A POSSIBLE EXPLANATION OF THE VARIATION OF THE DIFFERENTIAL CROSS SECTION FOR THE (p, α) *AND* (α, p) *REACTIONS IN THE LARGE ANGLE REGION*

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In order to explain the peaks in the differential cross sections of the (p, α) and (α, p) reactions in the large-angle region, a direct process with local interaction is proposed, that is, a process in which the impinging particle interacts with a small part of the initial nucleus. An approximate calculation of the angular distributions for this process is performed for the F^{19} (p, α) O^{16} reactions.

1. INTRODUCTION

It has been recently shown in a number of experiments that in the angular distributions of particles produced in the (p, α)[1-3] and (α, p)[4-6] reactions the differential cross section is observed to have a maximum in the region of 180°, i.e., in the direction opposite to the direction of motion of the incident beam (these maxima can be called backward peaks). The experimental data allowed us to draw several conclusions on the properties of the backward peaks: 1) the amplitudes of these peaks depend on the state of the product nucleus and on the energy of the bombarding particle, where in some cases the dependence on the energy is very strong (for example, in the F^{19} (p, α) O^{16} reaction[1,2]); 2) the amplitudes of the backward peaks can attain a very high value, exceeding the value of the differential cross section maxima in the low-angle region; 3) the increase in the cross section in the large-angle region is quite rapid (the peak frequently begins at angles greater than 120 — 140°), and the rate of rise apparently increases with the energy; 4) no strong change in the width of the backward peak occurs with an increase in the mass of the nucleus.

Since the backward peaks occur in many cases, including the (p, α) and (α, p) reactions, and since these peaks are especially evident over a comparatively wide energy region (from 5.5 to 14.5 MeV for the (p, α) reaction on fluorine and from 16 to 31 MeV for the (α, p) reaction on carbon), it is difficult to suggest that they can be explained by a reaction mechanism associated with the production of a compound nucleus or interference of this mechanism with ordinary direct processes. On the other hand, the direct processes of

Reaction	E_i	$\Delta(k_1R_1)$	$\Delta(k_2R_2)$
$C^{12}(\alpha, p)N^{15}$	16	1.8	1.3
	32	3.0	1.9
$F^{19}(p, \alpha)O^{16}$	6	1.3	1.1
	14	2.0	1.4
$Al^{27}(p, \alpha)Mg^{24}$	6	0.9	0.55
	14	1.4	0.8

the knock-out or capture type can not explain the occurrence of the backward peaks, at least not in the framework of the Born approximation with plane waves. Of the known direct processes, a backward peak of the cross section is characteristic only for the stripping of a heavy particle.[7] This mechanism, however, can not explain the sharp rise of the differential cross section in the 180° region as is observed in many experiments. In fact, in the case of the stripping of heavy particles, the angular distribution of the cross section is determined approximately by the squares of spherical Bessel functions with the arguments k_1R_1 and k_2R_2, where the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ have the form

$$\mathbf{k}_1 = \mathbf{k}_f + (m_f/M_i)\,\mathbf{k}_i, \qquad \mathbf{k}_2 = \mathbf{k}_i + (m_i/M_f)\,\mathbf{k}_f, \tag{1}$$

here M_i and M_f are the mass numbers of the initial and final nuclei, and m_i and m_f refer to the incident and produced particles. As the angle of emission changes from 180° to 0° the argument k_1R_1 increases by the value $\Delta(k_1R_1) = 2(m_f/M_i)k_1R_1$; similarly, $\Delta(k_2R_2) = 2(m_i/M_f)k_2R_2$. The values for several reactions are shown in the table (it has been assumed that $R_1 = R_2 = 6F$). Since the distance between the zeros of the spherical Bessel functions is greater than π, then the width of the maximum is about

180° only for the reaction $C^{12}(\alpha, p)N^{15}$ with $E_\alpha = 32$ MeV. For lower energies of the bombarding particles and for larger masses of the initial nuclei, the width of the maximum is greater. Thus, the stripping of a heavy particle can account only for a broad backward maximum whose width rapidly increases with the mass of the target nucleus.

Recently Kromminga and McCarthy[8] attempted to explain the backward peaks qualitatively by the ordinary stripping process. For this, they used the Born approximation with distorted waves.

2. PROCESS WITH LOCAL INTERACTION

For reactions of the type $M_i (m_i, m_f) M_f$, where $m_i < m_f$, three types of direct interactions are considered at the present time: capture, knock-out,[9] and the stripping of a heavy particle.[7] These processes, however, do not exhaust the possible forms of direct processes. One can, in particular, assume the existence of a direct process involving the interaction of the incident particle (m_i) with a comparatively small part of the initial nucleus, with it's "substructure" (m), where the emitted particle is not this "substructure" as in the case of knock-out and not the particle "formed" from the impinging particle and the "substructure" as in the case of capture, but some association of nucleons (m_f) occurring in the nucleus in a "ready" form. The emission of this association can take place owing to the interaction in the nucleus with the remaining part of the nucleus, in particular, with the substructure (m) with which the incident particle interacts. This process can be called a process with local interaction. Since in this process, as in the case of the stripping of a heavy particle, the emitted particle did not interact directly with the initial particle, it should be expected that the most probable direction of emission will be opposite to the direction of motion of the incident particle. However, since the interaction takes place with a small part of the initial nucleus, then the backward peak should be comparatively narrow. We shall now calculate the cross section for this process with a local interaction in the Born approximation with plane waves. The matrix element for the transition between the initial and final states can be represented in the form

$$\langle V \rangle = \int \psi^*_{im}(\mathbf{r})\, \psi^*_{0(im)}(\mathbf{s})\, V(r)\, \psi_{0f}(\mathbf{R})\, \psi_{m(0f)}(\boldsymbol{\rho}) \times \exp\left[i(\mathbf{kr} + \mathbf{q}\boldsymbol{\rho} + \mathbf{QR})\right] d\mathbf{r}\, d\boldsymbol{\rho}\, d\mathbf{R}. \qquad (2)$$

The subscripts i, f, and m refer to particles m_i, m_f, and m, respectively, and the subscript 0 to that part of the nucleus not taking a direct part in the reaction, so that $M_i = M_0 + m_f + m$, $M_f = M_0 + m_i + m$. In (2) we have also introduced the notation:

$$\mathbf{R} = \mathbf{r}_0 - \mathbf{r}_f, \qquad \boldsymbol{\rho} = \mathbf{r}_m - \frac{M_0\mathbf{r}_0 + m_f\mathbf{r}_f}{M_0 + m_f}, \qquad (3)$$

$$\mathbf{r} = \mathbf{r}_i - \mathbf{r}_m, \qquad \mathbf{S} = \mathbf{r}_0 - \frac{m_i\mathbf{r}_i + m\mathbf{r}_m}{m_i + m}.$$

These vectors are not independent and are related to one another by

$$\mathbf{S} = -\boldsymbol{\rho} + \frac{m_f}{M_i + m}\mathbf{R} - \frac{m_i}{m_i + m}\mathbf{r}. \qquad (4)$$

The wave vectors occurring in the exponent of the integrand have the form

$$\mathbf{k} = \mathbf{k}_i + \frac{m_i}{M_f}\mathbf{k}_f, \qquad \mathbf{q} = \frac{M_i - m}{M_i}\mathbf{k}_i + \frac{m_i + m}{M_f}\mathbf{k}_f,$$

$$\mathbf{Q} = \frac{M_0(M_i + m_i)}{M_f(M_i - m)}\mathbf{k}_f. \qquad (5)$$

Comparing these expressions with the expressions for the wave vectors in the case of the stripping of a heavy particle,[1] we can conclude that $\mathbf{k} = \mathbf{k}_2$ and the vector $\mathbf{q}$ has the same character as $\mathbf{k}_1$, but changes more rapidly with the angle. The rate at which $\mathbf{q}$ changes with the angle depends on the relation between the masses of the "substructure" M_0 and m. The stripping of a heavy particle is a special case of the process with a local interaction, where $M_0 = 0$. Then $\mathbf{q} = \mathbf{k}_1$ and $\mathbf{Q} = 0$.

We consider the special case of the (p, α) reaction, where we assume that the initial nucleus can be represented in the form of several α particles plus a triton. The calculation made under

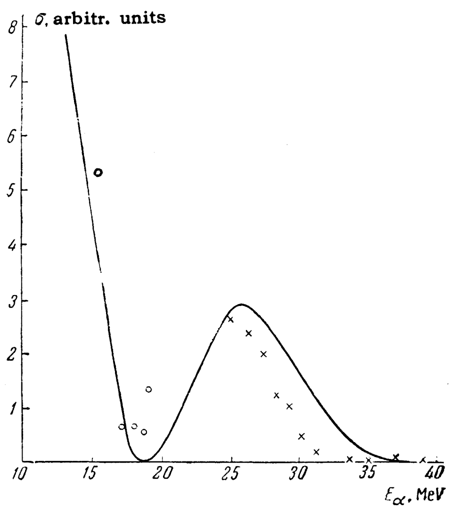

FIG. 1. Dependence of the cross section at 180° (solid curve) on the α-particle energy for the $C^{12}(\alpha, p)N^{15}$ reaction (R_0 = 6F, ρ_0 = 5F). The experimental points refer to the angle 170° (o – from [5], x – from [4]).

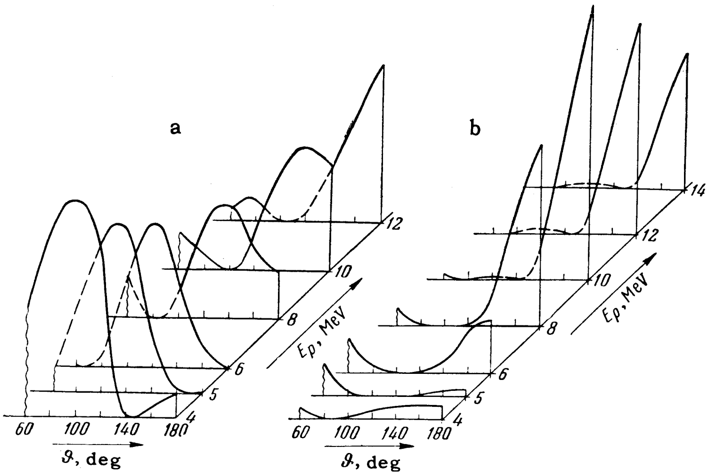

FIG. 2. Angular distributions of α particles for the $F^{19}(p,\alpha)O^{16}$ reaction for the following values of the parameters: a) $R_0 = 4.8\,F$, $\rho_0 = 5.7\,F$; b) $R_0 = \rho_0 = 4.8\,F$.

such conditions can be used, for example, for the (p, α) reaction on F^{19}, Al^{27}, N^{15} and also for the inverse reactions. For such reactions it is natural to assume that the impinging proton interacts with a "substructure" of the triton type and produces an α particle which remains in the nucleus. At the same time, a second α particle is emitted from the nucleus. In this case, $k_i = k_p$, $k_f = k_\alpha$, $m_i = 1$, $m = 3$, $m_f = m_\alpha = 4$.

In order to simplify expression (2) for the matrix element, we make several assumptions. We assume, first, that in the right-hand part of relation (6) we can neglect the term $[m_i/(m_i + m)]\,r = \frac{1}{4}r$. This is possible since the mean distance between the proton and triton in the α particle is comparatively small. As a consequence of this assumption, it proves to be possible to separate the integral over **r**. Second, we assume that the wave functions ψ occurring in (2) can be replaced by δ-functions, i.e., the moduli of the vectors **R**, **S**, and ρ can be considered to be constants ($R = S = R_0$, $\rho = \rho_0$). This assumption is a very crude one; keeping in mind, however, that the similar assumption in the case of ordinary stripping does not change qualitatively the form of the angular distributions, we can expect that in our case the qualitative picture also does not change. Finally, we limit ourselves to the case in which the final nucleus is produced in the ground state. Then the wave functions $\psi_{0\,(im)}$ and ψ_{0f} prove to be spherically symmetric.

Under these assumptions, we can readily carry out the integration in (2), and the expression for the matrix element takes the form

$$\langle V\rangle \sim (\varkappa^2 + k^2)\exp(-k^2/12\gamma^2)\sin\beta \sum_{L,l,M,m} i^{L+l}\sqrt{(2l+1)/(2L+1)},$$

$$P_L(\cos\beta)\, C^{ll_0L}_{000} C^{ll_0L}_{mm_0M}\, j_L(QR_0)\, j_l(q\rho_0)\, Y^M_L(\vartheta_Q, \varphi_Q)\, Y^{m*}_l(\vartheta_q, \varphi_q), \quad (6)$$

where

$$\varkappa^2 = \frac{2mm_i}{\hbar^2(m+m_i)}\varepsilon = \frac{3m_p}{2\hbar^2}\varepsilon,$$

$$\cos\beta = \frac{M_f R_0}{2m_\alpha \rho_0}\left(\frac{\rho_0^2}{R_0^2} - 1 + \frac{m_\alpha^2}{M_f^2}\right). \quad (7)$$

Here l_0 is the orbital angular momentum of the triton in the initial nucleus and ϵ is the binding energy of the protons in the α particle. The wave function of the α particle has been taken in the form[10]

$$\sim \exp\left(-\gamma^2 \sum_{i<j} r_{ij}^2\right) \quad (\gamma^{-1} = 4.5\,F)$$

With the aid of formula (6), we calculated the dependence of the cross section for the $C^{12}(\alpha, p)N^{15}$ reaction on the α-particle energy for protons emitted at 180° (Fig. 1) and also the angular distributions of α particles produced in the $F^{19}(p, \alpha)O^{16}$ reaction for several values of the proton energy between 4 and 14 MeV (Fig. 2). The values of the parameters were chosen as follows: $R_0 = 6.0$ F, $\rho_0 = 5.0$ F for the first reaction and $R_0 = 4.8$ F, $\rho_0 = 5.7$ F, and $R_0 = \rho_0 = 4.8$ F for the second reaction.

3. DISCUSSION

The angular distributions calculated for the $F^{19}(p, \alpha)O^{16}$ reaction (Fig. 2) indicate that the process involving a local interaction leads to the occurrence of backward peaks in the differential cross section. A more detailed picture of the behavior of the cross section in the large-angle region depends on the values of the parameters R_0 and ρ_0.

This is connected with the fact that the value of the angle β between the vectors **R** and ρ (see [7]), and, consequently, the relative weight of the individual terms of the sum in expression (6) depends on their ratio. When $R_0 = \rho_0$ (in which case $\cos\beta$

is close to zero), the angular distributions have peaks practically only at 180° (Fig. 2b). If the parameters R_0 and ρ_0 are not equal, then at some energies the peak in the cross section can occur at angles less than 180°, where the peak shifts towards the larger angles with an increase in the energy of the incident particles. The presence of such a peak was observed experimentally for the reaction on fluorine produced by 5 — 5.5 MeV protons.[2]

As seen from Fig. 1, the calculated energy dependence of $\sigma(180°)$ for the $C^{12}(\alpha, p)N^{15}$ reaction is in satisfactory agreement with the experimental data for the angle of emission 170°.[4,5]

Inasmuch as the calculations were made under very rough assumptions, the obtained results should be considered to be only qualitative. At the same time, these results indicate that the process with local interaction can explain the basic properties of the backward peaks mentioned in the introduction, at least for the type of reaction we have been considering (p, α). The estimate of the relative cross section for this process and the capture process indicates that their cross sections at the peaks can be of the same order. The amplitude of backward peaks due to processes with local interaction depends on the state of the final nucleus and also on the incident particle energy E_i. The dependence of $\sigma(180°)$ on E_i is mainly determined by the factor $j_L(QR_0)$ in (6), so that the distance between peaks is 5 — 15 MeV. According to the experimental data, the width of the backward peak decreases as the energy of the bombarding particle is increased. As regards the change of these peaks with the mass of the target nucleus, the process with local interaction should lead to their broadening with the mass, although, more weakly than in the case of stripping of a heavy particle.

Hence the process with local interaction can explain the occurrence of peaks in the differential cross section in the large-angle region. At the same time, the backward peaks can apparently be explained by the ordinary stripping process with distorted waves.[8] At the present time, however, no definite conclusions can be made as regards which process is responsible for the occurrence of backward peaks in different cases. To answer this question it is necessary to carry out a more detailed analysis of both processes and to increase the amount of experimental data.

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