EXCITATION OF COLLECTIVE NUCLEAR LEVELS IN HEAVY μ-MESIC ATOMS

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Effects connected with quadrupole interaction of a muon with the nucleus in heavy mesic atoms are investigated. The probabilities of excitation of collective nuclear levels are computed. The hyperfine splitting of the collective nuclear level, additional muon depolarization, and the change in circular polarization of the γ-quantum from the 2p-1s transition are considered.

1. INTRODUCTION

MESIC atoms are produced by capture of muons at highly excited states. The muon goes over subsequently by cascade to the ground state of the mesic atom. During the cascade transition, nuclear levels can be excited with noticeable probability. The excitation of the nucleus with transfer of the entire energy of the given muon transition to the nucleus (nonradiative excitation) was considered by the authors earlier. [1]

In addition to nonradiative excitation, there is appreciable probability of exciting the low-lying nuclear levels. In this case part of the energy of the given muon transition goes to the excitation of the nuclear levels, and a γ quantum of lower energy is emitted. The effect of excitation of the 2^+ rotational level as the muon goes through the 2p state was considered by Wilets and Jacobson. [2] This effect is due to the interaction between the muon and the quadrupole moment of the nucleus. The probability of excitation of these levels depends not only on the magnitude of the internal quadrupole moment, but also on its sign.

In the present paper we consider the probability of excitation of an arbitrary collective level with spin 2^+. It is obvious that in this case the excitation probability depends on the magnitude of the nuclear quadrupole moment in the excited state and on the reduced probability of the quadrupole transition for the given nuclear level.

For odd nuclei with a ground-state spin larger than $1/2$, the probability of excitation depends also on the quadrupole moment in the ground state. The value of B(E2) is determined from data on the Coulomb excitation. Consequently, it is possible in principle to determine the quadrupole moment Q_{22} of the nucleus in the excited state. On the other hand, the ratio $|Q_{22}|^2/B(E2)$ can be calculated with the aid of various nuclear models. Comparing the theoretical and experimental ratios we can conclude whether a particular nuclear model is applicable.

In heavy mesic atoms of the thorium and uranium type, the rotational states of the nuclei can be excited not only when the muon goes through the 2p state, but also when it goes through the 3d state. This effect is of interest in connection with the measurement of the sign of the quadrupole moment and the possibility of exciting the 4^+ rotational level. We also consider in this paper the effect of the additional muon depolarization resulting from the excitation of the nuclear collective levels. Account is taken of the change in the circular polarization of the γ quantum in the 2p-1s transition, due to the quadrupole interaction of the muon with the nucleus.

2. EXCITATION OF ARBITRARY COLLECTIVE LEVEL

We consider the excitation of the lowest collective nuclear levels. The effective excitation of the nuclear levels is appreciable if

$$|\Delta E_{12} - E_{\text{nuc}}| \lesssim |\langle\psi_0\varphi_1|H_{int}|\psi_I\varphi_2\rangle|, \tag{1}$$

where ΔE_{12} is the distance between the given muon levels 1 and 2, E_{nuc} is the energy of the nuclear level, ψ_0 and ψ_I are the wave functions of the nucleus in the ground and excited states respectively, φ_1 and φ_2 are the wave functions of the given muon levels in the spherically-symmetrical field of the nucleus,

$$H_{int} = \left\langle\psi_0\left|\sum_i \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_\mu|}\right|\psi_0\right\rangle - \sum_i \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_\mu|},$$

while $\mathbf{r}_i$ and $\mathbf{r}_\mu$ are the radius vectors of the proton and muon respectively. In the case of heavy nuclei, Eq. (1) is satisfied for the muon levels $2p_{3/2}$ and $2p_{1/2}$ and for the lowest nuclear levels with spin 2^+ over a wide range of atomic weights.

The Hamiltonian of the muon-nucleus system can be written in the form

$$H = H_\mu + H_{\text{nuc}} + H_{int},$$
$$H_\mu = -(\hbar^2/2m)\nabla^2 + V(r) + V_{\text{so}},$$
$$V(r) = \begin{cases} (Ze^2/2R_0)(r^2/R_0^2 - 3), & r < R_0, \\ -Ze^2/r, & r > R_0, \end{cases} \qquad (2)$$

where m —mass of the muon, Ze —charge of the nucleus, R_0 —radius of the nucleus, V_{so} —spin-orbit interaction, and H_{nuc} —Hamiltonian of the nucleus.

Since we are interested in the excitation of the 2^+ level, it is sufficient to separate in the interaction Hamiltonian H_{int} the term corresponding to the quadrupole interaction;

$$H_q = -e^2\frac{4\pi}{5}\begin{cases} \sum_{i,m} r_i^2 Y_{2m}^*(\theta_i\varphi_i)\, r^{-3} Y_{2m}(\theta, \varphi), & r \geqslant r_i, \\ \sum_{i,m} r^2 Y_{2m}(\theta, \varphi)\, r_i^{-3} Y_{2m}^*(\theta_i\varphi_i), & r \leqslant r_i. \end{cases} \qquad (3)$$

For simplicity we assume that the quadrupole part of the interaction between the muon and the nucleus is significant only in the $2p_{1/2}$ and $2p_{3/2}$ states. As will be shown later, an account of the quadrupole interaction in the 3d state yields a correction of approximately 20% to the excitation probability of the rotational 2^+ level for the heaviest nuclei of the thorium and uranium type. The basis wave functions of the muon-nucleus system can be written in the form

$$\Phi_{JM}^{Ij} = \sum_{mm_j} C_{Imjm_j}^{JM} \psi_{Im}\varphi_{jm_j}, \qquad (4)$$

where I, m and j, m_j —respectively the momenta and their projections for the nucleus and muon, J and M —the total momentum of the system and its projection; C —Clebsch-Gordan coefficients. The matrix elements of the operator H_q on the wave functions (4) are proportional, after integration over the muon coordinates, to the nuclear matrix elements

$$\langle \psi_{Im} | r_i^2 Y_{2\mu}(\theta_i, \varphi_i) f(r_i) | \psi_{I'm'} \rangle,$$
$$f(r_i) = \int_{r_i}^{\infty} |R_{nl}|^2 \frac{dr}{r} + \frac{1}{r_i^5}\int_0^{r_i} r^4 |R_{nl}|^2 dr, \qquad (5)$$

where R_{nl} —real part of the wave function of the muon in the state with quantum numbers n and l.

When the collective excitations can be considered in the adiabatic approximation with respect to the internal excitations, the wave function of the nucleus has the following form [3]

$$\psi_{Im} = \chi_0(\mathbf{r}_i, \alpha_\nu) K_{Im}(\alpha_\nu), \qquad (6)$$

where α_ν —collective coordinates, χ_0 —wave function of the internal state of the nucleus, and $K_{Im}(\alpha_\nu)$ —wave function of the collective degrees of freedom. Assuming the charge to be uniformly distributed over the volume of the nucleus, the matrix element (5) with account of (6) is proportional to the matrix element

$$\left\langle K_{Im} \left| \int Y_{2\mu}(\theta_i, \varphi_i)\, d\Omega_i \int_0^R r_i^4 f(r_i)\, dr_i \right| K_{I'm'} \right\rangle,$$
$$R = R_0\left[1 + \sum_\nu \alpha_{2\nu} Y_{2\nu}(\theta_i, \varphi_i)\right]. \qquad (7)$$

Inasmuch as $\alpha_{2\nu} \ll 1$, the form factor $f(r_i)$ can be factored out, using its value at the point $r_i = R_0$. Consequently, the matrix elements connected with the excitation of the collective levels of the nucleus by the muon turn out to be proportional to the nuclear matrix element for the quadrupole transition. As a result, the matrix elements of the operator H_q in representation (4) have the form

$$\langle j_2 l_2 I_2 JM | H_q | j_1 l_1 I_1 JM' \rangle = -e^2 f(R_0)(-1)^{l_2 - l_1 + I_1 - I_2}$$
$$\times \sqrt{4\pi(2l_2+1)(2I_2+1)(2j_1+1)(2j_2+1)/5}$$
$$\times W(2l_2 j_1 {}^1\!/_2;\ l_1 j_2)$$
$$\times W(I_1 2 J j_2;\ I_2 j_1)\, C_{l_2 020}^{l_1 0} \cdot \tfrac{1}{2}\sqrt{5/4\pi}\, Q_{I_2 I_1}^{(2)} \delta_{MM'}, \qquad (8)$$

where

$$Q_{I_2 I_1}^{(2)} = 2\sqrt{4\pi/5}\left\langle I_2 \left| \sum_i r_i Y_2(i) \right| I_1 \right\rangle,$$

W are Racah coefficients and l_1 and l_2 are the orbital momenta of the muon.

In the particular case of even-even nuclei, when the quadrupole interaction of the muon with the nucleus is essential only in the 2p-state, expression (8) simplifies greatly. In this case we have

$$\langle j_2 12 JM | H_q | j_1 10 JM \rangle$$
$$= \tfrac{2}{5}\sqrt{6\pi(2j_2+1)}\, e^2 f(R_0)\, W(21 j_1 {}^1\!/_2;\ 1 {}^1\!/_2)\, B^{1/2}(E2),$$
$$\langle j_2 12 JM | H_q | j_1 12 JM \rangle = \sqrt{3(2j_1+1)(2j_2+1)/2}\, e^2 f(R_0)$$
$$\times W(21 j_1 {}^1\!/_2;\ 1 {}^1\!/_2)\, W(22 J j_2;\ 2 j_1)\, Q_{22}^{(2)}. \qquad (9)$$

To find the wave functions and the energies in the states with $J = {}^3\!/_2$ and ${}^1\!/_2$ we must solve secular equations of third and second degree, respectively.

The relation between $B^{1/2}(E2)$ and Q_{22} is known for the rotational levels of an axially-symmetrical nucleus. However, there are several heavy nuclei whose collective levels do not correspond to the spacing rule established for axially symmetrical nuclei. Such levels have been interpreted recently as the rotational spectrum of an axially-asymmetrical rigid rotator.[4] In this case $|Q_{22}^{(2)}|^2$ is uniquely related with $B^{1/2}(E2)$ as follows:

$$\frac{|Q_{22}^{(2)}|^2}{B_{2\to 0}(E2)} = 32\pi\frac{6}{7}\frac{1-\sin^2 3\gamma}{9-8\sin^2 3\gamma}\left(1+\frac{3-2\sin^2 3\gamma}{\sqrt{9-8\sin^2 3\gamma}}\right)^{-1}, \quad (10)$$

where γ is the axial-symmetry parameter.

Within the framework of this model, we calculated for several nuclei the probabilities W_{2+} of exciting the first collective 2^+ levels. The magnitude of the spin-orbit splitting ΔE_{fs} and the form factor were calculated with the aid of the approximate muon wave functions.[5] The energies E_{2+} of the first 2^+ levels and the values of B(E2) and γ were taken from [6]. The results of the calculations are listed in Table I.

Table I

Nucleus	$B(E2)_{0\to 2}\cdot 10^5$ cm^4	γ, deg	$\frac{\lvert Q_{22}\rvert^2}{B_{0\to 2}(E2)}$	ΔE_{fs}, MeV	E_{2+}, MeV	W_{2+}
W^{186}	356	15.8	3.86	0.134	0.123	0.40*
Os^{190}	255	22.3	2.66	0.147	0.187	0.30
Hg^{198}	113	24.3	1.90	0.162	0.411	0.025

*The probability $W_{2}+$ is referred to the probability of the muon passing through the state 2p.

We also calculated the probability of excitation of the first 2^+ level for Os^{190}, within the framework of the vibration model. In this case $Q_{22} \approx 0$. The probability of excitation turned out to be 0.24. Thus, the excitation probability depends distinctly on the nature of the collective level.

3. EXCITATION OF ROTATIONAL LEVELS

If the quadrupole interaction of the muon with the nucleus is significant only in the 2p states, then only the 2^+ level can be excited for even-even nuclei. However, in heavy mesic atoms of the thorium and uranium type, this interaction turns out to be appreciable also for muon states with n = 3. As a result of the quadrupole interaction, the muon states $3s_{1/2}$, $3d_{5/2}$, and $3d_{3/2}$ can become intermixed. Owing to the finite dimensions of the nucleus, the $3s_{1/2}$ level lies about 0.4 Mev above the 3d levels, and it is therefore sufficient to take into account the mixing of the levels $3d_{3/2}$ and $3d_{5/2}$. This increases the probability of excitation of the 2^+ nuclear level. In addition, it becomes possible for the 4^+ level to become excited. Indeed, when the muon goes through the 3d states the nuclear level 2^+ is excited. The 3d-2p muon transition that follows can be accompanied by the transition of the nucleus from the 2^+ level to the 4^+ level.

Let us examine this effect. In the case of the rotational spectrum of axially-symmetrical nuclei, the expression for the matrix element (8) has the form

$$\langle j_1 l I_1 J \mid H_q \mid j_2 l I_2 J\rangle = \tfrac{1}{10} Q_0 e^2 f(R_0)\,\Lambda(l j_1 j_2 I_1 I_2 J), \quad (11)$$

where Q_0 is the internal quadrupole moment,

$$\Lambda(l j_1 j_2 I_1 I_2 J) = (-1)^{l+j_1+j_2+I_1+I_2-J-1/2} C^{20}_{I_1 0 I_2 0} \times\left[\frac{5l(l+1)(2l+1)}{(2l-1)(2l+3)}(2j_1+1)(2j_2+1)(2I_1+1)(2I_2+1)\right]^{1/2} \times W(l j_1 l j_2; 1/2\, 2)\, W(I_1 j_1 I_2 j_2; J2).$$

It must be noted that the H_q matrix for the 3d states, unlike the H_q matrix for the 2p states, has elements that are diagonal in all the quantum numbers. When the muon is in the 3d states, the wave functions of the muon-nucleus system have the form

$$|3d, J, M\rangle_\alpha = \sum_{Ij} A_{Ij}^{J\alpha}\,\Phi_{Ij,\,3d}^{JM}, \quad (12)$$

where $A_{Ij}^{J\alpha}$ are the coefficients of the expansion of the eigenfunctions of the Hamiltonian (2) $|3d, JM\rangle_\alpha$ in the basis functions (4). The quantum number α corresponds to the eigenvalues of the Hamiltonian (2). We obtain similarly for the 2p state

$$|2p, J', M'\rangle_\beta = \sum_{I'j'} A_{I'j'}^{J'\beta}\,\Phi_{I'j',\,2p}^{JM}. \quad (13)$$

The muon transition to the 3d state is from higher states, where the quadrupole interaction can be neglected. Consequently the population of the α states is

$$\omega(J, \alpha, 3d) = |A_{0j}^{J\alpha}|^2. \quad (14)$$

For the same reason, the total momentum J in the states (12) is $3/2$ or $5/2$.

The transition to the β states (13) is from the α states, where the nucleus is already excited. Consequently the population of the level (13) is determined by the coefficients $A_{Ij}^{J\alpha}$ and $A_{I'j'}^{J'\beta}$. It follows also that the total momentum of the muon-nucleus system in the states (13) can take on values $1/2$, $3/2$, $5/2$, and $7/2$.

The probability W_I of excitation of a nuclear state with spin I is determined from an analysis of all the possible transitions from the states (12) into (13) and from (13) into states corresponding to the motion of the muon on the K orbit, $\varphi_{1s}\psi_I$. As a result we obtain

$$W(I) = \sum_{JJ'\alpha\beta j''} \tfrac{1}{2}(2J+1)(2J'+1)\,|A_{0J}^{J\alpha}|^2\,|A_{Ij''}^{J'\beta}|^2\Big[\sum_{jj'I'} A_{I'j'}^{J'\beta} A_{I'j}^{J\alpha} \times\sqrt{(2j+1)(2j'+1)}\,W(2j1j'; 1/2\,1)\,W(jj'JJ'; 1I')\Big]^2. \quad (14')$$

The probabilities W_{2+} and W_{4+} for the excitation of the rotational level 2^+ and 4^+ were calculated for U^{238}. The results of the calculation are listed in Table II. The probabilities given in Table II for the excitation of the nuclear levels are referred to the probability of the muon passing through the 3d states. It is seen from the table that the excitation probability of the 4^+ level is more sensitive to the sign of the quadrupole moment of the nucleus than the excitation probability of the 2^+ level.

Table II

	Q_0, b	
	10	−10
ΔE^{2p}_{fs}, keV	226	
ΔE^{3d}_{fs}, keV	63	
E_{2+}, keV	44	
W_{2+}	0.56	0.51
W_{4+}	0.05	0.01

4. HYPERFINE SPLITTING IN HEAVY MESIC ATOMS

The interaction between the magnetic moment of the muon and the magnetic moment of the excited nucleus leads to a hyperfine splitting of the excited levels of the nucleus. The nuclear transition from the excited state into the ground state should therefore have a fine structure. In calculating the hyperfine splitting in heavy mesic atoms it is necessary to take into account the effect of the finite dimensions of the nucleus. It is therefore to be expected that the hyperfine splitting is determined not only by the magnitude of the magnetic moment of the nucleus, but also by the distribution of the current over the volume of the nucleus. The current density $\mathbf{j}(\mathbf{r})$ averaged over the internal motion of the nucleus has the form

$$\mathbf{j} = \gamma(r)\,[\mathbf{Ir}], \tag{15)*$$

where $\mathbf{I}$ —nuclear spin, $\gamma(\mathbf{r})$ —a scalar function connected with the magnitude of the nuclear magnetic moment μ_{nuc} by the relation

$$\mu_{nuc} = \frac{1}{3c}\int r^2\gamma(r)\,dr. \tag{16}$$

The integration in (16) is over the volume of the nucleus.

The hyperfine splitting is equal to the energy of interaction between the magnetic moment of the muon and the magnetic field, which in turn is determined by the nuclear current (15). Using (16) we have

*$[\mathbf{I}\,\mathbf{r}] = \mathbf{I}\times\mathbf{r}$

$$\Delta E_{hfs} = 2(2I+1)\,\mu_0\mu_{nuc}\int_0^{R_0} r\gamma(r)\,dr \times \int_0^{r} r_\mu^2 R_{1s}^2(r_\mu)\,dr_\mu \Big/ \int_0^{R_0} r^4\gamma(r)\,dr, \tag{17}$$

where μ_0 is the Bohr magneton of the muon. For a point nucleus ($R_0 \to 0$), formula (17) goes over into the well known expression for the hyperfine splitting of the s-electron level (see, for example, [7]).

If the collective level of the nucleus is rotational and the nuclear motion can be described as rotation of a rigid body, then $\gamma(r)$ = const. In this case the hyperfine splitting is determined by the magnitude of the magnetic moment of the excited level of the nucleus. In the general case, when $\gamma(r) \neq$ const, we can determine the moment $\gamma(r)$, which is given by the ratio of the integrals in (17), by measuring the hyperfine splitting and using the known μ_{nuc}. Table III lists the values of the hyperfine splitting in certain mesic atoms for the first rotational level, assuming γ = const; the gyromagnetic ratio for the rotational levels was assumed to equal Z/A.

Table III

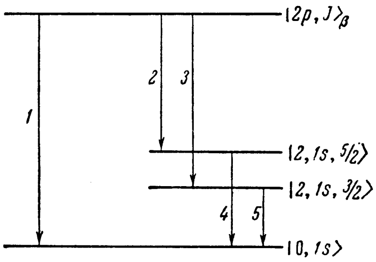
Nucleus	ΔE_{hfs} eV	E_{2+}, keV
Sm^{154}	740	82
Er^{168}	780	80
W^{182}	800	120
Th^{232}	840	50
U^{238}	840	44

Assuming that the nuclear current is concentrated on the surface, the values of the hyperfine splitting are less than those listed in the table by 20–25%. The value of the hyperfine splitting can be determined by observing the fine structure of the nuclear γ quanta or of the internal-conversion electrons corresponding to the nuclear transition.

5. POLARIZATION EFFECT IN HEAVY MESIC ATOMS

It is known that the muon becomes depolarized in a cascade transition in the mesic atom on the K shell. The residual polarization of the muon in the 1s state amounts in this case to about 16%.[8] The depolarization mechanism is connected with the spin-orbit interaction.[9] In those mesic atoms in which the collective levels of the nuclei are excited, additional depolarization is to be expected.[10] This effect is qualitatively connected with the partial transfer of the muon polarization to the excited nucleus.

Let us examine this effect in the case of even-even nuclei. The additional depolarization in the heavy mesic atoms is then determined by two effects: excitation of the nuclear collective levels as the muon passes through the 2p states, and interaction of the magnetic moment of the excited state with the magnetic moment of the muon on the K shell. Since the natural width of the nuclear level is much smaller than the magnitude of the hyperfine splitting ΔE_{hfs}, we can consider the radiative transitions from the state $|2p, J\rangle_\beta$ directly to the hyperfine splitting components $|2, 1s, F\rangle$ ($F = 5/2$ or $3/2$). The scheme of transitions from the state $|2p, J\rangle_\beta$ is shown in the figure. Transition 1 corresponds to the muon transition from the 2p to the 1s level without excitation of the nucleus. Transitions 2 and 3 correspond to muon transition with excitation of the nucleus. Transitions 4 and 5 are due to the nuclear transition with emission of a γ quantum or an internal-conversion electron in the ground state of the muon-nucleus system $|0, 1s\rangle$.



To calculate the depolarization it is convenient to use the concept of polarization moments. By definition (see, for example, [11]), the polarization moment P_l of the state of a system with spin J is equal to

$$P_l = \sum_M \rho_J(M)\, C^{JM}_{JMl0}, \tag{18}$$

where $\rho_J(M)$ is the polarization density matrix. If the system goes from a state with total momentum J_1 to a state with total momentum J_2, emitting a quantum of multipolarity L, then the polarization moments of the initial and final states are related by

$$P_l(J_2) = \sqrt{(2J_1+1)(2J_2+1)}\, W(J_1 l L J_2; J_1 J_2)\, P_l(J_1). \tag{19}$$

Consequently, the polarization moments in the states $|2p, j\rangle_\beta$ are determined by the polarization moments in the higher states, when the quadrupole interaction can be neglected. In states with excitation greater than 2p, the depolarization is due to the spin-orbit interaction. Consequently only the first polarization moment differs from zero in these states.

On the basis of (19) we can also conclude that

$$P_1(2p_{3/2}) = P_1(|2p, 3/2\rangle_\beta), \qquad P_1(2p_{1/2}) = P_1(|2p, 1/2\rangle_\beta), \tag{20}$$

where $P_1(2p_{3/2})$ and $P_1(2p_{1/2})$ are the polarization moments in the states $2p_{3/2}$ and $2p_{1/2}$, respectively, in the absence of quadrupole interaction. According to the depolarization mechanism based on the spin-orbit interaction, [9] we have

$$P_1(2p_{1/2}) = 0, \qquad P_1(2p_{3/2}) = \bar{\sigma}_0 \sqrt{3/5}, \tag{21}$$

where $\bar{\sigma}_0$ is the residual polarization of the muon on the K shell in the absence of quadrupole interaction. Consequently, it is sufficient to consider the transitions from the states $|2p, 3/2\rangle_\beta$.

Let us denote by $P_1(5/2)$ the partial polarization moment of the state $|0, 1s\rangle$, which corresponds to the dipole muon transition $|2p, 3/2\rangle_\beta \to |2, 1s, 5/2\rangle$ and the subsequent nuclear quadrupole transition $|2, 1s, 5/2\rangle \to |0, 1s\rangle$. With the aid of (19) we obtain

$$P_1(5/2) = 7/25\, \sqrt{5}\, P_1(|2p, 3/2\rangle_\beta). \tag{22}$$

Analogously, for $P_1(3/2)$, the partial polarization moment corresponding to the transitions 3 and 5, and for $P_1(1/2)$, corresponding to transition 1, we obtain

$$P_1(3/2) = -11/25\, \sqrt{5}\, P_1(|2p, 3/2\rangle_\beta),$$
$$P_1(1/2) = (\sqrt{5}/3)\, P_1(|2p, 3/2\rangle_\beta). \tag{23}$$

The probabilities of these transitions can be calculated if we know $A^{3/2\beta}_{lj}$ —the coefficient of expansion of the wave functions $|2p, 3/2\rangle_\beta$ in the basis system (4):

$$W_{0,\beta} = (A^{3/2\,\beta}_{0\,3/2})^2,$$
$$W_{F,\beta} = (2F+1) \times \Big[\sum_j (-1)^{j+1/2} \sqrt{2j+1}\, W(3/2\; 2\, 1\, 1/2;\, jF)\, A^{3/2\,\beta}_{2j}\Big]^2, \tag{24}$$

where $W_{0,\beta}$ is the probability of transition 1 and $W_{F,\beta}$ is the probability of transitions 2 and 3 (they correspond to $F = 5/2$ and $3/2$). The transition probabilities given in (24) satisfy the normalization condition

$$W_{0,\beta} + \sum_F W_{F,\beta} = 1. \tag{25}$$

Averaging over all the transitions indicated above, summing over all the levels β with account of W_β (the population of these levels), and taking (20) into account, we obtain P_1, the total polarization moment in the muon-nucleus system ground state $|0, 1s\rangle$:

$$\bar{P}_1 = P_1(2p_{3/2})\sum_\beta W_\beta \times \left(\frac{\sqrt{5}}{3}W_{0,\beta} - \frac{11}{5\sqrt{5}}W_{3/2,\beta} + \frac{7}{5\sqrt{5}}W_{5/2,\beta}\right). \tag{26}$$

$\bar{P}_1$ is simply related with σ, the polarization of the muon on the K shell, by

$$\bar{P}_1 = \bar{\sigma}/\sqrt{3}. \tag{27}$$

Using (21), (24), (26), and (27) we obtain as the final expression for σ

$$\bar{\sigma} = B_q\bar{\sigma}_0,$$

$$B_q = \sum_\beta (A_{0\,3/2}^{3/2\,\beta})^2\{(A_{0\,3/2}^{3/2\,\beta})^2 + \tfrac{1}{125}[73(A_{2\,1/2}^{3/2\,\beta})^2 - 128A_{2\,1/2}^{3/2\,\beta}A_{2\,3/2}^{3/2\,\beta} - 23(A_{2\,3/2}^{3/2\,\beta})^2]\}. \tag{28}$$

The factor B_q in (28) takes into account the additional depolarization due to the quadrupole and magnetic interaction of the muon with the nucleus. The values of B_q for several nuclei are listed in Table IV.

Table IV

Element	B_q	D_q
Gd^{158}	0.71	0.23
W^{184}	0.73	0.27
Th^{232}	0.42	0.43

The presence of a residual polarization of the muon in the 2p states causes the γ quantum of the 2p-1s transition to be circularly polarized.[12] Owing to the quadrupole interaction, a change takes place in the circular polarization of the 2p-1s transition quantum. The circular polarization ξ_2 of the dipole quantum (the Stokes parameter) for the transition of the system from the state with momentum J_1 to the state with J_2 is equal to

$$\xi_2 = 3\sqrt{3(2J_1+1)/2}\,W(11J_2J_1;\,1J_1)\,P_1(J_1)\cos\theta. \tag{29}$$

If the quadrupole interaction can be neglected, we get, on the basis of (21), $J_1 = 3/2$ and $J_2 = 1/2$. We therefore obtain in this case for the circular polarization $\xi_2^{(0)}$

$$\xi_2^{(0)} = \frac{\sqrt{15}}{2}P_1(2p_{3/2})\cos\theta = \frac{3}{2}\bar{\sigma}_0\cos\theta. \tag{30}$$

If the nuclear level 2^+ is excited, the spin J_2 of the transition 2p-1s can take on values $1/2$, $3/2$, $5/2$. Averaging over these transitions, we obtain the following expression for the circular polarization of the quantum $\bar{\xi}_2$:

$$\bar{\xi}_2 = D_q\xi_2^{(0)},$$

$$D_q = \sum_\beta (A_{0\,3/2}^{3/2\,\beta})^2\{(A_{0\,3/2}^{3/2\,\beta})^2 + \tfrac{1}{5}[(A_{2\,1/2}^{3/2\,\beta})^2 + 4A_{2\,1/2}^{3/2\,\beta}A_{2\,3/2}^{3/2\,\beta} - 2(A_{2\,3/2}^{3/2\,\beta})^2]\}. \tag{31}$$

The factor D_q takes into account the influence of the quadrupole interaction on the circular polarization of the γ quantum from the 2p-1s transition. The values of D_q for certain nuclei are listed in Table IV, from which it is seen that the excitation of the collective nuclear levels in mesic atoms greatly influences the magnitude of the residual polarization of the muon and the circular polarization of the γ quantum from the 2p-1s transition.

6. CONCLUSION

An experimental investigation of the effects considered in the present paper is of interest from the point of view of a study of the properties of the ground state of the nucleus and of the low-lying collective nuclear levels. Experiments with μ-mesic atoms make it also possible in principle to determine such fundamental characteristics of the nucleus as the sign and magnitude of the quadrupole moments of the excited states. In addition, it becomes possible to find a definite moment of current distribution over the nuclear volume.

In heavy mesic atoms of the uranium or thorium type the quadrupole excitation of the collective levels may be masked by non-radiative excitation. After the entire energy of the 2p-1s transition is transferred to the nucleus, the latter may go through the states of collective excitation by cascade radiation of γ quanta. Consequently, to separate the nonradiative excitation it is necessary to measure the nuclear transitions $4^+ \to 2^+$ and $2^+ \to 0^+$ by coincidence with the muon-transition γ quanta.

The additional depolarization of the muon, due to the quadrupole interaction, may be masked by the influence of the electron shell.[13] Consequently from the point of view of studying the polarization effects connected with the quadrupole interaction of the muon with the nucleus, it is of interest to investigate the circular polarization of the γ quantum from the 2p-1s transition, where the influence of the electron shell can be neglected.

One should also note that the quadrupole interaction effects influence also the line shapes of the muon transitions. Consequently the study of the fine structure of the spectral lines can yield information on the quadrupole interaction between the muon and the nucleus.

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