DIRECT NUCLEAR PHOTOEFFECT AND THE OPTICAL MODEL OF THE NUCLEUS

Yu. V. ORLOV

Nuclear Physics Institute, Moscow State University

Submitted to JETP editor August 4, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 42, 247-252 (January, 1962)

Simple formulas are obtained in the dipole approximation for the cross section and polarization of the direct nonresonant photonuclear effect, using the shell model with jj-coupling. The final state interaction is described by means of a complex optical model potential. For the square well potential, numerical computations are made which enable one to judge the role of various features of the interaction.

1. In the present paper we continue an investigation previously begun[1] of the direct nonresonant nuclear photoeffect.* As previously, the problem is treated within the framework of the jj-coupling shell model in the dipole approximation. The interaction of the emerging nucleon with the nucleus is described by means of the complex potential of the nuclear optical model. Rectangular potential wells are used for both the initial and final states. The purpose of this work was to study the influence of various features of the interaction on the cross section and polarization for the direct photonuclear effect.

The numerical computations enabled us to explain the role of spin-orbit interaction, the effect of the imaginary part of the potential, and the contribution of the region inside the nucleus to the dipole matrix element. The dependence of the computational results on the parameters of the potential, and especially on the nuclear radius R, turns out to be very strong. This imposes strict requirements on the choice of potential and indicates that computations which use too crude approximations are of little value.[2-4] Wherever possible we use the same notation as in [1].

2. The cross section for the direct photonuclear effect, which has previously been given in a complicated form,[1,5] can be written in a form which is more convenient for actual computations (with the dependence on l shown explicitly):

$$d\sigma/d\Omega = \sum_{(njl)} (d\sigma/d\Omega)_{njl}, \qquad j = l \pm 1/2;$$

$$\begin{aligned}
(d\sigma/d\Omega)_{njl} &= A_{njl} + B_{njl}\sin^2\theta \\
&= \beta\varepsilon^2 MKk\,[2\hbar^2(2l+1)^2]^{-1}(\bar{A}_{njl} + \bar{B}_{njl}\sin^2\theta);
\end{aligned}$$

$$\begin{aligned}
\bar{A}_{nl\pm 1/2 l} = {} & 2\,[2(l\pm1)+1]^{-2}\,[(2l+1)^2(l\pm1) \\
& \times (l\pm1+1)\langle 3/2, 3/2\rangle + 2\,[2(l+1)(l\pm1) \\
& \mp 1]\langle 1/2, 1/2\rangle - (2l+1)(2l+1\pm3)\langle 3/2, 1/2\rangle] \\
& + (2l+1\mp1)\,[2(l\pm1) \\
& + 1]^{-1}\,[(2l+1\pm3)(2l+1)\langle 3/2, -1/2\rangle \\
& + 2\langle 1/2, -1/2\rangle] + 2l(l+1)\langle -1/2, -1/2\rangle,
\end{aligned}$$

$$\begin{aligned}
\bar{B}_{nl\pm 1/2 l} = {} & [2(l\pm1)+1]^{-2}\,[(2l+1)^2(l\pm2) \\
& \times (l\pm2+1)\langle 3/2, 3/2\rangle \\
& - (2l+3\pm1)(2l-1\pm1)\langle 1/2, 1/2\rangle \\
& + 3(2l+1)(2l+1\pm3)\langle 3/2, 1/2\rangle] \\
& - 3/2\,(2l+1\mp1)\,[2(l\pm1) \\
& + 1]^{-1}\,[(2l+1\pm3)(2l+1)\langle 3/2, -1/2\rangle \\
& + 2\langle 1/2, -1/2\rangle] + (l\mp1)(l\mp1+1)\langle -1/2, -1/2\rangle;
\end{aligned}$$

$$\langle j_1, j_2\rangle \equiv \mathrm{Re}\,(a^*_{l\pm j_1} a_{l\pm j_2}), \qquad a_l \equiv \int_0^\infty \varphi^*_{lL}(r)\, R_{njl}(r)\, r^3 dr; \qquad (1)$$

$\beta = 1$ if the subshell contains one particle, $\beta = 2$ for the case of two particles in the subshell, $\beta = 2j$ if there is one "hole" in the subshell, and $\beta = 2j + 1$ if the subshell is filled.

The corresponding expression for the polarization of nucleons emerging from a given subshell (njl) also has a simple form:*

*In [1], the formulas for the cross section and the corresponding numbers in the table should be multiplied by ¼. In formula (2), $\sqrt{2EV_0}R$ should be replaced by $\sqrt{2E_{hl}V_0}R$. The numbers given in the table for E = 3 and 5 MeV are incorrect, the correct values being those shown in Fig. 2 of the present paper.

*The method for calculating the polarization of the nucleons in the photoeffect and some specific computations are given in [9]. A complicated expression for the polarization was given by Francis et al.[7]

$$P_{njl} = \sin 2\theta (2l+1) \{4 [2 (l \pm 1) + 1] (\bar{A}_{njl} + \bar{B}_{njl} \sin^2\theta)\}^{-1} \{-(2l+1 \mp 1) \times \overline{\langle 1/2, -1/2 \rangle} + (2l+1 \pm 3) [(2l+1 \mp 1) \times \overline{\langle 3/2, -1/2 \rangle} \pm \overline{\langle 3/2, 1/2 \rangle}]\},$$

$$j = l \pm 1/2; \qquad \overline{\langle j_1, j_2 \rangle} \equiv \mathrm{Im} (a^*_{l \pm j_1} a_{l \pm j_2}). \tag{2}$$

The polarization vector is perpendicular to the plane formed by the vectors **K** and **k** (the positive direction being that of **K** × **k**). Formula (2) reflects the interference nature of the polarization. In the absence of spin-orbit interaction, the occurrence of polarization is caused by interference of the waves with $L = l + 1$ and $L = l - 1$:

$$P_{njl} = \pm \sin 2\theta (2l+1) \{4 (\bar{A}_{njl} + \bar{B}_{njl} \sin^2\theta)\}^{-1} (2l + 1 \mp 1) \times \mathrm{Im} (a^*_{L=l+1} a_{L=l-1}), \tag{3}$$

$j = l \pm 1/2$. From formula (3) it follows immediately that for neutrons from the s-shell, the polarization is equal to zero if we neglect spin-orbit interaction.

For potentials V_0 and V_f of rectangular shape (we assume their radii are the same)

$$V_{0,f}(r) = -U_{0,f} [f(r) + (\lambda_{0,f}/R) (\sigma \mathbf{L}) \delta (r - R)],$$

$$f(r) = \begin{cases} 1, & 0 \leqslant r < R \\ 0, & r > R \end{cases} \tag{4}$$

the radial matrix element can be obtained in analytic form; because of their complexity, we shall not give the explicit expressions.

3. Numerical computations were made for various values of the parameters of the potential (cf. the table), for the case of the photoeffect on C^{12} with emission of neutrons with energies up to 16 MeV. The parameters of the potential well for the initial state were determined from the separation energy of the neutron.

Variant	R, F	U_0, MeV	λ_0, F^2	U_f, MeV
I_a	3.88	36.85	0.38	40
I_b	3.88	37.76	0	40
II	3.32	43.77	0	42

Two different sets of parameters in the complex potential were considered. In the first of these we used the data from the paper of Kawai et al.;[8] the energy dependence of the imaginary part of the potential was chosen in the form $W = \zeta U_f = -(AE^2 + BE + C)$. The values $A = -0.0085$ MeV $B = 0.613$, $C = -0.185$ MeV were determined by the method of least squares from the data of Kawai et al.[8] The parameters in the second variant were taken from the paper of Feshbach, Porter, and Weisskopf,[9] and a constant value of $\zeta = -0.15$ was used. The value $\lambda_f = 0.35 \times 10^{-26}$ cm^2 was taken from the work of Levintov.[10]

These two sets of parameters lead to results which differ in order of magnitude (cf. Figs. 1 and 2 where, as in all other figures, we use the orbital angular momentum l of the shell as the index in place of (njl). This marked dependence on the well parameters is apparently related to the use of rectangular potential shapes. Here the matrix element of the dipole operator is expressed in terms of spherical Bessel functions, which oscillate, and this can result in a sharp drop in cross section for certain parameter values. Precisely this situation arises when one uses the parameters of Kawai et al[8] and radii of 3.8 and 4 Fermis, which are ascribed by them to the C^{12} and O^{16} nuclei (we have neglected the difference in radius of C^{12} and C^{11}). Naturally in such cases the dependence on the parameters may be even stronger.

In Figs. 1 and 2 we show the effect of the imaginary part of the potential (the quantity B_0 is not shown in Fig. 2 because it is small compared to A_1 and B_1). Including the absorption of the neutron in nuclear matter leads to a significant reduction of the reaction cross section and can even change the shape of the spectrum. Spin-orbit in-

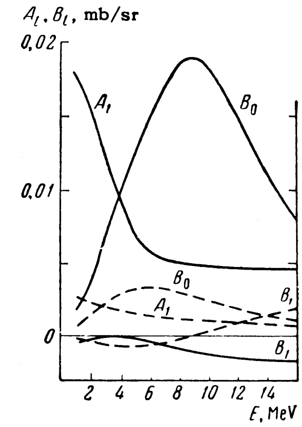

FIG. 1. Dependence of A_l and B_l in (1) on neutron energy for variant I_b ($\lambda_f = 0$). The solid curve is for $\zeta = 0$, the dashed curves for $\zeta \neq 0$.

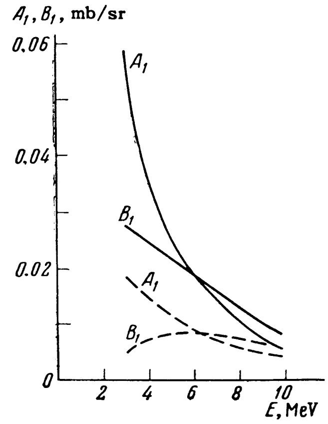

FIG. 2. Dependence of A_1 and B_1 on neutron energy for variant II ($\lambda_f = 0$). Solid curves for $\zeta = 0$, dotted curves for $\zeta = -0.15$.

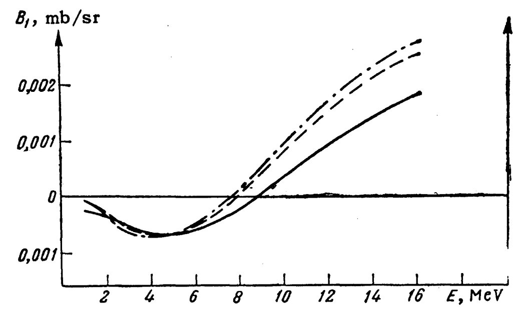

FIG. 3. Influence of spin-orbit interaction on the quantity B_1. The solid line is variant I_b, $\lambda_f = 0$; the dashed line is variant I_a, $\lambda_f = 0.35\,(\text{Fermi})^2$, the dot-dashed line is variant I_a, $\lambda_f = 0$.

teraction produces only a comparatively small change in the differential cross section for the direct photoeffect. (Cf. Fig. 3. The dependence of A_1 and B_0 on the spin-orbit interaction is weaker and is therefore not shown.) Even the polarization of the emerging neutrons, which should be most sensitive to this interaction, up to a neutron energy of ~ 10 MeV is changed only slightly by including it (Fig. 4). Possibly this is connected with the use of the spin-orbit potential (4) in the form of a δ function. Figure 4 shows the percentage polarization for $\theta = 45°$; for arbitrary θ it can be gotten from the formula

$$P_l(\theta) = P_l(45°)\sin 2\theta\left(A_l + \tfrac{1}{2}B_l\right)/(A_l + B_l\sin^2\theta). \qquad (5)$$

An estimate of the contribution to the direct nonresonant nuclear photoeffect from the regions inside and outside the nucleus shows (Fig. 5) that each region by itself does not give a contribution which is close to the total value.

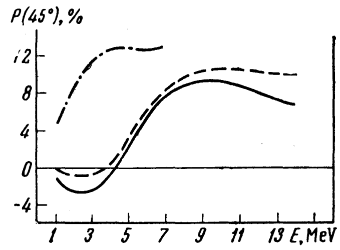

FIG. 4. Percentage polarization of neutrons from the direct photoeffect, emitted at 45° to the direction of propagation of the γ quantum (theoretical curves). The solid line is variant I_a, $\lambda_f = 0.35\,(\text{Fermi})^2$, the dashed line is variant I_a, $\lambda_f = 0$, the dot-dashed line is variant II.

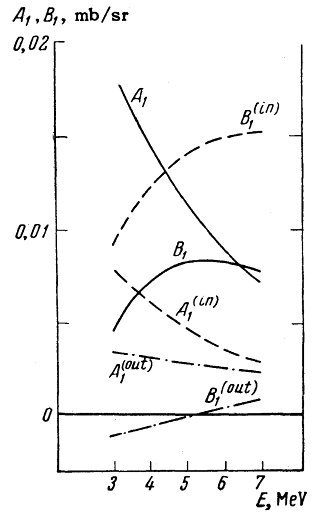

FIG. 5. Dependence of A_1 and B_1 on neutron energy for different regions of the radial integration. Solid line – region from zero to infinity; dashed line – the region inside the nucleus; the dot-dashed line – the region outside the nucleus.

In some papers[3,4,7] in computing the matrix element only the region outside the nucleus is included ("surface effect"). In doing this one usually relies on the attenuation of the wave function of the emergent particle inside the nucleus because of the absorption which is introduced via the imaginary part of the potential. However the rapid falloff of the wave function of the bound nucleon in the region outside the nucleus should in its turn lead to a reduction of the matrix element in the external region, which explains the result found in the present work from a direct computation.

Comparison of the computational results with experiment[11] shows that in the case of variant II (cf. the table) the computation reproduces the falling energy spectrum of the neutrons and gives a fair description of the angular distributions for energies of 3 – 7 MeV (Figs. 6 and 7). The absolute values of the cross sections are more than an order of magnitude smaller than the experimental values. This may arise from the important part played by the resonance effects, predicted by Wilkinson,[12] which are related to the existence of well separated quasistationary states in this region of energy of the γ quanta. But there are at least three factors which would apparently help give a larger cross section for the direct nonreso-

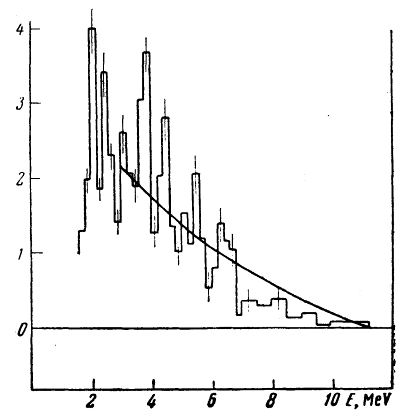

FIG. 6. Comparison of the theoretical shape of the energy spectrum of photoneutrons from the direct process (variant II) with experiment [11] (in arbitrary units). The theoretical curve is normalized to the area of the histogram starting from E = 3 MeV.

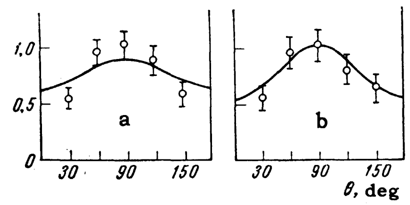

FIG. 7. Comparison of theoretical neutron angular distributions (variant II) with experiment [11] (in arbitrary units): a) $3 < E \leq 5$ MeV, $\sigma(\theta) \sim 1 + 0.45\sin^2\theta$; b) $5 < E \leq 7$ MeV, $\sigma(\theta) \sim 1 + 0.88\sin^2\theta$.

nant photoeffect on C^{12}. These are a further reduction of the nuclear radius (the radius $R = 3.32$ F corresponds to the formula $R = 1.45\, A^{1/3}$ F), taking into account the diffuseness of the nuclear boundary (cf., for example, [13]) and reducing the value of $|\zeta|$, which should occur for nuclei close to closed shells.[14]

From Fig. 2 it follows that reducing ζ to zero does not spoil the agreement of the angular distributions with experiment. Thus by suitably modifying the model we may hope to obtain better agreement with experiment, but the solution of this problem requires further investigation.

It should be noted that, according to the Wilkinson model, no polarization should result from the photonuclear reaction, since the emerging nucleon owes its appearance either to the decay of the compound nucleus or to a direct resonance effect in which the angular momentum of the emerging particle is fixed and consequently there is no interference between states of different angular momentum. Thus the investigation of the polarization of the photonucleons may give further valuable information about the mechanism of the photonuclear process.

In conclusion I take this opportunity to express my profound gratitude to Profs. I. S. Shapiro and V. V. Balashov for discussion of some of the results of the present work, and also to M. Ogareva who did the programming and computation of the quantities describing the neutron wave function in the complex potential.

[1] Yu. V. Orlov, JETP **37**, 1834 (1959), Soviet Phys. JETP **10**, 1294 (1960).

[2] S. Sueoka, Can. J. Phys. **37**, 232 (1959).

[3] V. DeSabbata and A. Tomasini, Nuovo cimento **13**, 1268 (1959).

[4] Agodi, Eberle, and Sertorio, Nuovo cimento **13**, 1279 (1959).

[5] S. Fujii, Progr. Theoret. Phys. (Kyoto) **21**, 511 (1959).

[6] W. Czyz and J. Sawicki, Nuovo cimento **3**, 864 (1956).

[7] Francis, Goldman, and Guth, Phys. Rev. **120**, 2175 (1960).

[8] M. Kawai et al., Progr. Theoret. Phys. (Kyoto) **18**, 66 (1957).

[9] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

[10] I. I. Levintov, JETP **30**, 987 (1956), Soviet Phys. JETP **3**, 796 (1956).

[11] Emma, Milone, and Rubbino, Phys. Rev. **118**, 1297 (1960).

[12] D. H. Wilkinson, Physica **22**, 1039 (1956).

[13] N. K. Glendenning, Phys. Rev. **114**, 1297 (1959).

[14] Lane, Lynn, Melkonian, and Rae, Phys. Rev. Letters **2**, 424 (1959).

Translated by M. Hamermesh
37