KINETICS OF NEUTRONIZATION AT ULTRA-HIGH DENSITY

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The rate is calculated of the transformation of atomic nuclei into neutrons as matter is compressed to extremely high densities. The matrix elements are those of the inverse β-decay process. In helium and carbon the neutronization processes require times of the order of ten-thousandths of a second, while in hydrogen and iron they continue many seconds after the thermodynamically required density limit is reached. The results of the calculation may be useful in analyzing the mechanism of the gravitational collapse of stars after all energy sources are exhausted.

WHEN the chemical potential of electrons in highly compressed matter exceeds a certain limit the electrons will combine with protons to form neutrons. The equilibrium condition for this reaction is well known. [1] We can expect neutronization processes to occur in nature under non-equilibrium conditions, when the catastrophic compression (collapse) of a star, following the exhaustion of all its energy sources, results in a stellar explosion (a supernova). Here the process is so rapid that equilibrium cannot be established. We shall therefore calculate the kinetics of the neutronization reaction.

We write the reaction in the form

$$A + e^- \rightarrow B + \nu - \Delta,$$

where A and B are nuclei and Δ is the energy difference, which like the electron energy ϵ will be given in units of mc^2. The probability W of this process, like the recombination probability, will be given in terms of the initial electron concentration n_E and the final neutrino state density ρ_E per unit energy interval:

$$dW = 2\pi\hbar^{-1} |\langle H'\rangle|^2 n_E \rho_E \, dE. \tag{1}$$

The matrix element $\langle H'\rangle$ of neutronization equals the matrix element of the inverse β-decay process

$$B \rightarrow A + e^- + \bar{\nu} + \Delta.$$

The energy required for neutronization exceeds the electron rest energy; therefore neutronization occurs in a degenerate electron gas. We shall here consider the simplest limiting case of complete degeneracy, when the thermal energy is small compared with the Fermi energy. In this case the chemical potential of an electron is simply equal to the Fermi energy ϵ_m.

The electron concentration n_E per unit energy interval in a completely degenerate gas is a power function which is zero at energies above ϵ_m, and which coincides with the density of possible electron states at energies below ϵ_m. Therefore in the limiting case of complete degeneracy the differential probability dW at energies below ϵ_m is expressed exactly as for β decay. However, the complete probability is obtained by integrating from Δ to ϵ_m, instead of from 0 to Δ as for β decay.

The probability of neutronization in the completely degenerate case is therefore expressible in terms of the reduced time of its inverse the β process:

$$W = \overline{W} / (ft_\beta), \tag{2}$$

where the dimensionless function $\overline{W}$ represents

$$\overline{W} = w(\varepsilon_m) - w(\Delta). \tag{3}$$

$$w(\varepsilon) = \int_1^\varepsilon \varepsilon \sqrt{\varepsilon^2 - 1}\, (\varepsilon^2 - \Delta)^2 d\varepsilon. \tag{4}$$

The integrand in (4) has the same form as the Fermi statistical function, but with different integration limits. The integral is found analytically to be

$$w(\varepsilon) = \tfrac{1}{60}\sqrt{\varepsilon^2 - 1}\,\{12\varepsilon^4 - 30\varepsilon^3\Delta + 4(5\Delta^2 - 1)\varepsilon^2 + 15\varepsilon\Delta - 4(5\Delta^2 + 2)\} + \tfrac{1}{4}\Delta \ln\left(\varepsilon + \sqrt{\varepsilon^2 - 1}\right). \tag{5}$$

At high densities the statistical function of neutronization approaches a limit:

$$\overline{W} \approx \tfrac{1}{5}(\varepsilon_m^5 - \Delta^5). \tag{6}$$

The value of Δ is given by the proton-neutron mass difference only for hydrogen. For all other nuclei the inverse β process, which produces a neutron that remains inside the nucleus, is not regarded as neutronization. The energy expenditure required for the liberation of a neutron is of at least the order of the neutron binding energy. Some specific values of Δ calculated from the binding energies are:

$$H^1 + e^- \to n + \nu - 0.783 \text{ MeV}, \quad \Delta = 2.53;$$
$$He^4 + e^- \to H^3 + n + \nu - 20.595 \text{ MeV}, \quad \Delta = 41.3;$$
$$C^{12} + e^- \to B^{11} + n + \nu - 16.740 \text{ MeV}, \quad \Delta = 33.8;$$
$$Fe^{56} + e^- \to Mn^{55} + n + \nu - 10.937 \text{ MeV}, \quad \Delta = 22.4.$$

These are typical values; for most nuclei Δ is about 20—22. Carbon and especially helium are relatively stable, while the neutronization of hydrogen is relatively very easy. The figure shows the dependence of $\log_{10} \overline{W}$ on $\log_{10} \epsilon_m$ for these four typical values of Δ; the asymptote (6) is represented by a dashed line.

The Fermi energy ϵ_m is conveniently represented as a function of the dimensionless electron concentration ν:

$$\varepsilon_m = \sqrt{1 + \nu^{2/3}}, \tag{7}$$

$$\nu = 3n\lambda_c^3/8\pi = 3\pi^2 n \lambda\!\!\!^{-}{}_c^3 \approx 10^{-30}\, n, \tag{8}$$

where $\lambda_c = h/mc$ is the Compton wavelength.

The neutronization process begins as soon as ϵ_m becomes equal to Δ. At equilibrium the electron concentration is such that $\epsilon_m = \Delta$. Neutronization is a single-stage process only in the case of hydrogen. In all other nuclei, the initial reaction must be followed by a chain of successive processes whereby each resultant nucleus is neutronized. The rate of the combined process will be determined by the slowest reaction in the chain. We shall use the kinetic characteristics (Δ, ft_β) of the slowest (controlling) stage. The succeeding stages are, as a rule, much more rapid, and we can assume that each occurrence of the controlling reaction results in the complete neutronization of the initial nucleus with charge Z and in the consumption of Z electrons.

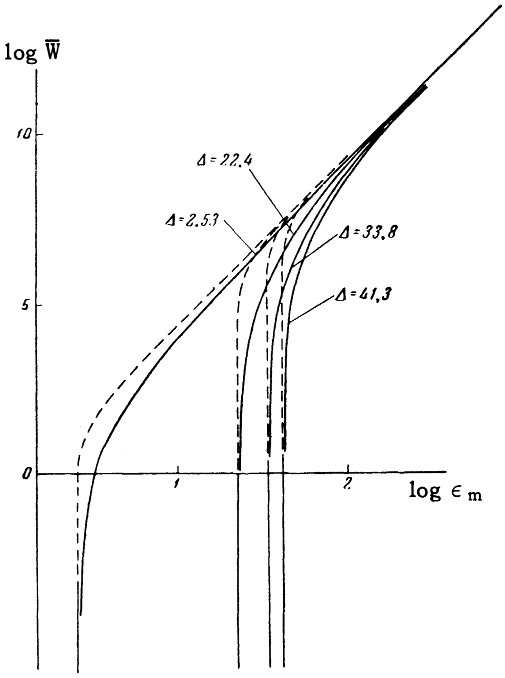
The course of neutronization can be represented by the kinetic equation

$$\frac{dN}{dt} = \frac{1}{Z}\frac{dn}{dt} = -WN = -\frac{\overline{W}}{ft_\beta}N, \tag{9}$$

where n is the electron concentration, N is the concentration of nuclei, f and t_β are the Fermi statistical function and the mean life for the inverse β decay, and $\overline{W}$ is the dimensionless function defined by Eqs. (3)—(4) and represented in the figure.

When $\Delta \gg 1$, $\overline{W}$ can be represented by the asymptotic formula (6), which is sufficiently accurate for all nuclei except hydrogen. The equation of neutronization kinetics then becomes

$$\frac{dN}{dt} = \frac{1}{Z}\frac{dn}{dt} \approx -\frac{\varepsilon_m^5 - \Delta^5}{5ft_\beta}N. \tag{10}$$



For large values of ϵ_m, (7) and (8) give

$$\varepsilon_m \approx \nu^{1/3} = \lambda\!\!\!^{-}{}_c (3\pi^2 n)^{1/3}. \tag{11}$$

We now introduce the equilibrium electron concentration n_* derived from the condition

$$\varepsilon_m(n_*) = \Delta. \tag{12}$$

Equation (10) can now be put into the form

$$\frac{dn}{dt} = -\frac{(3\pi^2)^{5/3}}{5}\frac{\lambda\!\!\!^{-}{}_c^5}{ft_\beta}\left(n^{5/3} - n_*^{5/3}\right)n. \tag{13}$$

This equation is easily integrated. For the time required to reduce the initial electron concentration n_0 to the running value n we finally obtain

$$t \approx \frac{3ft_\beta}{\Delta^5}\ln\frac{n_0^{5/3} - n_*^{5/3}}{n^{5/3} - n_*^{5/3}}. \tag{14}$$

This result shows that if the initial electron concentration is associated with a Fermi energy appreciably exceeding Δ, the neutronization time depends only logarithmically on the initial and final states.

The order of magnitude of the neutronization time is given by

$$t_n \sim 3ft_\beta\Delta^{-5}. \tag{15}$$

Equation (14) applies to all nuclei except hydrogen. The rough estimate (15) can always be used.

Estimated neutronization times for the foregoing four typical reactions are given in the table, along with the density limits ρ^* for which neutronization becomes possible.

Initial nucleus	Δ	ρ^*, g/cm^3	$ft_{1/2}$	Δ^5	$t_n = \frac{3ft_{1/2}}{\Delta^5 \ln 2}$
H	2.53	$2\cdot10^7$	$1.2\cdot10^3$	$1.0\cdot10^2$	17
He	41.3	$2\cdot10^{11}$	$1.0\cdot10^3$	$1.2\cdot10^8$	$\sim10^{-5}$
C	33.8	$1\cdot10^{11}$	$1.5\cdot10^4$	$4.4\cdot10^7$	$4.9\cdot10^{-4}$
Fe	22.4	$4\cdot10^{10}$	$1.55\cdot10^7$	$5.6\cdot10^6$	4.0

The values of $ft_{1/2}$ for hydrogen, carbon, and iron have been taken from the inverse β decays of the neutron,[2] B^{12} and Mn^{56}.[3] For helium, since the decay of H^4 cannot be used, we estimate the order of magnitude of the analogous tritium decay process. In the literature on β processes it is customary to use the half-life $t_{1/2}$, which is related to the mean life t_β by $t_{1/2} = t_\beta \ln 2$. Accordingly, we have $t_n = 3ft_{1/2}/\Delta^5 \ln 2$.

Thus, while the neutronization of carbon and helium is concluded in less than a millisecond, the neutronization of hydrogen and iron requires many seconds. The neutronization of hydrogen can hardly occur in actuality since hydrogen will probably be consumed through thermonuclear or "pycnonuclear" reactions[4,5] before the density required for neutronization is reached. The evaluation of the neutronization rate for helium, carbon, and iron is important for determining the mechanism involved in the gravitational collapse of stars deprived of energy sources.

[1] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Gostekhizdat, 1951, ch. 11.

[2] Sosnovskiĭ, Spivak, Prokof'ev, Kutikov, and Dobrynin, JETP **36**, 1012 (1959), Soviet Phys. JETP **9**, 717 (1959).

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[4] Ya. B. Zel'dovich, JETP **33**, 991 (1957), Soviet Phys. JETP **6**, 760 (1958).

[5] A. G. W. Cameron, Astrophys. J. **130**, 916 (1959).

Translated by I. Emin
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