THE TRANSITION OF LIQUID He^3 INTO THE SUPERFLUID STATE

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Submitted to JETP editor September 15, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 600-605 (February, 1962)

A study has been made of the Cooper effect in liquid He^3, showing that at sufficiently low temperatures He^3 undergoes a transition into the superfluid state. The problem of estimating the temperature of the transition is discussed. Various estimates yield transition temperatures ranging from 2×10^{-4} to 8×10^{-3} °K

1. INTRODUCTION

THE question of a possible transition of liquid He^3 into the superfluid state has been frequently discussed in the recent literature. [1,2]

As is well known, there act between isolated helium atoms at large distances attractive van der Waals forces with energy

$$U_0(r) = -V_0(R/r)^6, \tag{1}$$

where $V_0 \approx 41$°K and $R \approx 2.6$ A. As one of the authors has shown, [3] attraction at large distances also occurs between two excitations in liquid He^3. In this case, the potential energy of the interaction between the excitations differs from (1) only by a renormalization factor:

$$U = \Phi U_0, \tag{2}$$

$$\Phi = \{[(2\pi)^2/3mm^*c^2]\,(3N/8\pi)^{2/3}\}^2 = (mc_0^2/m^*c^2)^2, \tag{3}$$

where N is the number of atoms per unit volume, m is the mass of the He^3 atom, m^* is the effective mass of an excitation in He^3, c^{-2} is the compressibility of liquid He^3, and $c_0^{-2} = 3m^2(2\pi\hbar)^{-2}(8\pi/3N)^{2/3}$ is the compressibility of an ideal Fermi gas of mass m and density N. Since $N = 1.5 \times 10^{22}$ cm^{-3}; $c_0 \approx 93$ m/sec, $m^* = 2m$, and $c \approx 183$ m/sec $\approx 2c_0$, we obtain $\Phi_0 \approx 1.7 \times 10^{-2}$.

Large separations r provide the major contribution to the two-excitation scattering amplitude for scattering with large orbital momentum. Thus, two excitations in states of sufficiently large orbital momentum l must be attracted to one another, and may therefore combine into a pair, which leads in turn to a transformation of the He^3 into the superfluid state (the Cooper phenomenon). The present article is devoted to a more detailed study of the Cooper effect in He^3. We shall also consider the possibility of estimating the temperature of the transition of He^3 into the superfluid state.

2. COOPER EFFECT FOR LARGE ORBITAL MOMENTA

We shall begin with an investigation of Cooper pair formation in states of large orbital momentum. For the actual case of He^3 we cannot, obviously, base our calculations on some simplified model, but must rather proceed from the real situation of a strongly interacting fluid. Let us consider the Matsubara vertex component

$$\mathfrak{T}_{\alpha\beta\gamma\delta}(\mathbf{p}_1\,\omega_1,\ \mathbf{p}_2\,\omega_2;\ \mathbf{p}_3\omega_3,\ \mathbf{p}_4\omega_4), \qquad \omega_i = \pi T\,(2n+1)$$

($n = 0, \pm 1, \pm 2 \ldots$), and determine the temperature T_c at which this vertex component, with $\mathbf{q} = \mathbf{p}_1 + \mathbf{p}_2 = 0$, $\omega_0 = \omega_1 + \omega_2 + 0$, goes to infinity. This is the temperature at which the pairing will take place; i.e., the transition temperature.

In order to clarify the last assertion, we shall make use of the analogy with an ideal Bose gas. In a Bose gas, the Green's function

$$g(\omega\mathbf{q}) = \frac{1}{i\omega - q^2/2m + \mu} \qquad (\omega = 2\pi n T)$$

with $\omega = 0$ and $\mathbf{q} = 0$ first becomes infinite at the transition point; i.e., for $\mu = \mu(T_c) = 0$. At this point, an infinite number of particles start to accumulate in the ground state level, and "Bose condensation" begins. For a bound pair of Fermi particles the analog of the Green's function for Bose particles is the vertex component $\mathfrak{T}$. When, with $\mathbf{q} = \mathbf{p}_1 + \mathbf{p}_2 = 0$ and $\omega_0 = \omega_1 + \omega_2 + 0$, this component goes to infinity at some particular temperature, this indicates that at this point Cooper pairs form and "condense" into the state in which the momentum of the pairs as a whole is zero.

We recall that $T_{\alpha\beta\gamma\delta}(\mathbf{p}_1\omega_1,\ \mathbf{p}_2\omega_2;\ \mathbf{p}_3\omega_3,\ \mathbf{p}_4\omega_4)$ is determined from the Fourier component of the average of four Matsubara operators [4] $K(x_1, x_2;\ x_3, x_4) = \langle T(\psi_{1\alpha}\psi_{2\beta}\overset{+}{\psi}_{3\gamma}\overset{+}{\psi}_{4\delta})\rangle$ by means of the relation

$$K_{\alpha\beta\gamma\delta}\,(\mathbf{p}_1\omega_1,\ \mathbf{p}_2\omega_2;\quad \mathbf{p}_3\omega_3,\ \mathbf{p}_4\omega_4)$$
$$=\frac{(2\pi)^3}{T}\Big\{\frac{(2\pi)^3}{T}\,\mathfrak{G}\,(\omega_1\mathbf{p}_1)\,\mathfrak{G}\,(\omega_2\mathbf{p}_2)\,[\delta_{\omega_1\omega_3}\delta\,(\mathbf{p}_1-\mathbf{p}_3)\;\delta_{\alpha\gamma}\delta_{\beta\delta}$$
$$-\,\delta_{\omega_1\omega_4}\delta\,(\mathbf{p}_1-\mathbf{p}_4)\;\delta_{\alpha\delta}\;\delta_{\beta\gamma}]-\frac{1}{2}\,\mathfrak{G}\,(\omega_1\mathbf{p}_1)\;\mathfrak{G}\,(\omega_2\mathbf{p}_2)\;\mathfrak{G}\,(\omega_3\mathbf{p}_3)$$
$$\times\;\mathfrak{G}\,(\omega_4\mathbf{p}_4)\,\mathfrak{T}_{\alpha\beta\gamma\delta}\,(\omega_1\mathbf{p}_1,\ \omega_2\mathbf{p}_2;\ \omega_3\mathbf{p}_3,\ \omega_4\mathbf{p}_4)\Big\}$$
$$\times\;\delta_{\omega_1+\omega_2,\omega_3+\omega_4}\;\delta\,(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4). \tag{4}$$

Among the set of diagrams for the vertex component with small $\mathbf{q}$ a special part is played by the diagram in Fig. 1, which contains the integral of two functions $\mathfrak{G}$ in the form

$$T\sum_{\varepsilon}\int d^3p\,\mathfrak{G}\,(\varepsilon,\,\mathbf{p})\;\mathfrak{G}\,(\omega_0-\varepsilon,\ \mathbf{q}-\mathbf{p}). \tag{5}$$

Near the Fermi surface, the Green's function has a pole

$$\mathfrak{G}\,(\omega\mathbf{p})=\frac{a}{i\omega-\xi}+g\,(\mathbf{p}\omega) \tag{6}$$

[$g(\mathbf{p}\omega)$ has no singularity at the Fermi surface;

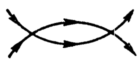

FIG. 1

$\xi = v(p-p_0)$, and $v = p_0/m^*$ is the velocity at the Fermi surface]. Substituting (6) into (5), we see that for $\mathbf{q}$ and ω_0 small (small T), the integration of (5) leads to a logarithmic singularity of the form*

$$a^2\int \mathrm{th}\,\frac{\xi}{2T}\,\frac{d\xi}{\xi}\,. \tag{7}$$

Diagrams of other types, with the exception of a ladder of diagrams of the form of Fig. 1, do not lead to singularities. We shall therefore write out the equation for the vertex component in such a form that integrals of the types appearing in (5) and (7) will be clearly separated out:

$$\mathfrak{T}_{\alpha\beta\gamma\delta}(\mathbf{p}_1\omega_1,\ \mathbf{p}_2\omega_2;\ \mathbf{p}_3\omega_3,\ \ \mathbf{p}_4\omega_4)=\widetilde{\mathfrak{T}}_{\alpha\beta\gamma\delta}\,(\mathbf{p}_1\omega_1,\ \mathbf{p}_2\omega_2;\ \mathbf{p}_3\omega_3,\ \ \mathbf{p}_4\omega_4)$$
$$-\frac{T}{2\,(2\pi)^3}\sum_{\varepsilon}\int\widetilde{\mathfrak{T}}_{\alpha\beta\zeta\eta}\,(\mathbf{p}_1\omega_1,\ \mathbf{p}_2\omega_2;\ \ \mathbf{k}\varepsilon,\ \ \mathbf{q}-\mathbf{k}\ \omega_0-\varepsilon)\ \mathfrak{G}(\varepsilon\mathbf{k})$$
$$\times\;\mathfrak{G}\,(\omega_0-\varepsilon,\ \mathbf{q}-\mathbf{k})\;\mathfrak{T}_{\zeta\eta\gamma\delta}$$
$$\times\,(\mathbf{k},\ \varepsilon;\ \omega_0-\varepsilon\ \ \mathbf{q}-\mathbf{k},\ \ \mathbf{p}_3\omega_3,\ \ \mathbf{p}_4\omega_4)\ d^3k. \tag{8}$$

Here $\widetilde{\mathfrak{T}}$ is the set of all diagrams which cannot be separated by a vertical line into two parts coupled by two lines whose arrows run in the same direction (see Fig. 2).

We shall now consider this equation for the case $\mathbf{q} = \mathbf{p}_1 + \mathbf{p}_2 = 0$, $\omega_0 = \omega_1 + \omega_2 = 0$. Here, the

*th = tanh.

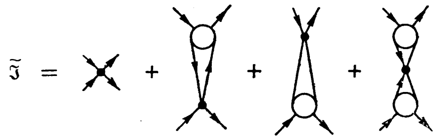

FIG. 2

vertex component is a function of two variables

$$\mathfrak{T}_{\alpha\beta\gamma\delta}\,(\mathbf{p}_1\omega_1;\ \mathbf{p}_2\omega_2;\ \mathbf{p}_3\omega_3,\ \mathbf{p}_4\omega_4)\equiv\mathfrak{T}_{\alpha\beta\gamma\delta}\,(\mathbf{p}_1\omega_1,\ \mathbf{p}_3\omega_3).$$

Let us expand the vertex component in a series of Legendre polynomials with respect to ϑ, the angle between $\mathbf{p}_1$ and $\mathbf{p}_3$. We shall have

$$\mathfrak{T}_{\alpha\beta\gamma\delta}\,(\mathbf{p}_1\omega_1;\ \mathbf{p}_3\omega_3)=\sum_{l}\mathfrak{T}^{l}\,(\,|\,\mathbf{p}_1\,|\,\omega_1,\ |\,\mathbf{p}_3\,|\ \omega_3)\ P_l\,(\cos\vartheta)\ S^{\pm}_{\alpha\beta\gamma\delta}\,, \tag{9}$$

where $S^{\pm}_{\alpha\beta\gamma\delta} = \delta_{\alpha\gamma}\delta_{\beta\delta} \pm \delta_{\alpha\delta}\delta_{\beta\gamma}$; in accordance with the Pauli principle, the plus sign corresponds to odd, and the minus to even, l. Equation (8) may be written out directly for the same harmonics:

$$\mathfrak{T}^{l}\,(\,|\,\mathbf{p}_1\,|\,\omega_1;\ |\,\mathbf{p}_3\,|\,\omega_3)=\widetilde{\mathfrak{T}}^{l}\,(\,|\,\mathbf{p}_1\,|\ \ \omega_1;\ \ |\,\mathbf{p}_3\,|\,\omega_3)$$
$$-\frac{2\,(2l+1)}{2\,(2\pi)^2}\,T\ \sum_{\varepsilon}\int\widetilde{\mathfrak{T}}^{l}\,(\,|\,\mathbf{p}_1\,|\,\omega_1;\ |\mathbf{k}\,|\,\varepsilon)\ \mathfrak{G}^2$$
$$\times\ \ (|\mathbf{k}|,\ \varepsilon)\ \mathfrak{T}^{l}\,(\,|\,\mathbf{k}\,|\,\varepsilon,\ \ |\,\mathbf{p}_3\,|\,\omega_3)\ k^2\ dk. \tag{8$'$}$$

Following what was said at the beginning of the article, let us consider Eq. (8′) in the limit of large l. We note here that for $l \gg 1$, $\mathfrak{T}^l$ falls rapidly with l. This fall is generally exponential, provided only that in $\mathfrak{T}^l_{\alpha\beta\gamma\delta}(\mathbf{p}_1\omega_1;\ \mathbf{p}_3\omega_3)$ there are no singularities in the square of the transfer $k^2 = (\mathbf{p}_3 - \mathbf{p}_1)^2$ for small values of k^2 (large values of l correspond to small transfers of momentum during scattering). When singularities are present this decrease follows a power law. The appearance of such special terms is due to the long-range component of the interaction (1).

The Fourier component of (1) is

$$U\,(k)=-\tfrac{1}{12}\,A\pi^2\,|\,\mathbf{p}_1-\mathbf{p}_3|^3. \tag{10}$$

In Eq. (8′) it will be sufficient for our purposes to limit ourselves in $\widetilde{\mathfrak{T}}^l\,(|\mathbf{p}\,|\,\omega;\ |\mathbf{k}\,|\,\epsilon)$ to terms of the first order in U. All the diagrams of this sort are illustrated in Fig. 2. The point in these diagrams represents the "priming" interaction (10), the circle represents the whole vertex component for the case of zero momentum transfer. As was shown in [3], the combination of all of the diagrams in Fig. 2 for the vertex component with small k and with $\mathbf{p}_1$ and $\mathbf{p}_2$ near the Fermi surface leads to renormalization of the interaction (10) by a factor Φ/a^2.

Upon integrating, the kernel of the integral equation (8′) gives rise, in accordance with (7), to logarithmic terms of the form $\ln(\epsilon_F/T)$. For small T the logarithm is large and may compensate for the small value of $\widetilde{\mathfrak{T}}^l$. In what follows, we shall compute the right-hand part of this equation out to and including terms of order $\widetilde{\mathfrak{T}}^l$. For this degree of accuracy it will be sufficient to limit ourselves in $\widetilde{\mathfrak{T}}$ to terms of the first order in (10). In fact, as one can readily verify, diagrams of the second order in (10) are proportional, for small momentum transfer, to $|\mathbf{p}_1-\mathbf{p}_3|^6$, and therefore have, for large l, an exponentially small component in the harmonic expansion (9).

Expanding the renormalized expression (10) in the series (9), we obtain for $\widetilde{\mathfrak{T}}^l(|\mathbf{p}_1|\,\omega_1, |\mathbf{p}_3|\,\omega_3)$ the relation

$$\widetilde{\mathfrak{T}}^l(|\mathbf{p}_1|, |\mathbf{p}_3|) = -\frac{A\pi^2 (p_1p_3)^{3/2}\Phi}{16a^2}\lambda^{l+1/2}\left\{\frac{\lambda^2}{(l+3/2)(l+5/2)} - \frac{2l+1}{(l+3/2)(l^2-1/4)} + \frac{\lambda^{-2}}{(l-1/2)(l-3/2)}\right\},$$

$$\lambda = (p_1^2+p_3^2)/2p_1p_3 - \sqrt{[(p_1^2+p_3^2)/2p_1p_3]^2-1}. \qquad (11)$$

When both of the momenta $\mathbf{p}_1$ and $\mathbf{p}_3$ lie near the Fermi surface,

$$\lambda \approx 1 - |\xi_1-\xi_3|/2\varepsilon_F,$$

where $\xi_{1,3} = v(p_{1,3}-p_0)$. For $l \gg 1$ and $|\xi_1-\xi_3|/\epsilon_F \ll 1$, one can write

$$\lambda^l \approx \exp\,[-l\,|\xi_1-\xi_3|/2\varepsilon_F].$$

Thus $\widetilde{\mathfrak{T}}^l$ falls off exponentially with distance from the Fermi surface for $|\xi_1-\xi_3| \gg \epsilon_F/l$.

In the vicinity of the Fermi surface we shall set

$$L(|\xi_1-\xi_3|) = \widetilde{\mathfrak{T}}^l(\omega_1|\mathbf{p}_1|, \omega_3|\mathbf{p}_3|)$$

and show that at some temperature T_c the solution to Eq. (8′) goes to infinity. Since near this point $\mathfrak{T}^l$ is very large, it is therefore sufficient to consider only the homogeneous equation (8′), and to seek $\mathfrak{T}^l$ in the form $\mathfrak{T}^l(p_1\omega_1; p_3\omega_3) = \varphi^l(\xi_1)\psi^l(\xi_3)$. A summation over frequency can be performed in the kernel of Eq. (8′). Since $L(|\xi_1-\xi_3|)$ cuts off at $|\xi_1-\xi_3| \sim \epsilon_F/l \ll \epsilon_F$ we may, in substituting the Green's function (6) into (8′), confine ourselves to the polar term. Thus

$$\varphi^l(\xi_1) = -a^2\frac{2l+1}{4(2\pi)^2}m^*p_0\int L(|\xi_1-\xi_3|)\,\mathrm{th}\frac{\xi_3}{2T}\varphi^l(\xi_3)\frac{d\xi_3}{\xi_3}.$$

This equation can be conveniently solved by Bogolyubov's method.[5] Integrating from the right by parts, we obtain

$$\varphi^l(\xi_1) = a^2\frac{(2l+1)m^*p_0}{4(2\pi)^2}\left\{\frac{1}{2T}\int_{-\infty}^{\infty} L(|\xi_1-\xi_3|)\varphi^l(\xi_3) \times \ln\frac{|\xi_3|}{\widetilde{\omega}}\frac{d\xi_3}{\mathrm{ch}^2(\xi_3/2T)} + \int_{-\infty}^{\infty}\ln\frac{|\xi_3|}{\widetilde{\omega}}\,\mathrm{th}\frac{\xi_3}{2T}\frac{d}{d\xi_3}[L(|\xi_1-\xi_3|)\varphi^l(\xi_3)]d\xi_3\right\},$$

where $\widetilde{\omega}$ is a constant, still unknown. We shall see below that $T_c \ll \widetilde{\omega}$ and that $\varphi^l(\xi)$ varies slowly for $\xi \sim T_c$. In the first integral, therefore, the argument ξ_3 in $L(|\xi_1-\xi_3|)$ and $\varphi^l(\xi_3)$ can be set equal to zero. The second integral, however, converges for $\xi_3 \gg T_c$ and varies slowly in the region $\xi_1 \sim T_c$. Therefore

$$\varphi^l(\xi_1) = a^2\frac{(2l+1)m^*p_0}{2(2\pi)^2}\varphi^l(0)L(|\xi_1|)\ln\frac{\pi T_c}{2\gamma\widetilde{\omega}} + \frac{(2l+1)m^*p_0a^2}{4(2\pi)^2}\int_{-\infty}^{\infty}\ln\frac{|\xi_3|}{\widetilde{\omega}}\frac{d}{d\xi}[L(|\xi_1-\xi_3|)\varphi^l(\xi_3)]\,d\xi_3$$

($\gamma \approx 1.78$ is the Euler constant). Since the logarithm is assumed to be large, $\psi^l(\xi)$ is in the first approximation determined by the first term. We shall select the constant $\widetilde{\omega}$ from the condition that for $\xi_1 \sim T_c$ the second term shall be zero:

$$-L^2(0)\ln\widetilde{\omega} = \int_0^{\infty}\ln|\xi|\frac{d}{d\xi}[L^2(\xi)]\,d\xi. \qquad (12)$$

This latter integral converges for $\xi \sim \epsilon_F/l \ll \epsilon_F$. Simplifying (11) with the necessary degree of accuracy, we obtain

$$L(|\xi_1-\xi_2|) = -\frac{A\pi^2\Phi p_0^3}{a^2 16 l^4}\exp\left(-\frac{l|\xi_1-\xi_2|}{2\varepsilon_F}\right) \times\left\{12 + 6\frac{l|\xi_1-\xi_2|}{\varepsilon_F} + \frac{l^2|\xi_1-\xi_2|^2}{\varepsilon_F^2}\right\}, \qquad (13)$$

or

$$\ln\widetilde{\omega} = \ln\frac{\varepsilon_F}{l} - \int_0^{\infty}\ln u\frac{d}{du}\left[\left(1+\frac{u}{2}+\frac{u^2}{12}\right)^2 e^{-u}\right]du.$$

We find, finally

$$\widetilde{\omega} = (\varepsilon_F/\gamma l)\,e^{-13/\gamma}.$$

Thus a non-zero solution for $\varphi^l(0)$ is found at a temperature

$$T_c^l = (2/\pi)\gamma\widetilde{\omega}e^{-1/\varkappa} = (p_0^2/m^*\pi l)\,e^{13/\gamma}e^{-1/\varkappa},$$

$$\varkappa = \frac{3m^*p_0^4(2l+1)A\Phi}{64}\left[\frac{2l+1}{(l+5/2)(l^2-1/4)(l^2-9/4)}\right] \approx 0.99\frac{(l+1/2)^2}{(l+5/2)(l^2-1/4)(l^2-9/4)} \qquad (14)$$

Since $1/\kappa$ appears in the exponent, it is necessary,

*ch = cosh.

in order to achieve pre-exponential accuracy in Eq. (14), to know $L(0)$ to a greater degree of precision than was required in (12) and (13).

It has thus been shown that the vertex component goes to infinity at the temperature T_c defined by (14). These results are of course asymptotic. They demonstrate once more that a transition of He^3 into the superfluid state must necessarily take place: if this transition does not occur at higher temperatures, due to the formation of pairs with $l \sim 1$, then, as is evident from the equations, it must in any case take place as a result of pair formation with $l \gg 1$. Pairing does in fact occur for l not excessively large (probably $l = 2$); i.e., at temperatures considerably higher than those given by the asymptotic formula (14), within its range of applicability.

3. THE POSSIBILITY OF ESTIMATING THE TRANSITION TEMPERATURE FOR He^3

The precise determination of the transition temperature for He^3 presents great difficulties. We have just stated that pairing does in fact take place in states of relatively small l. Small values of l correspond to interactions between excitations at distances in the atomic range. There is, generally speaking, no reason in this case to make use of the interaction forces existing between isolated helium atoms, since at such distances the forces cannot by any means be reduced to two-particle ones. An estimate of the transition temperature might be provided by the asymptotic formula (14). If, however, we consider large values of l, for which along this formula is valid, the temperature is found to be extremely small. Even when the value $l = 2$ is inserted into (14) (this is of course merely an extrapolation) we obtain

$$T_c \approx 2 \cdot 10^{-4}\,^\circ\mathrm{K}, \tag{15}$$

which is still quite low.

It is possible, however, that the renormalization factor Φ for the interaction between excitations is associated with screening of excitations at a few times the interatomic distance, and that the effect of this screening is not yet manifested in the scattering of excitations with $l = 2$—3. One might therefore attempt to make an estimate of T_c on the basis of a gas approximation, in which He^3 is regarded as a weakly non-ideal Fermi gas. (It is appreciated that the great departure of the renormalization factor Φ from unity does not support this picture. We wish, however, to emphasize that all of the estimates previously advanced[1,2] are also in fact based upon a gas approximation.)

Theory[6] provides for T_c the expression

$$T_c = (\gamma/\pi)\ (2/e)^{7/3} \exp\ (-\pi/2\,\delta_l), \tag{16}$$

where δ_l is the scattering phase. Two isolated He^3 atoms interact with an energy given by the interpolation formula

$$U(r) = V_0\ [(R/r)^2 - (R/r)^6]. \tag{17}$$

Computation of the scattering phase for the potential (17) shows that the largest phase having an attractive sign corresponds to $l = 2$. To gain an idea of the accuracy of the values of T obtained from (16), we present the values obtained for two cases: 1) when the mass of an excitation is taken as m, the intrinsic mass of the He^3 atom:

$$T_c = 8 \cdot 10^{-3}\,^\circ\mathrm{K},$$

2) when $m^* = 2m$, the effective mass:

$$T_c = 0.12\,^\circ\mathrm{K}.$$

These estimates differ by as much as two orders of magnitude (it is known at present[7] that $T_c < 10^{-2}\,^\circ\mathrm{K}$). Introduction of the renormalization factor (3) would shift these estimates into the range of unattainably low temperatures. It should be borne in mind, however, that for the ideal gas model it is logical to use m rather than m^*.

In any case, we should like to emphasize the fact that a strictly quantitative evaluation of the transition temperature in He^3 cannot be obtained from theory. As is evident from the estimates presented above, however, there is some basis for hoping that the temperature sought is within attainable limits, and that it lies between 8×10^{-3} and $2 \times 10^{-4}\,^\circ\mathrm{K}$.

In conclusion, the authors express their deep gratitude to their colleagues of the Mathematical Institute É. É. Shnol' and N. D. Vvedenskaya to whose kindness the authors are indebted for the calculation of the scattering phase. The authors are also obliged to S. P. Kapitza for his consideration of this work and to Academician L. D. Landau for his comments.

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Translated by S. D. Elliott
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