_1}{n_2}\frac{2}{\xi}\right)^{3/2} \exp\left\{-\frac{n_2}{n_1}\frac{\xi R(u_0)}{u_0 + s_2}\right\} \frac{u_0(u_0+s_2)^{3/2}}{R(u_0)\sqrt{R''(u_0)}}, \quad (5.5)$$

where

$$n_{22}^* = \exp\{-u_0^2\}, \quad U_{22}^* = \left(1 + \frac{u_0}{s_2}\right) n_{22}^*,$$

$$T_{22}^* = \frac{2}{3}\left[u_0 + s_2\left(1 - \frac{U}{U_2}\right)\right]^2 n_{22}^*,$$

$$n_{21}^* = \frac{n_1}{n_2}\left(\frac{h_1}{h_2}\right)^{3/2} \exp\left\{-\left[\sqrt{\frac{h_1}{h_2}}(u_0 + s_2) - s_1\right]^2\right\},$$

$$U_{21}^* = \left(1 + \frac{u_0}{s_2}\right) n_{21}^*, \quad T_{21}^* = \frac{2}{3}\left[u_0 + s_2\left(1 - \frac{U}{U_2}\right)\right]^2 n_{21}^*. \quad (5.6)$$

In the case (5.2) ($\sigma = \mathrm{const}$) we have

$$R''(u_0) = \frac{1}{u_0^3}\left\{\operatorname{erf} u_0 - \frac{2u_0}{\sqrt{\pi}}\exp\{-u_0^2\}\right\}, \quad (5.7)$$

and $u_0 = u_0(s_2)$ is determined from the expression

$$2\operatorname{erf} u_0 = s_2\left\{\frac{2}{\sqrt{\pi}}\exp\{-u_0^2\} + \left(2u_0 - \frac{1}{u_0}\right)\operatorname{erf} u_0\right\}. \quad (5.8)$$

We now examine n_{11} in a form analogous to (5.3):

$$n_{11} \approx \frac{2}{\sqrt{\pi}}\int_{s_1}^{\infty} \exp\left\{-u^2 - \frac{\xi R(u)}{u - s_1}\right\} \frac{u(u-s_1)^2}{\xi R(u)} du. \quad (5.9)$$

Using the case $\sigma = \mathrm{const}$ as a guide, we assume that $H(u, -s_1)$ assumes the lowest value in the integration interval only at infinity, with $H(u, -s_1) \approx 1 + s_1/u + \ldots$ Then, after factoring out $e^{-\xi}$, we are left with an integral of the same type as for $\gamma = 1$.

As a result we obtain

$$n_{11} \approx \frac{s_1}{\sqrt{3}}\left(\frac{s_1}{2}\xi\right)^{-1/3} \exp\left\{-\xi - 3\left(\frac{s_1}{2}\xi\right)^{2/3}\right\},$$

$$n_{12} \approx \frac{n_2}{n_1}\frac{s_1}{\sqrt{3}}\left(\frac{h_2}{h_1}\right)^{1/3}\left(\frac{s_1}{2}\xi\right)^{-1/3} \exp\left\{-\xi - 3\left(\frac{h_2}{h_1}\right)^{1/3}\left(\frac{s_1}{2}\xi\right)^{2/3}\right\},$$

$$U_{11} \approx \frac{-1}{s_1}\left(\frac{s_1}{2}\xi\right)^{1/3} n_{11}, \quad U_{12} \approx \frac{-1}{s_1}\left(\frac{h_1}{h_2}\frac{s_1}{2}\xi\right)^{1/3} n_{12},$$

$$T_{11} \approx \frac{2}{3}\left(\frac{s_1}{2}\xi\right)^{2/3} n_{11}, \quad T_{12} \approx \frac{2}{3}\left(\frac{h_1}{h_2}\frac{s_1}{2}\xi\right)^{2/3} n_{12}. \quad (5.10)$$

6. THE CASE $M_1 = \infty$

When the Mach number ahead of the discontinuity is infinite, we have

$$f_1(\mathbf{u}) = n_1\delta(\mathbf{u} - \mathrm{U}_1), \quad (6.1)$$

and the exact values of the integrals containing f_1 are taken. Then $n_{11} = U_{11} = T_1T_{11} = 0$, since the integration region $u_x < 0$ does not contain the point $u = U_1$.

Further,

$$n_{21} = (n_1/n_2)\exp\{-xQ_2(U_1 - U_2)/U_1\},$$

$$U_{21} = (U_1/U_2)\, n_{21}, \quad T_{21} = \frac{2}{3}h_2(U_1 - U)^2 n_{21}. \quad (6.2)$$

The asymptotic formulas for n_{22}, n_{12}, U_{22}, U_{12}, T_{22} and T_1T_{12}/T_2 are directly obtained from the corresponding formulas for $M_1 \neq \infty$ by taking the limit $h \to \infty$.

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