MEASUREMENT OF ELECTRON MOBILITY ACCORDING TO THE VARIATION OF PLASMA RESISTANCE IN A MAGNETIC FIELD

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Theoretical calculations and experimental data are presented which indicate the possibility of measuring the electron mobility on the basis of the change in the resistance of a plasma in thermodynamic equilibrium situated in a magnetic field.

INTRODUCTION

THE study of electron mobilities in gases presents great interest both for the theory of electron interaction with atoms and ions and for practical purposes. The Townsend method,[1] which is usually applicable for the direct measurement of the mobility, does not give sufficient accuracy, since it is necessary to make a number of inadequately based assumptions in the treatment of the measurement results.

The mobility can also be computed from a knowledge of the scattering cross section. However, direct measurement of these cross sections encounters great difficulties, especially for electrons with thermal velocities.[3]

We have developed a direct method for the measurement of the electrical conductivity of the plasma.[4] For calculation of the mobility by this method, it is necessary to know the electron concentration, and also the area of the working surface of the electrodes.

Although theoretical estimates show that the electron concentration [if the necessary conditions are satisfied (see [4,5])] is equal to the equilibrium concentration at a gas temperature equal to the electrode temperature, satisfaction of these conditions cannot, however, always be verified. Therefore, it is desirable to have an independent method which does not require a knowledge of the concentrations and exact geometric dimensions of the working surface of the electrodes. For this purpose, it proves to be possible to use the method of measurement of the mobility from a measurement of the resistance in a magnetic field, a method which has been widely used in the physics of semiconductors.

EXPERIMENTAL METHOD

We applied the method described above to the measurement of the electron mobility in weakly ionized cesium plasma. For the measurements, it was required to produce such an electrode configuration that the plasma in the interelectrode space was in a state of thermodynamic equilibrium, and its temperature was determined by the electrode temperature. We prepared special apparatus for this purpose. The apparatus consisted of a glass vessel fitted on a kovar base. A ceramic plate (Al_2O_3) was attached to the base, and a set of electrodes was mounted on it. The working electrodes were prepared from thin bands of tungsten, molybdenum, or tantalum (width 3.5 — 4 mm, length 12 — 14 mm, thickness 50 μ). The electrodes were attached on conducting rods, one under the other, and the bands were set at right angles to one another. The working sections were in between the bands. The interelectrode gap was set between the limits 0.2 and 0.8 mm; to fix the gap, the bands were stretched by means of special tungsten springs. Means of regulation were provided in the construction of the rods to guarantee good plane parallel conditions of the electrodes. The control of the assembly geometry of the electrodes was maintained by visual observation with fourfold magnification. Such construction assured the constancy of the gap for all operating temperatures of the electrodes. The value of the interelectrode distance was systematically monitored by photographing with fourfold magnification. The electrodes could be heated simultaneously from two independent single half-wave rectifiers. Metallic cesium was introduced in the apparatus after careful degassing of the flask and of the fittings, and separation from

the vacuum system was carried out at a pressure of $\sim(1-2)\times 10^{-7}$ mm mercury.

A magnetic field perpendicular to the flow of emitted electrons was produced by placing the apparatus in a solenoid, after which the location of the apparatus was rigidly fixed. The dimensions of the solenoid guaranteed good homogeneity of the field in a space which appreciably exceeded the dimensions of the heated electrodes. The magnetic field at the location of the electrodes was previously calibrated with a fluxmeter. In the measurements, the apparatus was placed together with the solenoid in a thermostatic oven, whose temperature determined the saturated vapor pressure of the cesium in the apparatus.

The measurement of the resistance in the presence and absence of the magnetic field was carried out during the non-conducting half-cycle by the method described by us elsewhere.[4] The high frequency of the supply voltage (400 cps) guaranteed the constancy of the electrode temperature during the cycle, with an accuracy to within 0.5 per cent. The electrode temperatures was measured by an optical pyrometer. In this case, the difference in the temperatures of the two electrodes did not exceed $10-15°$ K.

THEORY

With the described geometry of the apparatus, the condition of the plasma differs from the corresponding conditions in the semiconducting specimens usually employed in the measurement of the conductivity σ_H in the magnetic field. In the first place, the semiconducting specimens were so prepared that the lengths of the specimens far exceeded their transverse dimensions. Therefore, in these specimens, the transverse Hall current is equal to zero while the resulting Hall voltage must be taken into consideration in the calculation of σ_H.[6] In our case, the opposite relation holds, since the distance between the electrodes is much less than the transverse dimensions of the operating part. Therefore, the Hall current is shorted by the electrodes and the Hall voltage is practically zero.

In the second place, in contrast with the semiconductor, the plasma can move freely between the electrodes under the action of ponderomotive forces which arise in the magnetic field. In the motion of the plasma, an additional Lorentz force acts on the electron which, in principle, must be taken into consideration in the calculation of σ_H. For this it is necessary to find a common solution of the kinetic equation for electrons in a moving plasma and the equation of motion of the plasma.

The kinetic equation which determines the distribution function of the electrons f has the form

$$\mathbf{v}\nabla f+\frac{e}{m}\nabla\varphi\nabla_v f_0-\frac{e}{cm}[\mathbf{vH}]\nabla_v f=-\frac{f-f_0}{\tau}, \qquad (1)^*$$

where f_0 is the equilibrium distribution function in a system of coordinates moving with the plasma:

$$f_0=e^{(\mu-\varepsilon')/kT}, \qquad \varepsilon'=\frac{m}{2}\sum_i (v_i')^2, \qquad \mathbf{v}'=\mathbf{v}-\mathbf{v}_0.$$

Here $\mathbf{v}$ is the velocity of the electrons in the fixed system of coordinates, $\mathbf{v}_0$ is the hydrodynamic velocity of the plasma, μ is the chemical potential of the electrons, $\mathbf{E}=-\nabla\varphi$ is the electric field, $\mathbf{H}$ the magnetic field, and τ the relaxation time of the electrons. The first term in (1) gives a contribution to f of the order of $(v\tau/d)f\approx lf/d$, where l is the mean free path and d is the interelectrode distance.

As is shown in [7], account of this term leads to corrections of the order of l/d and for $l \ll d$ it can be neglected. Then, putting f in the form $f=f_0+f_1$,

$$\frac{e}{m}\left(\mathbf{E}'+\frac{1}{c}[\mathbf{v}_0\mathbf{H}]\right)\nabla_v f_0+\frac{e}{mc}[\mathbf{vH}]\nabla_v f_1=\frac{1}{\tau}f_1, \qquad (2)$$

which differs from the usual kinetic equation in the fact that, in place of $\mathbf{e}\cdot\mathbf{E}$, we have the quantity

$$e\mathbf{E}'=e\mathbf{E}+(e/c)[\mathbf{v}_0\mathbf{H}]. \qquad (3)$$

We seek f_1 in the form

$$f_1=(\mathbf{v}\chi(\varepsilon))\,\partial f_0/\partial\varepsilon'; \qquad (4)$$

in this case, it is necessary to take it into account that both χ and $\mathbf{v}_0$ are proportional to $\mathbf{E}$. Omitting the terms that are quadratic in E, we get the usual equation

$$e\mathbf{E}'+(e/mc)[\mathbf{H}\chi]=\chi/\tau. \qquad (5)$$

The solution of this equation has the form

$$\chi=\frac{e\tau}{1+(\omega\tau)^2}[\mathbf{E}'+\tau[\boldsymbol{\omega}\mathbf{E}]+\tau^2\boldsymbol{\omega}(\boldsymbol{\omega}\mathbf{E})], \qquad (6)$$

where

$$\boldsymbol{\omega}=e\mathbf{H}/mc.$$

The resulting current produced by the electrons (after deduction of the current $\mathbf{j}_0=-en\mathbf{v}_0$, which compensates the corresponding ion current), is given by the expression

$$\mathbf{j}=\int f_1\mathbf{v}\,d^3v=\int(\mathbf{v}\chi)\mathbf{v}\frac{\partial f_0}{\partial\varepsilon'}d^3v, \qquad \int f_0 d^3v=n, \qquad (7)$$

where n is the electron concentration and is equal

*$[\mathbf{vH}]=\mathbf{v}\times\mathbf{H}$

to the ion concentration. Hence the ratio of the quantity $\sigma'_{xx} = j_x/E'_x$ and the electrical conductivity σ_0 in the absence of the magnetic field is equal to

$$\sigma'_{xx}/\sigma_0 = \left\langle \varepsilon\tau \frac{1+(\omega_x\tau)^2}{1+(\omega\tau)^2} \right\rangle \Big/ \langle \varepsilon\tau \rangle, \quad (8)$$

where

$$\sigma_0 = enu, \qquad u = (e/m)\langle \varepsilon\tau\rangle/\langle\varepsilon\rangle, \quad (8a)$$

and the symbol $\langle \ldots \rangle$ denotes the average over the Maxwellian distribution; u is the mobility.

Under the conditions of our experiment, $H = H_z$ and the Hall field $E_y = 0$. Therefore, for computation of the electrical conductivity in what follows, we need only the quantity

$$\sigma'_H/\sigma_0 = \left\langle \frac{\varepsilon\tau}{1+(\omega\tau)^2} \right\rangle \Big/ \langle \varepsilon\tau \rangle. \quad (9)$$

In a weak magnetic field, i.e., for $uH/c \ll 1$, we have

$$\sigma'_H/\sigma_0 = 1 - \gamma\,(uH/c)^2, \qquad \gamma = \langle\varepsilon\tau^3\rangle\langle\varepsilon\rangle^2/\langle\varepsilon\tau\rangle^3. \quad (10)$$

If $\tau \sim \epsilon^n$, then

$$\gamma = 9\,(3n + 3/2)!\,(1/2)!^2/4\,(n + 3/2)!^2. \quad (11)$$

In scattering by atoms, if the scattering cross section Q does not depend on the energy, we have

$$n = -1/2, \qquad \gamma_{-1/2} = 9\pi/16 = 1.77. \quad (12)$$

In scattering by ions,

$$n = 3/2, \quad \gamma_{3/2} = 15\pi/8 = 5.9. \quad (13)$$

However, to increase the accuracy of the experiment, it is desirable to work with much stronger fields, in order that the relative change of the resistance $\Delta\rho_H/\rho_0$ be closer to unity. In this case it is necessary to make use of the general formula (9). In scattering by atoms, if Q = const, we get

$$\frac{\sigma'_H}{\sigma_0} = J(\theta), \qquad J(\theta) = \int_0^\infty e^{-t}\frac{t^2\,dt}{\theta + t}, \quad (14)$$

where

$$\theta = \gamma_{-1/2}\,(uH/c)^2, \qquad u = 4el/3\sqrt{2\pi mkT}. \quad (15)$$

The ratio σ_H/σ_0 is plotted in Fig. 1 as a function of uH. Making use of this curve, we can find u from the measured value of σ_H/σ_0, and hence determine the free path of the electrons l and the scattering cross section Q. However, the measured value of the electrical conductivity,

$$\sigma_H = jd/V = j/E = \sigma'_H E'/E \quad (16)$$

is in principle not identical with σ'_H, since the effective field **E′** also includes the Lorentz force in addition to **E**. For calculation of **E′**, it is necessary to find $\mathbf{v}_0$. The equation of motion for a weakly ionized plasma has the usual form

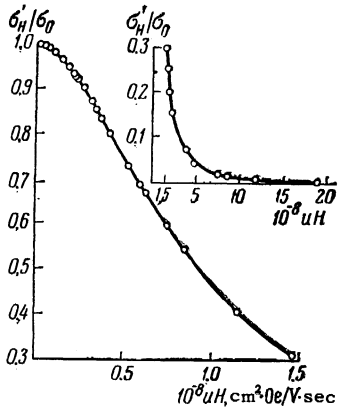

FIG. 1.

$$-\eta\nabla^2 \mathbf{v}_0 = \frac{1}{c}[\mathbf{jH}], \quad (17)$$

where η is the viscosity of the gas. In this case it is assumed that the gas flow is laminar. In the case of turbulent flow, the mean velocity of the gas will be less and, correspondingly, the Lorentz force will be less.

In our case $j = j_x$, $H = H_z$, and the solution of this equation for the boundary conditions $v_0 = 0$ for x = 0 and x = d has the following form:

$$v_0 = -(jH/2\eta c)\,x\,(d - x). \quad (18)$$

Consequently, according to (4),

$$E' = E - (jH^2/2\eta c^2)\,x\,(d - x). \quad (19)$$

Since the current density j is constant, then E′ no longer depends on x. Substituting $j = \sigma'_H E'$ and integrating this equation over x from zero to d, we obtain

$$E/E' = (1 + \sigma_H H^2 d^2/12\eta c^2)^{-1}. \quad (20)$$

We have not considered here the contribution of the ion current to the total current. This is permissible for weak magnetic fields, where the electron current far exceeds the ion current, i.e., for $uH/c \ll (M/m)^{1/4}$, where M is the ion mass. Consequently, according to (16) and (20),

$$\sigma_H/\sigma_0 = [\sigma_0/\sigma'_H + \sigma_0 H^2 d^2/12\eta c^2]^{-1}, \quad (21)$$

where σ'_H/σ_0 is determined by Eq. (9).

For an estimate of the role of the second term in the square brackets in (21), which is connected with the hydrodynamic motion of the plasma, we consider the limiting cases. For weak magnetic fields,

$$\sigma_H/\sigma_0 = -\gamma\,(uH/c^2)\,(1 + end^2/12\eta u\gamma). \quad (22)$$

In strong magnetic fields, ($uH/c \gg 1$) for l = const,

$$\sigma'_H/\sigma_0 = (2/\gamma_{-1/2})\,(c/uH)^2, \quad (23)$$

whence

$$\frac{\sigma_0}{\sigma_H} = \frac{\gamma}{2}\left(\frac{uH}{c}\right)^2\left(1 + \frac{end^2}{6\eta u\gamma}\right). \quad (24)$$

Consequently, both here and in the other case the relative contribution of the second term is characterized in practice by the quantity

$$\zeta = end^2/12\eta u\gamma. \quad (25)$$

For an estimate of this quantity, we make use of the formula

$$\eta = \tfrac{1}{5} N \langle \tau^2\varepsilon^2\rangle/\langle\tau\varepsilon\rangle, \quad (26)$$

where N is the concentration of the atoms and τ is their relaxation time. If the free path of the atoms $l_a = 1/NQ_a$ does not depend on the energy, then

$$\eta = \tfrac{3}{20} N l_a \sqrt{2\pi MkT}. \quad (27)$$

If we substitute u in the form (15), then

$$\zeta = (20/27\,\pi)\,(n/N)\,(m/M)^{1/2} d^2/l l_a.$$

For cesium vapor, $(m/M)^{1/2} = 1/500$, and $\zeta = 4.7 \times 10^{-4} nNQ_aQ_ed^2$. For cesium, $Q_a \approx 2.35 \times 10^{-13}$ cm^2,[8] $Q_e \approx 2 \times 10^{-14}$ cm. Under the conditions of our experiment, even under the "worst" conditions, when p = 2 mm mercury, T = 2300° K and $d = 4 \times 10^{-2}$ cm, we get $\zeta \approx 10^{-2}$, i.e., the contribution of the second term in (21) is insignificant; we neglect it and assume

$$\sigma_H/\sigma_0 = \sigma'_H/\sigma_0.$$

At the same time, for very high pressures and temperatures, and also for large gaps, this term can become the principal one. Thus, for p = 10 mm mercury, T ~ 2500° K, we have $\zeta = 1$ if d = 1 mm. For $\zeta \gg 1$, the value of σ_H/σ_0 should depend exponentially on the temperature and should also be proportional to the square of the interelectrode distance d, while for $\zeta \ll 1$ the ratio σ_H/σ_0 does not depend on the value of the gap and depends weakly on the temperature (thus, for l = const and $uH/c \ll 1$, we have $\Delta\sigma/\sigma \sim 1/T$). Therefore the presence of the described effect can easily be detected.

EXPERIMENTAL RESULTS

We carried out measurements of the resistance of cesium plasma as a function of the magnetic field at different pressures and temperatures. These experiments showed that the dependence of σ_H/σ_0 on the magnetic field is in excellent agreement with theory. Values of the quantity u, computed from Eq. (14) from three values of σ_H/σ_0, corresponding to three different magnetic fields (for T = 1625° K and p_{Cs} = 0.4 mm mercury) are shown in the table. While the value of uH in these experiments was varied by a factor of two, the values of u remain the same within ten per cent, i.e., within the limits of accuracy of the experiment.

	H=66 Oe	H=90 Oe	H=126 Oe
σ_H/σ_0	0.96	0,93	0.86
10^{-8} uH, cm^2·Oe/V·sec	0,161	0,22	0,33
10^{-5} u, cm^2/V·sec	2,4	2,4	2,6

The experimentally obtained values of the mobilities are plotted in Fig. 2 as a function of the temperature. These measurements were carried out for a cesium vapor pressure of 0.93 mm mercury and an interelectrode gap equal to 0.38 mm. The amplitude of the ac voltage applied between the electrodes amounted to 0.1 — 0.15 V. In this case, a linear dependence of the current on the voltage was observed over the entire interval on the oscillogram. The maximum value of the current changed from 1 mA at low temperature to 100 mA at high temperatures.

The crosses in Fig. 2 denote the values of the mobility u_R computed [according to Eq. (15)] from the directly measured values of the electrical conductivity in the absence of a magnetic field, and the circles denote the values of u_H found from measurement of the resistance in the magnetic field [according to Eqs. (21) and (14)]. The term that takes into account the motion of the plasma in (21) has been neglected.

It is seen that both methods give practically the same results in the region of temperatures up to 1800° K. For higher temperatures, the values of the mobility u_R begin to become appreciably smaller while the u_H decrease much more slowly. The great decrease in the mobility with increase in temperature can in principle be associated with the significant contribution made by scattering on ions, as was assumed in [9]. However, in this case, the mobility determined from measurements of the

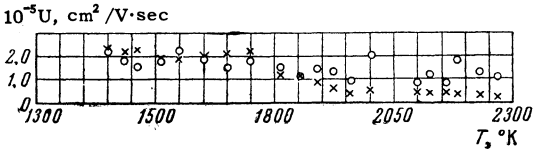

FIG. 2.

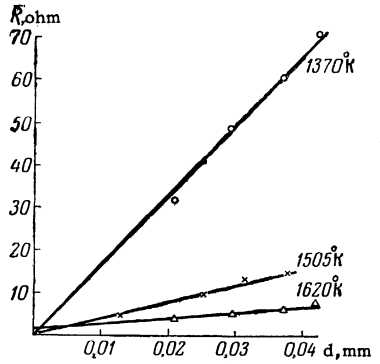

FIG. 3.

resistance in the magnetic field should also decrease appreciably. For a significant contribution of scattering from ions, Eqs. (23) and (15) will no longer be valid, and to calculate u_R and u_H it is necessary to make use of the more general formulas (8a) and (9). Therefore, as is seen for example from (12) and (13), the values of u_R and u_H calculated from approximate formulas can differ by a factor of 1.5 — 2, but their temperature dependence should be approximately the same. Actually, the difference between u_R and u_H is stronger for temperatures $\sim 2000°$ K. Moreover, the actual decrease in σ_0 is much greater than follows from the theoretical estimates of the scattering cross sections on ions.[10]

To clarify the reason for the decrease in the mobility with increase in the temperature, we measured the dependence of the resistance of the interelectrode region on the value of the gap d. For this purpose, special apparatus was prepared in which the distance between the electrodes could be varied over wide limits. The dependence of the resistance R on d for different temperatures is shown in Fig. 3. It is seen from the graph that for temperatures 1300 — 1500° K the value of R increases linearly with d and the straight lines R(d) go to zero for d = 0. For T = 1600° K and above, the straight lines R(d) intercept the R axis at a point $R_0 > 0$. These data show that the resistance measured at low temperatures is actually the volume resistance of the plasma.

As to the additional resistance R_0 which occurs upon increase in temperature, it is explained by the effect of the layers next to the electrode.

As already noted,[5,4] in measurements of the resistance of the plasma it is necessary that the work function of the electrode be close to the chemical potential of the plasma. More detailed calculations, carried out with the help of E. Sonin, showed that these measurements give the correct value of the mobility even when the work function of the electrodes is less than the chemical potential of the plasma, i.e., when the pre-electrode layers are enriched with electrons. In the opposite case, i.e., for large work function of the electrodes, layers that are deficient in electrons are formed near them which can make a significant contribution to the measured resistance. For low temperatures, the work function χ of tungsten covered with cesium is less than the chemical potential of the electrons μ. Upon increase in temperature, the adsorption coefficient of cesium decreases, the work function of tungsten increases, and for $\chi > \mu$ an additional resistance R_0 appears, which increases with increase in the difference $\chi - \mu$. The corresponding calculations, and also a detailed description of the experiments on the measurement of the dependence of the resistance on the value of the gap, will be published separately. Here we note only that the resistance of the pre-electrode layer R_0 should change much less in the magnetic field than the resistance of the volume R', since the thickness of this layer is much less than the mean free path of the electrons. Therefore, one can assume that R_0 does not depend on H. For measurements in the region T > 1600° K, the condition $uH/c \ll 1$ is satisfied; consequently, according to (22), the ratio of the cross section Q_H determined from the change in the resistance in the magnetic field to the true cross section Q_0 is equal to $Q_H/Q_0 = (R/R')^{1/2}$, where $R = R_0 + R'$ is the measured total resistance. At the same time, for the cross section Q_R determined from resistance, we have $Q_R/Q_0 = R/R'$, whence

$$Q_H/Q_0 = (Q_R/Q_0)^{1/2}. \qquad (28)$$

Figure 4 shows the dependence of the scattering cross section on the temperature, computed according to Eq. (15) from the data of Fig. 2. The crosses denote the values of Q_R and the circles Q_H. For T < 1600° K, both methods give a value $Q_0 \approx (3-4) \times 10^{-14}$ cm^2.

A sharp rise is observed in Q_R and a weaker rise in Q_H for increase in the temperature. We extrapolated the curve $Q_R(T)$ by the solid curve in Fig. 4 and, using it, have plotted $Q_H(T)$ according to Eq. (28) for $Q_0 = 3 \times 10^{-14}$ cm^2. The corresponding curve is shown in Fig. 4 by a dashed line. It is seen that the experimental points $Q_H(T)$ actually lie close to this curve.

The noticeable scatter in the values of the cross section, especially those obtained from the value of the electrical conductivity, is connected in a fundamental way with the error in the determination of the equilibrium concentration as a consequence of the possible errors in the measurement of the

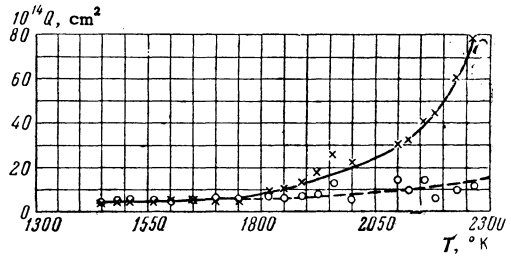

FIG. 4.

electrode temperatures, and also of the saturated vapor pressure of cesium. Thus, for $T \sim 2000°$ K, a 1 per cent error in the measurement of the electrode temperature leads to a ~ 10 per cent error in n. The errors in the determination of Q_H are connected fundamentally with errors in the measurement of small changes in the resistance for weak magnetic fields; to reduce them, it is necessary to work with such fields that the value of the change in the resistance $\Delta\rho_H$ would be of the same order as ρ_0.

At the present time we are carrying out additional research on increasing the accuracy of the experiment.

The calculation carried out above shows that one can find R_0 and determine the real mobility by measuring ΔR_H. However, a knowledge of the concentration of carriers is required for this purpose. Therefore, it is desirable to carry out measurements under such conditions when layers that are deficient in electrons are not formed near the electrodes. Under these conditions, as the data obtained by us confirm, the mobility can be determined directly from the change in the resistance of the plasma in a magnetic field. The true value of the mobility can be determined even in the case when such layers exist, from the slope of the lines $R(d)$ or $(\Delta R/R)_H = f(d)$. We note that these methods can be employed for study of the electron mobility in other gases, for which it is necessary to employ their mixture with cesium vapors as was assumed in [4].

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