ON DECELERATION OF ANTIPROTONS IN MATTER

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Slowing down of antiprotons in matter from nonrelativistic energies to energies $\sim Z^2\epsilon^4\mu/2\hbar^2$ is considered. It is shown that in this energy interval the antiprotons are mainly absorbed by nuclei. It is also shown that the number of antiprotons changes only slightly during the slowing down. The behavior of the cross section for the capture of low-energy antiprotons in atomic shells is also considered.

THE slowing down of antiprotons in matter is accompanied by their absorption. If the number of antiprotons of initial energy E_i is N_0, then the number of antiprotons slowed down to the energy E is determined by the formula

$$N = N_0 \exp\left[-\int\limits_{a(E_i)}^{b(E)} \rho\sigma dx\right] = N_0 \exp\left[\int\limits_{E}^{E_i} \rho\sigma \frac{dx}{dE} dE\right], \qquad (1)$$

where ρ is the density of the medium and σ is the absorption cross section for antiprotons. The integration is carried out over the antiproton path, whose length is determined by the ionization loss dE/dx.

The antiproton absorption cross section σ consists of the cross section for capture by nuclei (σ_c) and the cross section for capture in atomic shells (σ_{at}):

$$\sigma = \sigma_c + \sigma_{at}. \qquad (2)$$

In [1] it was shown that if the antiproton energy E is much smaller than the absolute value of its Coulomb energy at the boundary of the nucleus ($E \ll |V_Q(R)|$, then $\sigma_c = \text{const}/E$. If $E \gg |V_Q(R)|$, then $\sigma_c = \pi R^2$ (R is the nuclear radius), since the nucleus can be considered as perfectly black for antiprotons. In the intermediate energy region, we have

$$\sigma_c = \pi R^2 \left(1 + \frac{\text{const}}{\pi R^2 E}\right). \qquad (3)$$

The capture of an antiproton in one of the atomic shells can take place 1) by radiative capture, 2) by the knocking out of an atomic electron (Auger effect), 3) by the emission of a π^0 meson. Thus

$$\sigma_{at} = \sigma_{rad} + \sigma_{Auger} + \sigma_{\pi^0}. \qquad (4)$$

The calculations shown below on the cross sections for the capture in atomic shells are valid for light elements and, under certain conditions, for medium elements, namely, for the cases in which we can neglect the finite size of the nucleus and use the Coulomb wave functions for a point charge. It is assumed here that the atomic mass of the medium is large in comparison with the antiproton mass and that the antiproton energy E is less than the threshold for the production of a π^0 meson:

$$E < m_\pi c^2 - Z^2\varepsilon^4 M/2\hbar^2 n^2, \qquad (5)$$

where m_π is the π^0-meson mass, ϵ is the elementary charge, M is the antiproton mass, n is the principal quantum number of the antiproton in the bound state. In this case, the last term in (4) drops out.

The cross section for radiative capture of the antiproton in one of the atomic shells is determined from the formulas for radiative processes in the first approximation.[2] Here, the captured antiproton will lie deep in the electronic shells of the atom, since the Bohr radius for an antiproton is much smaller than the Bohr radius for an electron.

As the wave function of the initial state of the antiproton, we choose the Coulomb function (see [3]).

$$\psi(\mathbf{r}) = \frac{\sqrt{2\pi\hbar}}{2k} \sum_{l'=0}^{\infty} (2l'+1)\, i^{l'} P_{l'}(\cos\theta) \frac{\Gamma(l'+1-in')}{|\Gamma(l'+1-in')|} R_{El'}(r). \qquad (6)$$

Here, $R_{El'}$ is the radial function for a continuous spectrum and $n' = Z/k$. The energy $E = \frac{1}{2}k^2$ is measured in units of $\epsilon^4 M/\hbar^2$, and the length in units of $\hbar^2/M\epsilon^2$. The function (6) is normalized to unit flux density. It approximates sufficiently well the true initial wave function on the antiproton not only in the region $r \ll \hbar^2/\mu\epsilon^2 Z^{1/3}$ (μ is the electron mass) in which the Coulomb potential of the nucleus acts, but also for $r \gtrsim \hbar^2/\mu\epsilon^2 Z^{1/3}$ if $E \gg |\epsilon\varphi(\hbar^2/\mu\epsilon^2 Z^{1/3})|$; φ is the Thomas-Fermi potential. In fact, in this case the antiproton motion is almost free and the true initial wave function is almost a plane wave in the region $r \gtrsim \hbar^2/\mu\epsilon^2 Z^{1/3}$. On the other hand, for such energies and distances, the function (6) is almost a plane wave, since $kr \gg 1$. Henceforth, it will be assumed that the antiproton energy satisfies a condition stricter than that given above:

$$E \gg Z^2\varepsilon^4\mu/2\hbar^2, \quad n' \ll \sqrt{M/\mu}. \tag{7}$$

As the wave function for the antiproton final state, we choose the normalized function for a discrete spectrum of a hydrogen-like atom.

The cross section σ_{nlm}^{rad} obtained for radiative capture of the antiproton in the state nlm drops rapidly with n; σ_{nlm}^{rad} is most important for $n \ll n'$. With such n and n′, the largest cross section for fixed n occurs when $l = n - 1$. The value of σ_{nlm}^{rad} is zero if $m \neq 0, \pm 1$, We note that $\sigma_{n,n-1,0} \approx \sigma_{n,n-1,\pm 1}$.

If we take into account the finite size of the nucleus in the calculation of the radiative-capture cross sections, we find that the absolute value of the antiproton bound-state energy decreases. This decrease in energy, in turn, leads to a decrease in the radiative-capture cross section in states with small n.

The numerical estimate of the total cross sections for radiative capture in all of the levels σ_{rad} indicates that for carbon and copper σ_{rad}/σ_c is 4×10^{-5} and 27×10^{-5}, respectively, if $E < 0.5$ MeV. This ratio is still smaller for $E > 0.5$ MeV. Hence, for light and medium elements

$$\sigma_{rad} \ll \sigma_c \tag{8}$$

in the antiproton energy interval under consideration.

The Auger-effect cross section for antiproton capture in one of the atomic shells is determined from the perturbation theory formulas given in [4]. As the initial and final wave functions of the antiproton, we used the same functions that were used for radiative capture.

The Auger effect was considered for K-shell electrons, since each of them is sufficiently well described by hydrogen-like wave functions. Along with the Auger effect for electrons of the K shell, we made a rough estimate of the Auger effect for the L-shell electrons. This estimate showed that the cross section for the Auger effect for K-shell electrons is one order of magnitude greater than for L-shell electrons. We therefore assumed that the Auger effect involves mainly K-shell electrons.

The Auger-effect cross section for the capture of an antiproton in the nlm state is most important for $n \ll n'$. It equals zero if $m \neq 0$. For $n \ll n'$, the greatest cross section for a given n occurs when $l = n - 1$, as is the case with the cross section for radiative capture. With an increase in n, the value of σ_{Auger} drops.

If we take into account the finite size of the nucleus for the calculation of the Auger-effect cross section we find that the cross section decreases for small n, as in the case of radiative capture.

A numerical estimate of the total cross section of the Auger effect shows that for carbon and copper the ratio σ_{Auger}/σ_c is no greater than 3×10^{-6} and 10^{-6}, respectively, for $E < 0.5$ Mev; for $E > 0.5$ MeV, it is still smaller. Thus, for light and medium elements

$$\sigma_{Auger} \ll \sigma_c \tag{9}$$

in the antiproton energy interval under consideration.

On the basis of (8) and (9), we assume that the antiprotons in light and medium elements are absorbed primarily by nuclei: $\sigma \approx \sigma_c$.

To calculate the attenuation of the antiproton beam we turn to their ionization losses. It is known that if the velocity of the particles is much greater than $Z\epsilon^2/\hbar$, then the ionization loss is

$$-\frac{dE}{dx} = \frac{4\pi\varepsilon^4}{\mu v^2} NZ \ln\frac{2\mu v^2}{I}, \tag{10}$$

where N is the number of atoms of the medium per cm^3, $I \approx 10\,Z$ is the mean ionization potential of the atoms of the medium in eV. Formula (10) can also be used for $v \lesssim Z\epsilon^2/\hbar$ (see [5]). For $v \ll v_0$ (v_0 is the maximum velocity in a degenerate electron gas; $v_0 > \epsilon^2/\hbar$), the ionization loss of the particle in a condensed medium is described rather well by the formula (see [6])

$$-dE/dx \approx v\mu^2\varepsilon^4/\hbar. \tag{11}$$

The numerical calculation of the antiproton-beam attenuation by formula (1) shows that for the slowing down of antiprotons in carbon and copper from $E_i = 50$ MeV to $E = 50$ keV the beam is attenuated by a factor of $e^{0.05}$ and $e^{0.037}$, respectively. Hence the beam attenuation is very small in light and medium elements.

It was mentioned above that the results of the calculations of the cross sections for capture in atomic shells σ_{rad} and σ_{Auger} were obtained for antiproton energies E satisfying (7). If

$$E \ll Z^2\varepsilon^4\mu/2\hbar^2, \quad n' \gg \sqrt{M/\mu}, \tag{12}$$

then the true initial wave function of the antiproton is no longer approximated by function (6) for any r. Owing to the smallness of the antiproton Bohr radius, however, the main role in the matrix elements determining the radiative-capture and Auger-effect cross sections is played by the region with dimensions much smaller than $\hbar^2/\mu\epsilon^2 Z^{1/3}$. In these regions, the true initial antiproton wave function is satisfactorily approximated by function (6) for small E. Hence, function (6) permits an estimate of σ_{rad} and σ_{Auger} as $E \to 0$.

The radiative-capture cross section under condition (12) is described by the same formulas as in the case of condition (7). Thus, as $E \to 0$ the largest radiative-capture cross section for a given n has the form

$$\sigma^{\text{rad}}_{n,n-1,0} \sim \pi \frac{\varepsilon^2}{Mc^2}\frac{\hbar}{Mc}\frac{2}{3}\sqrt{\frac{\pi}{n}}\left(\frac{2}{e}\right)^{2n}(n')^2 \qquad (n \ll n'). \quad (13)$$

For the Auger effect, the situation is different. From the law of conservation of energy in this process, it follows from (12) that the energy of the emitted electron is

$$E_e \approx Z^2\varepsilon^4 M/2\hbar^2 n^2 - Z^2\varepsilon^4\mu/2\hbar^2. \quad (14)$$

Formula (14), in turn, leads to $n_{\max} \approx \sqrt{M/\mu}$, which did not occur earlier. The greatest Auger-effect cross section for a given n now has the form

$$\sigma^{\text{Auger}}_{n,n-1,0} \sim \pi\left(\frac{\hbar^2}{M\varepsilon^2}\right)^2\left(\frac{\mu}{M}\right)^3\frac{(2n)^3}{3Z^4}\sqrt{\frac{\pi}{n}}\left(\frac{2}{e}\right)^{2n}\frac{(n')^2}{\sqrt{M/\mu - n^2}}. \quad (15)$$

It follows from (3), (14), and (15) that, as $E \to 0$, the Auger effect outweighs radiative capture as well as capture by a nucleus, since the Auger-effect cross section becomes very large when $n \sim \sqrt{M/\mu}$. We note that the cross section for capture by a nucleus σ_c increases as $E \to 0$ more slowly than 1/E, owing to the screening of the nucleus by atomic electrons.[1] It also follows from (15) that at small energies the antiproton capture occurs primarily in states with $n \sim \sqrt{M/\mu}$ and large l, which is in agreement with the results of other authors.[7,8]

It was shown above that the antiprotons, upon being slowed down to energies $E \gg Z^2\epsilon^4\mu/2\hbar^2$ in light and medium elements are absorbed only slightly. The antiproton absorption becomes considerable only for $E \ll Z^2\epsilon^4\mu/2\hbar^2$, when the Auger-effect cross section is very large. Hence, most of the antiprotons are slowed down to small energies and are then captured on atomic shells to form antiprotonium.

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