RADIATION FROM ELECTRON BUNCHES IN A MICROTRON

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The coherent radiation produced by electron bunches moving in circular orbits in a magnetic field gives rise to retarding forces that can remove the electrons from the acceleration mode. These forces set an upper limit on the current of accelerated particles in a microtron. We calculate the radiative retarding force acting on an ultrarelativistic electron bunch that moves in a circular orbit and has a spherically symmetric charge distribution (in the laboratory coordinate system). The limiting current in the microtron is estimated to be approximately one ampere.

FORMULATION OF THE PROBLEM

An electron moving in a circle of radius a with velocity $c\beta$ radiates a power

$$P_0 = \frac{2e^2c}{3a^2}\beta^4\gamma^4 \qquad (\gamma = 1/\sqrt{1-\beta^2}) \tag{1}$$

and is subject to a retarding force

$$F_\varphi = -\frac{2e^2}{3a^2}\beta^3\gamma^4. \tag{2}$$

Now consider a bunch consisting of N electrons moving in a circular orbit; if the bunch is small enough (point charge) the radiated power is increased by a factor of N^2 while F_φ, the force acting on each electron, is increased by a factor N. The radiated power for a bunch of finite dimensions is

$$P = N^2\frac{2e^2c}{3a^2}\beta^4\gamma^4\Theta, \tag{3}$$

where the factor $\Theta < 1$ approaches unity as the coherence of the radiation produced by the electrons in the bunch increases, i.e., as the bunch dimensions become smaller; in what follows we shall call Θ the coherence factor. Since the retarding force is not the same for different electrons in the bunch, it is convenient to introduce a mean retarding force

$$\overline{F}_\varphi = -N\frac{2e^2}{3a^2}\beta^3\gamma^4\Theta \tag{4}$$

such that $N\overline{F}_\varphi$ is the force acting on the entire bunch. The quantities P and $\overline{F}_\varphi$ are then related by

$$P = -N\overline{F}_\varphi c\beta \tag{5}$$

and the factor Θ is the same in (3) and (4).

We shall be interested in the radiation retarding force acting on electron bunches in the microtron. Calculation [1] and direct measurements [2] indicate that the dimensions of these bunches are approximately the same in all directions in the laboratory frame (l.s.) so that the problem can be simplified by assuming that the charge distribution is spherically symmetric (in the l.s.) and that the bunch translates like a solid body with velocity $c\beta$, describing a circle of radius a. Using these assumptions it is relatively easy to compute the coherence factor and to estimate the effect of the radiation retardation on the motion of electrons in the microtron; in particular, it is possible to determine the value of N at which the radiation retardation removes electrons from the acceleration mode.

Rabinovich and Ioganseh [3-5] and other authors have investigated the coherent radiation reaction on the electron bunches in a synchrotron assuming that the bunches extended along circular arcs and were of small transverse dimensions. To the best of our knowledge, however, the problem has not been treated for the case of spherically symmetric bunches. We shall solve this problem for ultrarelativistic electrons, taking $\beta \approx 1$ and $\gamma^2 \gg 1$.

FIRST METHOD FOR COMPUTING Θ

It is well known that the radiation of a single ultrarelativistic electron is quasi-continuous, with the power radiated in a spectral interval $(\omega, \omega + d\omega)$ given by

$$P(\omega)\,d\omega = P_0\psi(t)\,dt, \tag{6}$$

where P_0 is the total radiated power (1), t is an auxiliary variable related to the angular frequency ω by

$$\omega = \frac{c}{a}\gamma^3 t^{3/2},$$

$$d\omega = \frac{3c}{2a}\gamma^3 \sqrt{t}\,dt, \qquad (7)$$

and the function $\psi(t)$ is given by

$$\psi(t) = -\frac{9}{2\sqrt{\pi}}\,t\left[v'(t) + \frac{t}{2}\int_t^\infty v(\tau)\,d\tau\right], \qquad (8)$$

where $v(t)$ is the Airy function.[6] The function $\psi(t)$ that satisfies the relation

$$\int_0^\infty \psi(t)\,dt = 1, \qquad (9)$$

is shown in Fig. 1.

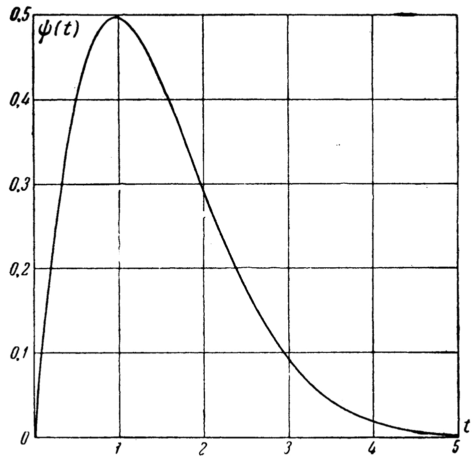

FIG. 1

In going from a single electron to a bunch consisting of N electrons the radiation field in the direction of the wave vector (k_x, k_y, k_z) is multiplied by the factor $Nf(k_x, k_y, k_z)$ where

$$f(k_x, k_y, k_z) = \frac{1}{Ne}\iiint_{-\infty}^{\infty} e^{-i(k_x\xi + k_y\eta + k_z\zeta)}\rho(\xi, \eta, \zeta)\,d\xi\,d\eta\,d\zeta \qquad (10)$$

is the form factor for the bunch (cf [7] or [8]) and $\rho(\xi, \eta, \zeta)$ is the volume charge density in the system of coordinates ξ, η, ζ, which are given in terms of the laboratory coordinates x, y, z by the relations

$$\xi = x - \bar{x}, \quad \eta = y - \bar{y}, \quad \zeta = z - \bar{z} \qquad (11)$$

where $\bar{x}$, $\bar{y}$, $\bar{z}$ are the coordinates of the center of gravity of the bunch. If the bunch is spherically symmetric then ρ and f each depend on a single dimensionless parameter

$$\rho = \rho(u), \quad u = r/r_0; \quad f = f(\nu), \quad \nu = \omega r_0/c, \qquad (12)$$

where

$$r = \sqrt{\xi^2 + \eta^2 + \zeta^2}, \quad \omega/c = \sqrt{k_x^2 + k_y^2 + k_z^2}, \qquad (13)$$

and r_0 is the "radius" of the bunch [see (15) and (16) below]. The functions in (12) are related by the expression

$$f(\nu) = \frac{4\pi c r_0^2}{Ne\omega}\int_0^\infty \rho(u)\,u\sin\nu u\,du, \qquad (14)$$

so that a Gaussian charge distribution yields

$$\rho(u) = (2\pi)^{-3/2} r_0^{-3} Ne e^{-u^2/2}, \quad f(\nu) = e^{-\nu^2/2}, \qquad (15)$$

while a uniform charge distribution inside a sphere of radius r_0 gives

$$\rho(u) = 3Ne/4\pi r_0^3 \text{ when } u < 1, \quad \rho(u) = 0 \text{ when } u > 1;$$
$$f(\nu) = 3(\sin\nu - \nu\cos\nu)/\nu^3. \qquad (16)$$

Using (7) we can write ν in the form

$$\nu = s t^{3/2} \quad (s = \gamma^3 r_0/a). \qquad (17)$$

In the case of spherical symmetry f depends only on the frequency and not the direction of the radiation so that the power radiated by the bunch in the spectral interval $(\omega, \omega + d\omega)$ is

$$P(\omega)\,d\omega = N^2 P_0 |f(st^{3/2})|^2 \psi(t)\,dt, \qquad (18)$$

while the total power is

$$P = \int_0^\infty P(\omega)\,d\omega = N^2 P_0 \int_0^\infty |f(st^{3/2})|^2 \psi(t)\,dt \qquad (19)$$

and the coherence factor becomes

$$\Theta = \int_0^\infty |f(st^{3/2})|^2 \psi(t)\,dt. \qquad (20)$$

The coherence factor depends only on the dimensionless parameter s, which is proportional to the radius of the bunch r_0, and satisfies the limiting relations

$$\Theta \to 1 \text{ as } s \to 0, \quad \Theta = \varkappa/s^{4/3} \text{ as } s \to \infty, \qquad (21)$$

where

$$\varkappa = \frac{2}{3}\psi'(0)\int_0^\infty |f(\nu)|^2 \nu^{1/3} d\nu, \quad \psi'(0) = -\frac{9}{2\sqrt{\pi}}v'(0) \qquad (22)$$

in particular, for a Gaussian bunch (15)

$$\varkappa = 6^{2/3}\Gamma(5/6)/4\sqrt{\pi} = 0.527, \qquad (23)$$

while for the uniform bunch (16)

$$\varkappa = 243 \cdot 6^{2/3}/560 = 1.43. \qquad (24)$$

The expressions in (21) can be made more exact for a Gaussian bunch:

$$\Theta = 1 - 7s^2 + \ldots \text{ when } s \ll 1,$$
$$\Theta = \varkappa/s^{4/3} - 1/4s^2 + \ldots \text{ when } s \gg 1 \qquad (25)$$

and the calculations can be carried out directly with (20) up to the point at which they join the limiting expression in (25). It is also possible to find Θ by numerical quadratures from (20) for a uniform bunch, but it will be found easier to use relations that will be derived below.

SECOND METHOD OF COMPUTING Θ

Assume a linear bunch in which N electrons are distributed around a circle of radius a in such a way that the charge in an angular interval $(\chi, \chi + d\chi)$ is $Neg(\chi)d\chi$ where

$$\chi = \varphi - \overline{\varphi} \tag{26}$$

is the polar angle computed from the position of the center of gravity of the bunch. If this bunch moves around the circle with velocity $c\beta$ the power radiated at the m-th harmonic (frequency $\omega = mc\beta/a$) differs from the power radiated by a single electron by the factor $N^2|g_m|^2$ where

$$g_m = \int_{-\pi}^{\pi} e^{im\chi} g(\chi)\, d\chi, \qquad g_0 = 1 \tag{27}$$

is the form factor for the linear bunch. If we use the relation

$$g_m = f(m\beta r_0/a), \tag{28}$$

then the coherence factors are the same for the linear bunch and a spherically symmetric bunch. Using (14) we can show that when (28) is satisfied it is sufficient to take $g(\chi)$ as an even function which is defined by the following expression for $\chi > 0$;

$$g(\chi) = \frac{2\pi a}{Ne} \int_{\chi/\chi_0}^{\infty} \rho(u)\, u du, \tag{29}$$

where

$$\chi = \beta r/a, \quad \chi_0 = \beta r_0/a. \tag{30}$$

Thus, a Gaussian bunch (15) corresponds to a linear bunch for which

$$g(\chi) = \frac{1}{\sqrt{2\pi}\,\chi_0} e^{-\chi^2/2\chi_0^2}, \tag{31}$$

that is to say, the linear charge density also follows a Gaussian distribution. A uniform bunch (16) corresponds to a linear bunch with a parabolic distribution

$$g(\chi) = \frac{3}{4\chi_0}\left(1 - \frac{\chi^2}{\chi_0^2}\right) \text{ for } |\chi| < \chi_0,$$
$$g(\chi) = 0 \qquad \text{for } |\chi| > \chi_0. \tag{32}$$

However, if the entire bunch is concentrated in a sphere of radius r_0 the corresponding linear bunch has a rectangular distribution.

$$g(\chi) = 1/2\chi_0 \quad \text{for } |\chi| < \chi_0,$$
$$g(\chi) = 0 \quad \text{for } |\chi| > \chi_0. \tag{33}$$

The correspondence between a spherically symmetric bunch and a linear bunch holds for any β and requires only that the condition $\exp\{-\pi^2/2\chi_0^2\} \ll 1$ be satisfied for a Gaussian bunch and that the condition $\chi_0 < \pi$ be satisfied for a bunch with sharp boundaries, (32) or (33).

The coherence factor for a linear bunch can also be computed in a different way by starting with (4) and using the interaction law for charges moving on the same circle of radius a, as has been done in [3-5]. We omit the calculations, giving only the final result for the coherence factor:

$$\Theta = -\frac{3}{2\beta^3\gamma^4} \int_0^{2\pi} G'(\chi) \frac{1 - \beta^2 \cos\psi}{2|\sin\psi/2|(1 - \beta\cos\psi/2)} d\chi$$
$$= -\frac{3}{2\beta^3\gamma^4} \int_{-\pi}^{\pi} G'(\chi) \frac{1 - \beta^2\cos\psi}{2|\sin\psi/2|} d\psi, \tag{34}$$

where $G(\chi)$ is the convolution of the function $g(\chi)$:

$$G(\chi) = \int_{-\pi}^{\pi} g(\chi - \mu)\, g(\mu)\, d\mu, \tag{35}$$

and the angle ψ is given by the equation

$$\psi - 2\beta|\sin\psi/2| = \chi. \tag{36}$$

The relation in (34) applies for any β. We compute the integral (34) for ultrarelativistic ($\beta \approx 1$) short ($\chi_0 \ll 1$) bunches assuming that for positive values of χ and ψ

$$\chi = \frac{\psi}{2\gamma^2} + \frac{\psi^3}{24}, \quad 1 - \beta^2\cos\psi = \frac{1}{\gamma^2} + \frac{\psi^2}{2}, \quad 2\sin\frac{\psi}{2} = \psi, \tag{37}$$

while for negative values of χ and ψ

$$\chi = 2\psi, \quad 1 - \beta^2\cos\psi = 1/\gamma^2, \quad 2|\sin\psi/2| = -\psi. \tag{38}$$

Introducing the even function $H(u)$ by means of the relation

$$G'(\chi) = -\frac{u}{\chi_0^2} H(u), \quad u = \frac{\chi}{\chi_0}, \tag{39}$$

we have

$$\Theta = \frac{1}{4s^3} \int_0^{\infty} H\left(\frac{\tau + \tau^3/12}{2s}\right)\left(\tau^2 + \frac{\tau^4}{8}\right) d\tau, \tag{40}$$

where

$$s = \gamma^3\chi_0 = \gamma^3 r_0/a \text{ and } \tau = \gamma\psi. \tag{41}$$

Equations (20) and (40) are equivalent but it is difficult to prove this equivalence directly. The factor Θ can be computed for a uniform bunch by means of (40). The expression in (32) yields

$$H(u) = \frac{3}{2}\left(1 - \frac{3u}{4} + \frac{u^3}{16}\right) \text{ for } 0 < u < 2,$$
$$H(u) = 0 \qquad \text{for } u > 2. \tag{42}$$

The integral in (40) is computed and we have

$$\Theta = \frac{t^3}{8s^3}\left[1 + \frac{3t^2}{40} - \frac{9t}{32s}\left(1 + \frac{5t^2}{36} + \frac{t^4}{192}\right)\right.$$
$$\left. + \frac{t^3}{256s^3}\left(1 + \frac{9t^2}{32} + \frac{t^4}{32} + \frac{11t^6}{6912} + \frac{t^8}{35146}\right)\right], \tag{43}$$

where the auxiliary parameter t is related to s by the expression

$$s = \frac{1}{4}(t + t^3/12). \tag{44}$$

Curve a of Fig. 2 (log-log scale) shows the dependence of Θ on s for a uniform bunch as computed with (20) and (43); curve b shows the same relation for a Gaussian bunch as computed with (20) and (25). The dashed lines show the asymptotes corresponding to the second formula (21) with (23) and (24) taken into account. As s increases the factor Θ decreases monotonically and the asymptote is approached at relatively large values of s.

For the distribution given by (33)

$$H(u) = 1/4u \quad \text{for} \quad 0 < u < 2, \qquad H(u) = 0 \quad \text{for} \quad u > 2 \tag{45}$$

and the coherence factor is given by the simple relation

$$\Theta = \frac{3}{8s^2}\left[\frac{t^2}{4} - \ln\left(1 + \frac{t^2}{12}\right)\right], \tag{46}$$

where s is related to t by Eq. (44) as before.

The formulas above indicate that $\Theta \to 0$ as $s \to \infty$. Actually $\Theta \to 1/N$ as $s \to \infty$, because the bunch is discrete. However, since $N \gtrsim 10^7$ for bunches in the microtron, the discrete nature of the bunch is not important.

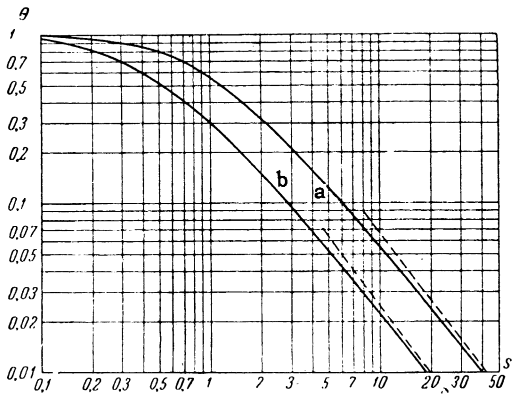

FIG. 2.

LIMITING CURRENT IN THE MICROTRON

In order to estimate the effect of the coherent radiation on the motion of electron bunches in the microtron we assume for simplicity that the time of flight of an electron through the accelerating gap (high-frequency field) can be neglected. Introducing the notation

$$\Omega = \omega_c/\omega, \qquad \omega_c = eH_0/mc, \tag{47}$$

where H_0 is the constant magnetic field, ω_c is the cyclotron frequency, and ω is the angular frequency of the accelerating field, we can write down the value of γ (dimensionless electron energy) for the k-th transit through the accelerating gap

$$\gamma_k = (g_i + gk)\Omega + \tilde{\gamma}_k, \tag{48}$$

where the numbers $g_i = 0, 1, 2 \ldots$ and $g = 1, 2 \ldots$ denote the multiplicity of the acceleration and give the energy of particles with equilibrium phase φ_s; the quantity $\tilde{\gamma}_k$ represents a correction for radiation retardation and the fact that the phase deviates from the equilibrium value by an amount $\tilde{\varphi}_k$ in the k-th transit. Generalizing the relations given by Kolomenskiĭ [9] to include radiation retardation, we obtain the finite-difference equations

$$\Delta\tilde{\varphi}_k \equiv \tilde{\varphi}_{k+1} - \tilde{\varphi}_k = 2\pi\gamma_k/\Omega + \delta\varphi_k,$$
$$\Delta\tilde{\gamma}_k \equiv \tilde{\gamma}_{k+1} - \tilde{\gamma}_k = g\Omega[\cos(\varphi_s + \tilde{\varphi}_{k+1})/\cos\varphi_s - 1] + \delta\gamma_k, \tag{49}$$

where $\delta\gamma_k < 0$ determines the energy loss due to radiation in the circular trajectory between the k-th and the (k + 1)-st transits through the accelerating gap while

$$\delta\varphi_k = \pi\delta\gamma_k/\Omega < 0 \tag{50}$$

is the phase increment in the same path caused by the radiation. [If the energy $\delta\gamma_k$ were lost at the beginning of the path, instead of over the entire path, we would use 2π instead of the factor π in (50).]

The two equations in (49) are equivalent to the single second-order nonlinear difference equation

$$\Delta^2\tilde{\varphi}_k = 2\pi g[\cos(\varphi_s + \tilde{\varphi}_{k+1})/\cos\varphi_s - 1] + \delta\varphi_k + \delta\varphi_{k+1}. \tag{51}$$

If $\delta\varphi_k$ does not vary greatly from orbit to orbit [this is the case within the limits of applicability of (66)] the radiation retardation simply causes a gradual displacement of the equilibrium phase. In this case we have

$$\tilde{\varphi}_k = \varphi_{s,k} - \varphi_s + \psi_k, \tag{52}$$

where $\varphi_{s,k}$ is the equilibrium phase at the k-th orbit (taking account of retardation), given by the relation

$$\cos\varphi_{s,k} = [1 - (\delta\varphi_k + \delta\varphi_{k+1})/2\pi g]\cos\varphi_s, \tag{53}$$

while ψ_k is the deviation of the phase from the equilibrium value $\varphi_{s,k}$. When $|\psi_k| \ll 1$ we can write a linear finite-difference equation for ψ_k:

$$\Delta^2\psi_k + 2\pi g[1 - (\delta\varphi_k + \delta\varphi_{k+1})/2\pi g]\,\mathrm{tg}\,\varphi_{s,k}\cdot\psi_k = -\Delta^2\varphi_{s,k}, \tag{54*}$$

where the coefficient for ψ_k and the small right

*tg = tan.

side are weak functions of k. This equation corresponds to a stable oscillation when

$$0 < \operatorname{tg} \varphi_{s,k} < \frac{2}{\pi g}\left(1 - \frac{\delta\varphi_k + \delta\varphi_{k+1}}{2\pi g}\right)^{-1}. \quad (54a)$$

The radiation retardation is usually negligible in the first orbits ($-\delta\varphi_k/\pi g \ll 1$) so that

$$\varphi_{s,k} \approx \varphi_s, \quad 0 < \varphi_s < \operatorname{arc\,tg}(2/\pi g). \quad (55)^*$$

As k increases the absolute value of $\delta\varphi_k$ increases gradually while the equilibrium phase is gradually reduced ($0 < \varphi_{s,k} < \varphi_s$, since $\delta\varphi_k < 0$). It is evident from (54) that stability is lost when $\varphi_{s,k} = 0$ so that microtron acceleration is no longer possible. It then follows that the limiting value of $-\delta\varphi_k$ (at which acceleration is still possible) is

$$(-\delta\varphi)_{max} = \pi g(1 - \cos\varphi_s)/\cos\varphi_s \quad (56)$$

and if φ_s is the optimum value $\varphi_s^0 = \tan^{-1}(1/\pi g)$ then

$$(-\delta\varphi)_{max} = \operatorname{tg}\varphi_s^0/2 = 0{,}155 \quad \text{for} \quad g = 1. \quad (57)$$

We can make further use of this estimate. If all the $\delta\varphi_k$ were the same the radiation retardation of the bunch as a whole could be easily compensated by choosing φ_s judiciously from (53) and (54), and the radiation force would only smear out the bunch, an effect which is much more difficult to compute.

The average energy lost per orbit by an electron is

$$-mc^2\delta\gamma = PT/N, \quad (58)$$

where the orbit radius a and the period of rotation T are related to the dimensionless energy γ by the expressions

$$a = c\beta\gamma/\omega_c, \quad T = 2\pi\gamma/\omega_c. \quad (59)$$

Hence, when $\beta \approx 1$

$$\delta\gamma = -\frac{4\pi Ne}{3J_0}\omega_c\Theta\gamma^3, \quad J_0 = \frac{mc^3}{e} = 17\,000\,\text{A}. \quad (60)$$

We replace the number of particles in a bunch by the corresponding current

$$J = \omega Ne/2\pi, \quad (61)$$

so that (50) and (60) yield

$$-\delta\varphi = \frac{8\pi^3 J}{3J_0}\Theta\gamma^3, \quad (62)$$

where, by virtue of (17), (47), and (59), the coherence factor Θ depends on the parameter

$$s = s_1\gamma^2, \text{ where } s_1 = \Omega\omega r_0/c. \quad (63)$$

Using (57) we can write an expression for the limiting current in the microtron

$$J_{max}/J_1 = 1/\Theta\gamma^3, \quad (64)$$

where

$$J_1 = \frac{3J_0}{8\pi^3}(-\delta\varphi)_{max} = 32\,\text{A}. \quad (65)$$

In the first few orbits the right sides of (62) and (64) vary rather strongly from orbit to orbit because s is small; however, when $s \gg 1$ this dependence is weakened and (64) becomes

$$J_{max}/J_1 = s_1^{4/3}/\varkappa\gamma^{1/3} = \Omega^{4/3}(\omega r_0/c)^{4/3}/\varkappa\gamma^{1/3}. \quad (66)$$

In Fig. 3 we show the dependence of J_{max}/J_1 on γ for the values Ω = 1, 2, and 4. In accordance with the measurements reported by Bykov[2] (carried out at $\Omega \approx 1$) the charge distribution in the bunch has been approximated by (15) and it has been assumed that

$$\omega r_0/c = 2\pi r_0/\lambda = 0{,}12. \quad (67)$$

These results apply for other modes of operation under the condition that the parameter s_1 assumes the values 0.12, 0.24, and 0.48.

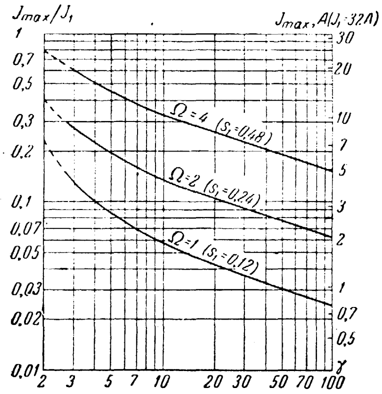

FIG. 3.

It is evident that the limiting current and the maximum number of electrons in a bunch are reduced monotonically with increasing γ and increased monotonically with increasing Ω. The relation in (64) becomes that in (66) at high values of γ, corresponding to the linear portions of the curves in Fig. 3.

By increasing the energy of a given electron bunch or by increasing the current J for a fixed electron energy we increase the effect of radiation forces on the motion of electrons in a bunch, which will then not only be deformed but also tend to deviate (as a whole) in phase. As we have indicated above, this mechanism can remove the bunch from the acceleration mode. More precisely, if the current J is increased for a given γ, then with J appreciably smaller than J_{max} both the equilibrium phase $\varphi_{s,k}$, which determines the position of the center of the bunch at the k-th orbit, and the region of phase stability, which determines

*arc tg = $\tan^{-1}$.

the longitudinal dimensions of the bunch (when $\varphi_{s,k} \rightarrow 0$ the region of phase stability contracts to a point), will both be changed. In this case the retarded interaction of the electrons must be considered in calculating the electron motion. This calculation is extremely difficult and the results do not justify the effort. The considerations given by us above leading to (64) obviously give only a rough estimate of the limiting current in the microtron.

CONCLUSIONS

The radiation retardation, which determines the limiting current of accelerated particles in a microtron, is due to the coherent radiation of the electron bunches in their circular motion. The spectrum of this radiation can be computed from (18); this computation shows that even at relatively low energies ($mc^2\gamma \gtrsim 5$ MeV, i.e., $\gamma \gtrsim 10$) the spectral width is determined by the dimensions of the bunch and is only a weak function of its energy. At these and higher energies, the limiting current (Fig. 3) can be estimated from the asymptotic formula (66), which always applies for actual microtrons. It can be shown that the walls have only a small effect on this spectrum.

We have assumed the radiation spectrum to be quasi-continuous. Actually, however, it consists of discrete lines whose frequencies are multiples of the frequency of the accelerating field (not the rotational frequency); this is due to the fact that there are several bunches moving simultaneously in each circular orbit. Hence the coherent radiation in the microtron may be regarded as the result of frequency multiplication of the accelerating field.

At the present time a current $J = 25$ mA has been reached in the microtron at the Institute for Physics Problems of the U.S.S.R. Academy of Sciences, The estimates given above show that the radiation retardation of the bunches will not affect the operation of the microtron even if this current is increased several times. The current would have to be increased by a factor of 15—30 or more for the effect of radiation forces to become evident.

We have neglected the electrodynamic interaction of different bunches in this calculation. Close to the accelerating cavity, however, all the circular orbits are tangent to each other and bunches moving on different orbits pass through the cavity simultaneously. In this case the radiation produced is sensitive to the properties of the cavity and also tends to limit the current in the microtron; however, this problem is beyond the scope of the present paper.

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