SPACING OF NUCLEAR ENERGY SURFACES

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Submitted to JETP editor June 17, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 1852-1858 (December, 1961)

A study of the distances between nuclear energy levels derived from experimental data reveals that the mean distances decrease approximately as $A^{-1/2}$. It is confirmed that, on the average, between an even-even surface and the surface of a nucleus with odd mass number A is greater than the distance between an odd-odd surface and the surface of a nucleus with odd A. These mean distances are given by the empirical formula (11). The distances between the surfaces are found to increase for magic numbers and on the boundaries of regions of deformed nuclei.

NUCLEAR energy surfaces are surfaces in a space with coordinates Z, N, and E_b where Z — number of protons, N — number of neutrons, and E_b — binding energy of the given nucleus. The binding energy E_b is the sum of the masses of the nucleons comprising the nucleus minus the mass of the nucleus. We distinguish between four energy surfaces, depending on the parities of Z and N: even-even, even-odd, odd-even, and odd-odd. As is well known, the second and third energy surfaces almost merge into one in the case of nuclei with odd mass number A, whereas the even-even and odd-odd surfaces are located at a considerable distance from the surface for odd A. We consider here only the energy surfaces obtained from experimental data.

The experimental values of the binding energy E_b have been calculated from the best nuclide mass values known on June 1, 1960. The binding energy of light nuclei with $A \leq 70$ are taken from the tables of Everling et al, [1] while those for medium masses are taken from the tables of Wapstra [2] with certain modifications. The binding energies of the nuclei from Xe to Eu were taken from the paper of Johnson and Nier, [3] while the values from Hf to Fr are taken from tables [4] calculated by the author from mass-spectroscopic measurements [5,6] and from the reaction and decay energies. The binding energies of nuclei heavier than Fr are taken from the tables of Foreman and Seaborg. Although listed in Wapstra's tables, the masses of the nuclides in the region from Ru to Xe were insufficiently accurate for use.

Nuclei with different parities have different binding energies because of the presence of pairing energy. The neutron pairing energy (the energy released when the (N + 2)-nd and the (N + 1)-st neutrons form a pair) is defined as

$$P_n(Z, N+2) = B_n(Z, N+2) - B_n(Z, N+1), \quad (1)$$

where Z and N are even and $B_n(Z, N)$ is the binding energy of the last (N-th) neutron or the energy necessary to remove this neutron from a nucleus with Z protons. The neutron pairing energy can be expressed in different fashion, directly in terms of the total binding energies:

$$P_n(Z, N+2) = 2\left\{\tfrac{1}{2}[E_{ee}(Z, N+2) + E_{ee}(Z, N)] - E_{eo}(Z, N+1)\right\} = 2S_n(Z, N+1), \quad (2)$$

where E_{ee} — binding energy of even-even nuclei and E_{eo} — binding energy of even-odd nuclei.

As shown in [7], the neutron pairing energy provides an approximate expression for double the distance S_n between the even-even and even-odd surfaces. Figure 1 shows the intersection between the plane Z = 26 and the even-even and odd-odd surfaces, with reduced slope. The ordinates (in Mev) are

$$E_0 - E_b = 9(Z + N) - E_b(Z, N). \quad (3)$$

It is seen from Fig. 1 that $S_n(Z, N+1) = AL$ is the distance between the even-even and even-odd surfaces in the plane Z = const at the point N+1 = 31, provided we approximate the arc CBF by the line CF.

In the same fashion we introduce the distance

$$S_n(Z, N) = E_{ee}(Z, N) - \tfrac{1}{2}[E_{eo}(Z, N+1) + E_{eo}(Z, N-1)], \quad (4)$$

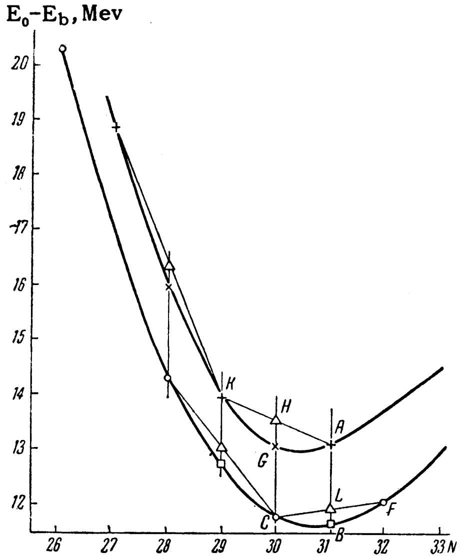

FIG. 1. Traces of the even-even and even-odd energy surfaces with decreasing slope on the plane Z = 26 (Fe): ordinate $E_0 - E_b = 9(Z + N) - E_b(26, N)$[Mev]; the segment AL = $S_n(26, 31) = (1/2)P_n(26, 32)$; the segment CH = $S_n(26, 30)$.

represented in the figure by the segment CH at the point N = 30. It is seen from Fig. 1 that the exact distance between these surfaces in the plane Z = const is greater than $S_n(Z, N+1)$ and smaller than $S_n(Z, N)$. We can therefore assume that the distance between the surfaces along the section Z = const is very close to the arithmetic mean of $S_n(X, N+1)$ and $S_n(Z, N)$:

$$D_n(Z, N) = \frac{1}{2}[S_n(Z, N+1) + S_n(Z, N)]$$

$$= \frac{1}{4}[3E_{ee}(Z, N) - 3E_{eo}(Z, N+1)$$

$$+ E_{ee}(Z, N+2) - E_{eo}(Z, N-1)]. \qquad (5)$$

The existence of the energy gap Δ enabled Bohr, Mottelson, and Pines [8] and Belyaev [9] to apply the theory of superconductivity to nuclear matter. Migdal [10] derived a formula for the energy gap Δ in terms of the nuclear masses, identical to formula (5) for $D_n(Z, N)$. It follows therefore that the distance $D_n(Z, N)$ coincides with the energy gap Δ.

Expanding some of the binding energies E_{ee} and E_{eo} in a Taylor series, we can show that the exact distance between the surfaces, along the section Z = const on the portion from N to N+1, differs from $D_n(Z, N)$ only by the amount

$$\frac{1}{2\cdot 2!}\left\{\frac{\partial^2 E_{ee}(Z, N)}{\partial N^2} - \frac{\partial^2 E_{eo}(Z, N-1)}{\partial N^2}\right\}$$

$$+ \frac{3}{2\cdot 3!}\left\{\frac{\partial^3 E_{ee}(Z, N)}{\partial N^3} - \frac{\partial^3 E_{eo}(Z, N-1)}{\partial N^3}\right\} + \cdots \qquad (6)$$

The intersections between the even-even and even-odd surfaces and the plane Z = const are actually at almost equal distance from each other, and therefore expression (6) is approximately equal to zero. Consequently, $D_n(Z, N)$ is a good approximate expression for the distance between the even-even and even-odd surfaces in the plane Z = const in the interval between N and N+1.

We can similarly express the distance between the traces of the even-even and odd-even surfaces on the surface N = const in the interval between Z and Z+1:

$$D_p(Z, N) = \frac{1}{2}[S_p(Z+1, N) + S_p(Z, N)]$$

$$= \frac{1}{4}[3E_{ee}(Z, N) - 3E_{oe}(Z+1, N)$$

$$+ E_{ee}(Z+2, N) - E_{oe}(Z-1, N)]. \qquad (7)$$

The distance between the odd-even and odd-odd surfaces on Z+1 = const in the interval from N to N+1 will be

$$D_n(Z+1, N) = \frac{1}{2}[S_n(Z+1, N+1) + S_n(Z+1, N)]$$

$$= \frac{1}{4}[3E_{oe}(Z+1, N) - 3E_{oo}(Z+1, N+1)$$

$$+ E_{oe}(Z+1, N+2) - E_{oo}(Z+1, N-1)]. \qquad (8)$$

The distance from the even-odd surface to the odd-odd surface on the section N+1 = const in the interval from Z to Z+1 is

$$D_p(Z, N+1) = \frac{1}{2}[S_p(Z+1, N+1) + S_p(Z, N+1)]$$

$$= \frac{1}{4}[3E_{eo}(Z, N+1) - 3E_{oo}(Z+1, N+1)$$

$$+ E_{eo}(Z+2, N+1) - E_{oo}(Z-1, N+1)]. \qquad (9)$$

Figures 2 and 3 show the dependence on the number of neutrons N of the distances $D_n(Z, N)$ and $D_n(Z+1, N)$ between energy surfaces of different parity, at the sections Z = const and Z+1 = const respectively. The distances are calculated from the experimental data by means of formulas (5) and (8). Figures 4 and 5 show the dependence of the distances $D_p(Z, N)$ and $D_p(Z, N+1)$ between energy surfaces of different parity at the sections N = const and N+1 = const on the number of protons Z. The distances were calculated from the experimental data by means of formulas (7) and (9).

A study of Figs. 2—5 shows that the distances between energy surfaces of different parity depend on the number of nucleons in a rather complicated manner. From the well known semi-empirical Weizsäcker-Fermi formula we obtain for this distance

$$D = \delta A^{-3/4}, \qquad (10)$$

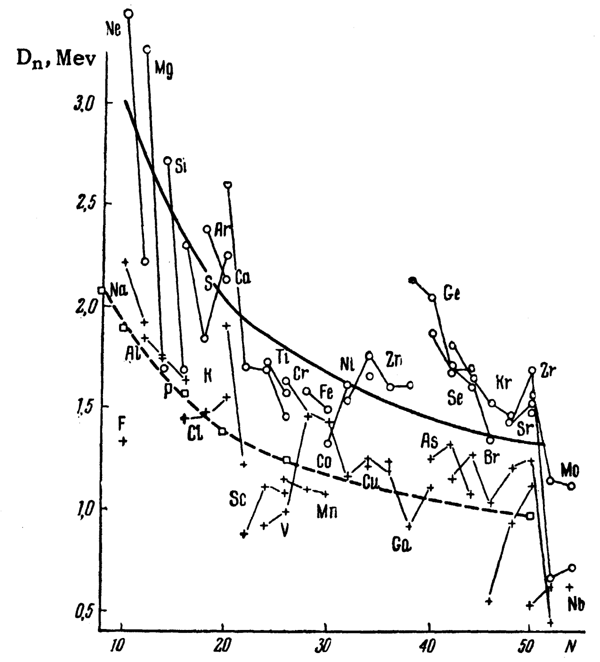

FIG. 2. Distance D_n between traces of energy surfaces of different parity on the surface Z = const as a function of the number of neutrons N, for $N \leq 54$. The points pertaining to identical sections are connected by a broken line, tagged by the element symbol. o – points for distance between even-even surface and even-odd surface; + – points for distance between odd-even and odd-odd surfaces. Solid curve – for average distances D_e dashed curve – for average distances D_0, calculated by formula (11).

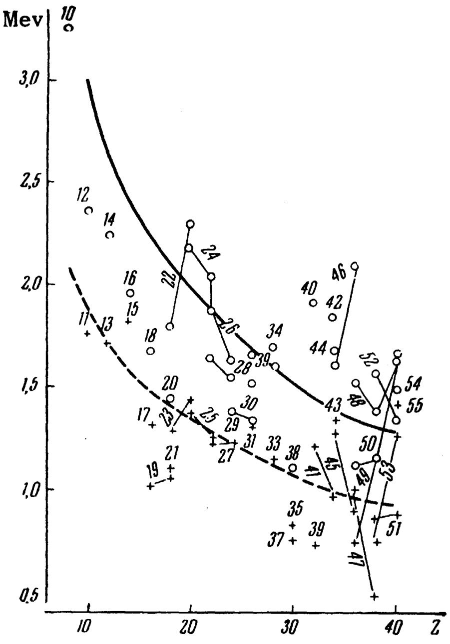

FIG. 4. Distances D_p between energy surfaces of different parity on the sections N = const as a function of the number of protons Z, for $Z \leq 40$. The points pertaining to identical sections are joined by a broken line marked by the number of neutrons N. o – points for distances between even-even and odd-even surfaces; + – points for distances between even-odd and odd-odd surfaces. Continuous curve – for mean values of D_e calculated from formulas (11); dashed curve – the same for D_0.

where A = Z + N – mass number and δ – constant. This formula is suitable both for the distance between the even-even surface and the surface for nuclei with odd A, and for the distance between the surface for nuclei with odd A and the odd-odd surface. Figures 2–5 show that the distance D_e between the even-even surface and the surface for odd A (designated by circles in the figures) are in the mean greater than the distances D_o between the surface for odd A and the odd-odd surface (designated by crosses).

We determined the average empirical values of the distances D_e and D_o, calculated by formulas (5) and (7)–(9). For this purpose we calculated the coefficients of the equation that relates log D with log A. The least-squares calculations were made separately for the distance D_e between the even-even surface and the surface for nuclei with

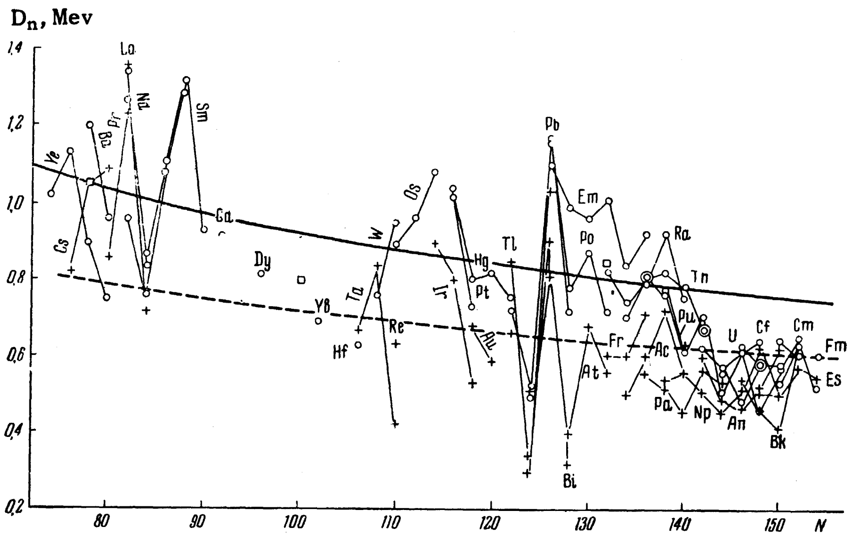

FIG. 3. The same as Fig. 2, but for $74 \leq N \leq 154$; notation the same as in Fig. 2.

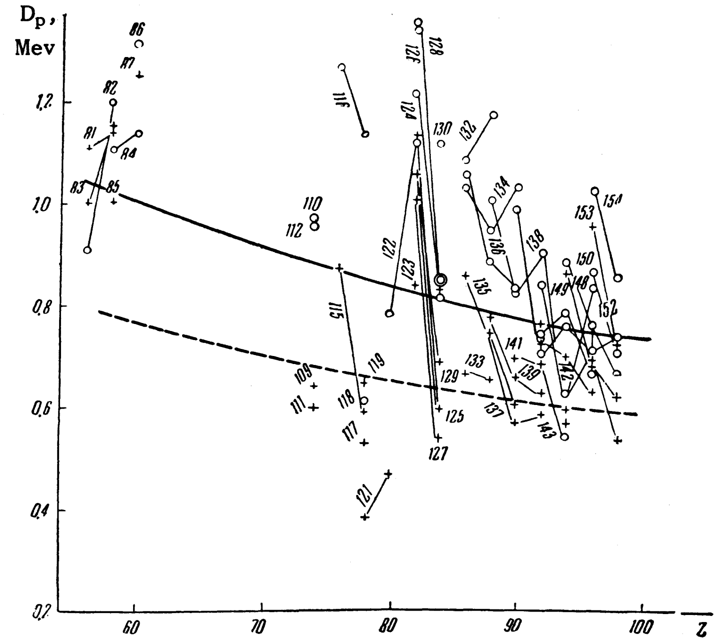

FIG. 5. The same as in Fig. 4, but for $54 \leq Z \leq 98$; notation the same as in Fig. 4.

odd A and for the distances D_o from the surface for nuclei with odd A to the even-even surface. The coefficients for the first distances (D_e) were calculated from 228 experimental values of these distances. The coefficients for the second distance (D_o) were calculated from 198 experimental values. These calculations yield the mean distances as functions of the mass number A:

$$D_e = 14.78 \cdot A^{-0.54} \text{ Mev.} \quad D_o = 7.25 \cdot 10^{-0.45} \text{ Mev,} \tag{11}$$

Comparing these formulas with the eariler expression (10) for D, contained in the Weizsäcker-Fermi formula, we see that the distances D decrease with A more slowly than previously assumed (approximately as $A^{-1/2}$). It is obvious that D depends on A in the same manner as the pairing energy, which, as shown earlier by the author,[7] also apparently decreases as $A^{-1/2}$. By studying the variation of the mean difference of the distances between surfaces $D_e - D_o$ we can see that as A increases this difference decreases uniformly from $D_e - D_o = 1.05$ Mev for light nuclei (A = 20) to $D_e - D_o = 0.15$ Mev for the heaviest nuclei (A = 250).

The mean values of the distances are shown in Figs. 2—5. The fact that the distance between the even-even surface and the surface for nuclei with odd A is greater than the distance between the surface for nuclei with odd A and the even-even surface was pointed out by Cameron.[11] But the distances between energy surfaces of different parity are excluded from Cameron's new mass formula. These distances, combined by Cameron with the shell effects are given in the form of a table of numerical corrections. Studying the course of the experimental points on Figs. 2—5, we see that at the magic and semi-magic numbers N = 20, 38 or 40, 50, 82 and 126 and Z = 20, 40 and 82 a certain increase is noted in these distances, owing to the shell effect. This increase occurs for both the even D_e and the odd D_o. Figure 3 shows also for the distances at N = 88 and N = 116 two maxima, probably connected with the start and the end of the region of deformed nuclei of rare-earth atoms. In the intervals between these two maxima we see an appreciable decrease in the distance, with a minimum in the second half of the region of deformed nuclei.

As indicated earlier, the pairing energies are connected with the distances between energy surfaces of different parity [see, for example, formulas (2) and (5)]. As indicated by Giese and Benson[12] and by the author,[7] the pairing energy has minima, near the magic numbers, while the distance between the energy surfaces increases. This increase in the distances between energy surfaces of different parity is due to the formation of shell grooves on the energy surfaces, the existence of which was pointed out by the author earlier[13] (see Fig. 6). As can be seen from Fig. 6, the formation of the shell groove leads to an increase in the distance between the energy surfaces at a magic number, and at the same time, the segment $S_n(Z, 125)$, equal to twice the pairing energy, decreases sharply when N = 125. The increase in the distances between the energy surfaces on both boundaries of the

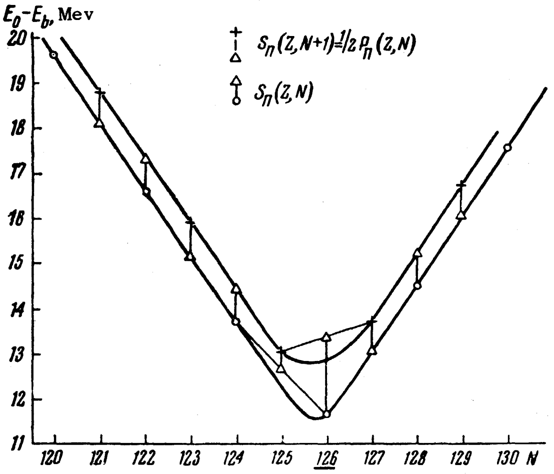

FIG. 6. Intersection of even-even and even-odd energy surfaces of decreasing slope with the plane Z = 82(Pb). Ordinate $E_0 - E_b = 400 + 6(Z + N) - E_b(82, N)$[Mev]; $S_n(Z, N)$ and $S_n(Z, N + 1)$ are determined by formulas (2) and (4).

rare-earth region of deformed nuclei corresponds to an increase in the pairing energy, as was shown in [7].

It can be noted on Figs. 2 and 4 that D_n and D_p increase considerably above their mean values when N and Z are close to 40. This increase is greater and broader than expected for the semi-magic number 40. A similar increase in the pairing energies P in the same region is seen also in the curves of [7]. Unfortunately, the nuclide masses from Ga to Ru have not been measured with sufficient accuracy, and any conclusion concerning this increase in D and P would be premature. It would be interesting to measure the masses of the nuclides in this region with greater accuracy.

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Translated by J. G. Adashko
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