NEUTRINO PRODUCTION IN THE ATMOSPHERE

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The energy spectra and angular distribution of neutrinos produced in the atmosphere in the $\pi \rightarrow \mu + \nu$ and $\mu \rightarrow e + \nu + \bar{\nu}$ decays are calculated taking the μ-meson energy losses at neutrino energies $\varepsilon = 10^9$—3×10^{11} ev into account. It is shown that the neutrino flux from the $\mu \rightarrow e + \nu + \bar{\nu}$ decay is comparable with that from the $\pi \rightarrow \mu + \nu$ decay. μ-meson energy losses only weakly affect neutrino production. K mesons produce neutrinos more efficiently than do π mesons. An experimental arrangement for detecting high-energy cosmic ray neutrinos is proposed.

1. INTRODUCTION

CONSIDERABLY increased attention has recently been given to possible experiments on the detection of neutrinos in cosmic rays.[1 5] This interest is due primarily to the possible investigation of weak interactions at high energies, and to the possibility of finding an explanation to a number of basic theoretical questions (the behavior of the weak interaction cross section with increasing energy,[4-6] the existence of an intermediate vector meson responsible for weak interactions, the existence of two pairs of neutral leptons: muonic and electronic,[7,6] etc.). On the other hand, cosmic-ray neutrino experiments open a new approach to a number of astrophysical problems.

The main object of the investigation in the underground cosmic-ray neutrino experiment as proposed by Markov, and in a modified form by us,[8] are the neutrino-nucleon reactions of the type

$$\nu + N \rightarrow N' + \mu,$$
$$\nu + N \rightarrow N' + \mu + n\pi \qquad (1)$$

(where N denotes a baryon). This is so because the cross sections of these processes are considerably larger than the cross sections of lepton-lepton interactions[6,9,4] and because of the remarkable interaction properties of the μ meson.[4,5] The character of the energy dependence of the interaction cross section of neutrinos with nucleons may vary with the transferred momentum corresponding to the nucleon size $\hbar/M_N c$. The study of interactions at neutrino energies $\varepsilon \gtrsim 1$ Bev is therefore of special interest.

The neutrino flux in cosmic rays consists basically of two components of different origin. One comprises the truly cosmic neutrinos, i.e., those coming to earth from outer space, of galactic or metagalactic origin. The other component comprises the neutrinos produced by cosmic rays in the atmosphere of the earth. If we make the natural assumption that the "intrinsic" high-energy neutrinos are due to cosmic rays only, then their intensity should be of the same order of magnitude as the intensity of photons of corresponding energies incident upon the earth from outer space (if we neglect the proton absorption processes). Up to now, photons have not been found in the primary cosmic radiation, and the upper limit of the acceptable flux obtained by the Bristol group[10] amounts to 10^{-3} of the cosmic-ray particle intensity. The possible flux of photons and neutrinos calculated from commonly accepted astro-physical values turns out to be two orders of magnitude lower.[3] As has been shown by our calculations, the flux of atmospheric high-energy neutrinos amounts to $\sim 10^{-1}$ of the intensity of primary cosmic rays, i.e., three orders of magnitude greater than the expected flux of intrinsic cosmic neutrinos. Therefore, if we do not take various far-reaching hypotheses[11] into consideration, we can suppose that in the experiment on cosmic-ray neutrinos we shall have to deal with neutrinos of atmospheric origin.

The atmospheric neutrinos are convenient for experiments devoted to the study of high-energy weak interactions, since their energy spectrum and angular distributions in the atmosphere can be calculated very exactly. This permits us to calculate the spectrum and angular distribution of the detected products of neutrino reactions with matter in the apparatus for each variant of the theory. Any deviations can be observed and interpreted. The low intensity, which imposes difficult requirements upon the experiment is, however, a drawback.

The estimate of the flux of atmospheric neutrinos from the $\pi \rightarrow \mu + \nu$ decay was carried out by Markov and Zheleznykh.[5] Exact calculations of the neutrino spectrum have not been carried out so far. We have calculated the spectrum of neutrinos from the $\pi \rightarrow \mu + \nu$ and $\mu \rightarrow e + \nu + \bar{\nu}$ decays, taking into account the energy losses of μ mesons and the angular distributions of neutrino flux in the atmosphere. The calculation carried out gives higher neutrino fluxes than the earlier estimates, and therefore supports the optimistic conclusions of Markov and Zheleznykh[4,5] on the feasibility of the experiments with high-energy cosmic-ray neutrinos.

2. NEUTRINOS FROM PION DECAY

Pions produced in the collisions of primary radiation with air nuclei produce both μ mesons and neutrinos in their decay. The energy spectrum of such neutrinos can easily be calculated from the π-meson spectrum, and, in the case where the μ mesons are produced only by π mesons, is determined without ambiguity by the well-known μ-meson spectrum.

Let us consider the one-dimensional problem, assuming that all secondary particles conserve the direction of the primary ones. The path x of the particles is measured in mass units and is calculated from the top of the atmosphere. Let $P^{\pi}(x, E, \theta)$ be the π-meson spectrum at the depth x at a zenith angle θ. The function of the neutrino source is then given by the expression

$$G_{\pi}^{\nu}(x, \varepsilon, \theta) = \int_{E_{min}(\varepsilon)}^{E_{max}(\varepsilon)} [l_{\pi}(E)\, \rho(x, \theta)]^{-1} P^{\pi}(x, E, \theta)\, D_{\pi\nu}(E, \varepsilon)\, dE, \quad (2)$$

where ε is the neutrino energy, $l_{\pi}(E)$ is the linear decay mean free path of a π meson with energy E, $\rho(x, \theta)$ is the air density, and $D_{\pi\nu}(E, \varepsilon)d\varepsilon$ is the spectrum of neutrinos originating from the decay of π mesons with energy E in the laboratory system (l.s.). The values E_{min} and E_{max} and the function $D_{\pi\nu}(E, \varepsilon)$ are found from the kinematics of the $\pi \rightarrow \mu + \nu$ decay:

$$E_{min} = \varepsilon (1 - m^2/M^2)^{-1}, \qquad E_{max} = \infty,$$

$$D_{\pi\nu}(E, \varepsilon)\, d\varepsilon = d\varepsilon\, [E(1 - m^2/M^2)]^{-1} \text{ for } (M/E)^2 \ll 1,$$

where m and M are the masses of the μ and π mesons, respectively.

The spectrum $P^{\pi}(x, E, \theta)$ and the function $G_{\pi}^{\nu}(x, \varepsilon, \theta)$ are calculated as described in the previous article.[12] By integrating the function $G_{\pi}^{\nu}(x, \varepsilon, \theta)$ over the depth x, we find the neutrino spectrum:

$$P_{\pi}^{\nu}(x, \varepsilon, \theta) = (1 - e^{-x})\, F^{\nu}(\varepsilon, \theta) \approx \frac{I_{\pi} A_{\pi\nu}\, \varepsilon^{-(\gamma+1)}}{1 + 3.28\, \varepsilon / E_{\pi}(\theta)} (1 - e^{-x}),$$

$$F^{\nu}(\varepsilon, \theta) = \frac{I_{\pi}}{1 - m^2/M^2} \int_{\varepsilon(1-m^2/M^2)^{-1}}^{\infty} \frac{E^{-(\gamma+2)}\, dE}{1 + E/E_{\pi}(\theta)},$$

$$A_{\pi\nu} = \frac{1}{1+\gamma}\left(1 - \frac{m^2}{M^2}\right)^{\gamma}, \quad (3)$$

where x is expressed in the units of λ (interaction mean free path of π mesons), I_{π} is the intensity of π-meson production at E = 1 (energy expressed in Bev), γ is the exponent of the integral production spectrum of the π mesons, and $E_{\pi}(\theta)$ is the critical π-meson energy for which the decay probability of a π meson at the depth x = 1 is equal to the probability of nuclear interaction. The quantity $E_{\pi}(\theta)$ was calculated earlier.[12] To simplify the calculations, we have assumed $\lambda = \lambda_n$, where λ_n is the absorption mean free path in the atmosphere of the π-meson producing component; according to experimental data, $\lambda_n = 120$ g/cm^2.

At sea level, $(x \gg 1)$, the vertical-flux spectrum can be approximated by the function (in units of cm^{-2} sec^{-1} sr^{-1} Bev^{-1})

$$P_{\pi}^{\nu}(\varepsilon, 0)\, d\varepsilon = \begin{cases} 1.85 \cdot 10^{-2}(0.08 + \varepsilon)^{-2.80} d\varepsilon, & 1 \leqslant \varepsilon \leqslant 10 \\ 6.65 \cdot 10^{-2}(1.1 + \varepsilon)^{-3.22} d\varepsilon, & 10 \leqslant \varepsilon \leqslant 300 \end{cases}. \quad (4)$$

The total neutrino flux with energy >1 Bev is equal to $P_{\pi}^{\nu}(>1.0) = 8.9 \times 10^{-3}$ cm^{-2} sec^{-1} sr^{-1}. In the article of Zheleznykh and Markov,[4] this flux was found to be 4×10^{-3} cm^{-2} sec^{-1} sr^{-1}. Equation (3) permits us to calculate the angular distributions in the atmosphere of the neutrinos from the $\pi \rightarrow \mu + \nu$ decay.

3. NEUTRINOS ORIGINATING IN MUON DECAY

Muons also contribute to the flux of atmospheric neutrinos. Owing to the large path traversed by μ mesons in the atmosphere, and the large energy fraction transferred in their decay to the neutrinos, muons, in spite of their long lifetime, produce a neutrino flux comparable to that from the $\pi \rightarrow \mu + \nu$ decay. The spectrum of neutrinos and antineutrinos from the $\mu^{\pm}$ meson decay is found by integrating the neutrino source function $G_{\mu}^{\nu}(x, \varepsilon, \theta)$ for the $\mu \rightarrow e + \nu + \bar{\nu}$ decay over x.

The function $G_{\mu}^{\nu}(x, \varepsilon, \theta)$ is constructed similarly to the function (2), provided the μ-meson spectrum $P^{\mu}(x, E, \theta)$ is known:

$$G^{\nu}_{\mu}(x, \varepsilon, \theta) = \int\limits_{E_{min}(\varepsilon)}^{E_{max}(\varepsilon)} [l_{\mu}(E)\, \rho(x, \theta)]^{-1} P^{\mu}(x, E, \theta)\, R_{\mu\nu}(E, \varepsilon)\, dE, \tag{5}$$

$$R_{\mu\nu}(E, \varepsilon,) = R^{(-)}_{\mu\nu}(E, \varepsilon) + R^{(+)}_{\mu\nu}(E, \varepsilon), \tag{6}$$

where $l_{\mu}(E)$ is the decay mean free path of μ mesons with energy E; $R^{(-)}_{\mu\nu}(E, \varepsilon)\,d\varepsilon$ and $R^{(+)}_{\mu\nu}(E, \varepsilon)\,d\varepsilon$ are the neutrino spectra from the decay of μ^- and μ^+ mesons with energy E, respectively, in the l.s.

From the decay kinematics, we find the values of $E_{min}(\varepsilon)$ and $E_{max}(\varepsilon)$:

$$E_{min} = \varepsilon, \qquad E_{max} = \infty \quad \text{for } (m/E)^2 \ll 1.$$

The spectra $R^{(-)}_{\mu\nu}(E, \varepsilon)$ and $R^{(+)}_{\mu\nu}(E, \varepsilon)$ are calculated in the Appendix [Eqs. (18) and (19)]. For antineutrinos we have $R^{(-)}_{\mu\bar{\nu}} = R^{(+)}_{\mu\nu}$ and $R^{(+)}_{\mu\bar{\nu}} = R^{(-)}_{\mu\nu}$.

The expression for the spectrum $P^{\nu}_{\mu}(x, \varepsilon, \theta)$ can be conveniently written in the form:

$$P^{\nu}_{\mu}(x, \varepsilon, \theta) = \int\limits_{\varepsilon}^{\infty} P^{\mu}_{d}(x, E, \theta)\, R_{\mu\nu}(E, \varepsilon)\, dE, \tag{7}$$

where

$$P^{\mu}_{d}(x, E, \theta) = \int\limits_{0}^{x} [l_{\mu}(E)\, \rho(t, \theta)]^{-1} P^{\mu}(t, E, \theta)\, dt \tag{8}$$

represents the total number of μ mesons which decayed while moving in the atmosphere at a zenith angle θ between the top of the atmosphere and the level x, and which had an energy E at the moment of decay.

A general expression for the μ-meson spectrum $P^{\mu}(x, E, \theta)$ was obtained by us earlier[12] taking the decay and the energy loss of μ mesons in a spherical atmosphere into account. For the neutrino spectrum at energies $\varepsilon \lesssim 10^{11}$ ev, the expression simplifies somewhat since we can restrict ourselves to ionization losses only (see Fig. 1). $P^{\mu}(x, E, \theta)$ is then of the form

$$P^{\mu}(x, E, \theta) = I_{\pi} A_{\pi\mu} E^{-(\gamma+1)} \int\limits_{0}^{x} e^{u-t} \left[1 + \frac{\beta}{E}(x - t)\right]^{-(\gamma+1)} \times \left\{1 + \frac{1{,}22E}{E_{\pi}(\theta)} \left[1 + \frac{\beta}{E}(x - t)\right]\right\}^{-1} dt, \tag{9}$$

where

$$A_{\pi\mu} = \frac{1 - (m/M)^{2(\gamma+1)}}{(1+\gamma)(1 - m^2/M^2)}, \qquad u = \frac{mc}{\tau_{0\mu}} \int\limits_{x}^{t} \frac{dz}{\rho(z, \theta)\,[E + \beta(x - z)]}, \tag{9a}$$

and β is the ionization energy loss of the μ mesons per unit path. For angles $0.1 < \cos\theta \leq 1$, the atmosphere can, with sufficient accuracy, be regarded as flat, so that $\rho(x, \theta) = h_0^{-1} x \cos\theta$ (h_0 is the height of the homogeneous atmosphere) and

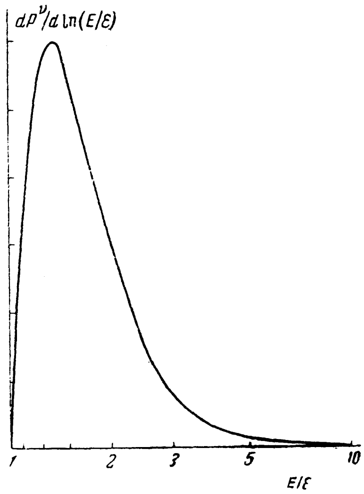

FIG. 1. Contribution of μ mesons of different energies to the flux of neutrinos with energy ε.

$$e^{u} = (t/x)^{\delta}\,[1 + (\beta/E)(x - t)]^{-\delta},$$

$$\delta = mch_0/\tau_{0\mu}\,[E + \beta x] \cos\theta = h_0/l_{\mu}(E + \beta x)\cos\theta. \tag{9b}$$

Substituting (9) and (8) and integrating numerically, we obtained the function $P^{\mu}_{d}(x, E, \theta)$. Using relation (7), we then found the spectrum and the angular distribution in the atmosphere of neutrinos from the $\mu \to e + \nu + \bar{\nu}$ decay, taking the energy loss of the μ mesons into account.

If we neglect the μ-meson energy loss, i.e., if we set $\beta = 0$, then $(x \gg 1)$

$$P^{\mu}_{d}(x, E, \theta) = \{1 - \exp[-L(x, x_{\text{eff}}, \theta)/l_{\mu}(E)] \times \Gamma(1 + \delta)\}\, F^{\mu}_{1}(E, \theta), \tag{10}$$

where $L(x, x_{\text{eff}}, \theta)$ is the effective linear path traversed by the μ mesons in a spherical atmosphere from the point of production to sea level. The values of $L(x, x_{\text{eff}}, \theta)$ for various θ and the function F^{μ}_{1} have been found earlier.[12] $\Gamma(z)$ is the gamma function.

Substituting (10) into (7) we find $P^{\nu}_{\mu}(x, \varepsilon, \theta)$. The calculated spectrum is found to differ little from the neutrino spectrum calculated with account of the energy loss of the μ mesons. The losses lead to a neutrino flux decrease that is small and varies little with energy. The role of the losses determined by the factor $1 - P^{\nu}/P^{\nu}_{\text{(without loss)}}$ varies from $\sim 25\%$ for neutrino energy $\varepsilon = 1$ Bev to $\sim 12\%$ for $\varepsilon = 100$ Bev.

Since anyway the role of the losses varies little with increasing zenith angle θ, the angular distributions calculated with and without account of the energy loss differ little. The angular distribution in the atmosphere of neutrinos from the $\pi \to \mu + \nu$ and $\mu \to e + \nu + \bar{\nu}$ decays is shown in Fig. 2.

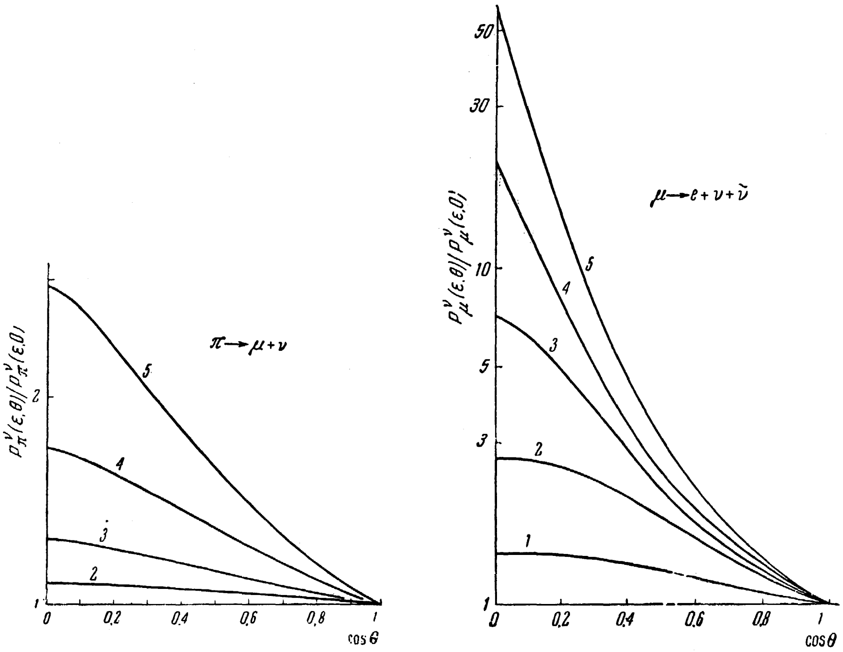

FIG. 2. Distributions of neutrinos of different energies in the atmosphere for the two production mechanisms: curve 1 – $\varepsilon = 1 \times 10^9$ ev, curve 2 – $\varepsilon = 3 \times 10^9$ ev, curve 3 – $\varepsilon = 1 \times 10^{10}$ ev, curve 4 – $\varepsilon = 3 \times 10^{10}$ ev, curve 5 – $\varepsilon = 1 \times 10^{11}$ ev.

The neutrino spectrum from the μ-meson decay, with account of the μ-meson energy loss in the vertical flux at sea level is approximated by the function (in units of $\mathrm{cm^{-2}\,sec^{-1}\,sr^{-1}\,Bev^{-1}}$)

$$P_\mu^\nu(\varepsilon, 0)\, d\varepsilon = \begin{cases} 7{,}65 \cdot 10^{-2}\,(0{,}37 + \varepsilon)^{-3{,}75}\, d\varepsilon, & 1 \leqslant \varepsilon \leqslant 10 \\ 1{,}48\,(3{,}5 + \varepsilon)^{-4{,}51}\, d\varepsilon, & 10 \leqslant \varepsilon \leqslant 100 \end{cases} \quad (11)$$

(we do not differentiate here between muonic and electronic neutrinos). The total neutrino flux with energy greater than 1 Bev is equal to

$$P_\mu^\nu(> 1{,}0) = 1{,}17 \cdot 10^{-2}\ \mathrm{cm^{-2}\,sec^{-1}\,sr^{-1}}.$$

The total spectrum of the neutrinos from the decay of π and μ mesons $P^\nu = P_\pi^\nu + P_\mu^\nu$ in the vertical flux is approximated by the function (in the same units)

$$P^\nu(\varepsilon, 0)\, d\varepsilon = \begin{cases} 6{,}0 \cdot 10^{-2}\,(0{,}15 + \varepsilon)^{-3{,}15}\, d\varepsilon, & 1 \leqslant \varepsilon \leqslant 10 \\ 0{,}12\,(0{,}9 + \varepsilon)^{-3{,}34}\, d\varepsilon, & 10 \leqslant \varepsilon \leqslant 300. \end{cases} \quad (12)$$

The total flux of neutrinos with energy greater than 1 Bev is then

$$P^\nu(> 1{,}0) = 2{,}06 \cdot 10^{-2}\ \mathrm{cm^{-2}\,sec^{-1}\,sr^{-1}}.$$

The spectra of electronic ν_e and muonic ν_μ neutrinos are given by the relations

$$P_{\nu_e} = P_\mu^\nu/2, \qquad P_{\nu_\mu} = P_\pi^\nu + P_\mu^\nu/2.$$

The energy spectra of neutrinos in the vertical and horizontal flux are shown in Fig. 3. The angular distribution in the atmosphere of the total neutrino flux from the $\pi \to \mu + \nu$ and $\mu \to e + \nu + \bar{\nu}$ decays is shown in Fig. 4.

So far we have always had the total flux of neutrinos and antineutrinos in mind when speaking about neutrinos. Moreover, it was not necessary to take the positive excess of π and μ mesons in the atmosphere into account, and $P^\pi(x, E, \theta)$ and $P^\mu(x, E, \theta)$ denoted the total intensity of mesons of both signs. In order to calculate the partial intensity of ν and $\bar{\nu}$, it is necessary to define $P^\pi(x, E, \theta)$ and $P^\mu(x, E, \theta)$ as the spectrum of mesons of one sign. To calculate the spectrum of electronic or muonic neutrinos and antineutrinos from the $\mu \to e + \nu + \bar{\nu}$ decay, it is also necessary to use the function $R_{\mu\nu}^{(+)}$ of the form (19) or $R_{\mu\bar{\nu}}^{(-)}$ from Eq. (18) instead of $R_{\mu\nu}$ from Eq. (6). Thus, say for electronic neutrinos, the expression (7) will now have the form

$$P_{\nu_e}(x, \varepsilon, \theta) = \int_\varepsilon^\infty P_d^{\mu(+)}(x, E, \theta)\, R_{\mu\nu}^{(+)}(E, \varepsilon)\, dE. \quad (13)$$

Using this expression, let us estimate the spectrum of electronic neutrinos and antineutrinos $k_\nu(\kappa)$ and $k_{\bar{\nu}}(\kappa)$ determined by the relations

$$P_\nu(\varepsilon) = k_\nu(\varkappa)\, P_d^{\mu(+)}(\varepsilon), \quad P_{\bar{\nu}}(\varepsilon) = k_{\bar{\nu}}(\varkappa)\, P_d^{\mu(-)}(\varepsilon),$$

where κ is the exponent of the integral spectrum of decaying μ^+ (μ^-) mesons. For simplicity, let us write

$$P_d^{\mu(+)}(E) = P_d^{\mu(-)}(E) = AE^{-(\varkappa+1)}.$$

Substituting $R_{\mu\nu}^{(+)}$ from Eq. (19) and $R_{\mu\bar{\nu}}^{(-)}$ from Eq. (18) into (13) for ν_e and $\bar{\nu}_e$ respectively, we find

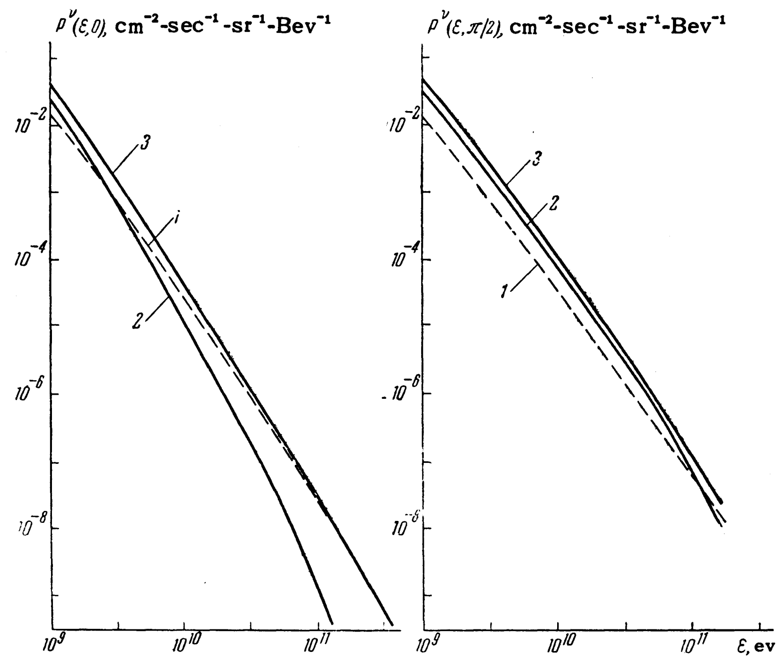

FIG. 3. Different energy spectra of neutrinos from $\pi \to \mu + \nu$ (curve 1) and $\mu \to e + \nu + \bar{\nu}$ (curve 2) and the total spectrum (curve 3) for the angle $\theta = 0$ and $\theta = \pi/2$.

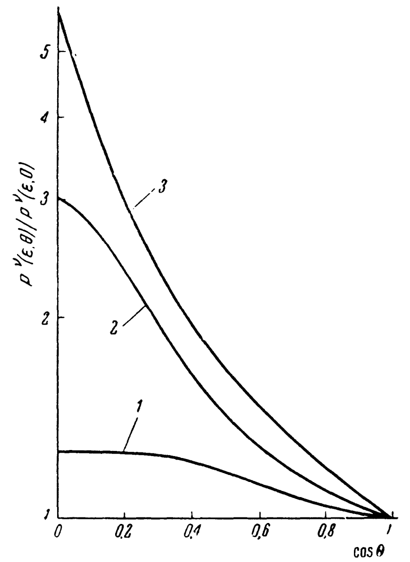

FIG. 4. Angular distribution in the atmosphere of the total neutrino flux from the $\pi \to \mu + \nu$ and $\mu \to e + \nu + \bar{\nu}$ decays at neutrino energies 1 – $\varepsilon = 1 \times 10^9$ ev, curve 2 – $\varepsilon = 1 \times 10^{10}$ ev, curve 3 – 1×10^{11} ev. The curves are normalized to the intensity of the vertical flux.

$$k_\nu(\varkappa) = \frac{5}{3}\frac{1}{\varkappa+1} - \frac{3}{\varkappa+3} + \frac{4}{3}\frac{1}{\varkappa+4},$$

$$k_{\bar{\nu}}(\varkappa) = 2\left(\frac{1}{\varkappa+1} - \frac{3}{\varkappa+3} + \frac{2}{\varkappa+4}\right).$$

For $\kappa = 2$, the ratio $k_\nu/k_{\bar{\nu}}$ amounts to $4/3$. This means that, replacing the quantity $R_{\mu\nu}$ in Eq. (7) by, say $2R_{\mu\nu}^{(-)}$, the flux of neutrinos and antineutrinos (electric and muonic) will increase by 2/1.75 times, i.e., by 14%.

It should be noted that we have neglected the polarization of cosmic-ray μ mesons. Taking the polarization into account increases the neutrino intensity from μ-meson decay by $\sim 5\%$.

4. NORMALIZATION OF THE NEUTRINO SPECTRA

The neutrino spectra from the π and μ meson decay were normalized to experimental spectra of the μ mesons at sea level.[13,14] For this, we have calculated the spectrum of the μ mesons at sea level taking into account the decay and energy loss of the μ mesons according to Eqs. (9) and (9b) for different values of γ and I_π.

For $I_\pi = 0.159$ cm^{-2}sec^{-1}sr^{-1}Bev^{-1} and for $\gamma = 1.62$, the calculated spectrum of the μ mesons agrees with the experimental results within the errors of the experiment in the 10^9—10^{12} ev energy range. It should be noted that the values of I_π and γ chosen are close to those obtained by Paine, Davison, and Greisen;[13] $I_\pi = 0.156$ cm^{-2}sec^{-1}sr^{-1}Bev^{-1}, $\gamma = 1.64$.

5. DISCUSSION OF RESULTS

The above calculations show that the $\pi \to \mu + \nu$ $\mu \to e + \nu + \bar{\nu}$ decays produce comparable neutrino fluxes. Account of the energy loss by the μ mesons causes little change in the intensity of the neutrino flux from the $\mu \to e + \nu + \bar{\nu}$ decay. The neutrino fluxes in the atmosphere are anisotropically distributed, and the degree of anisotropy $P^\nu(\varepsilon, \pi/2)/P^\nu(\varepsilon, 0)$ increases with increasing neutrino energy, tending to about 10 for the $\pi \to \mu + \nu$ decay[12] and to $10\, L(x, x_{eff}, \pi/2)/L(x, x_{eff}, 0) \sim 300$ for the $\mu \to e + \nu + \bar{\nu}$ decay for $\varepsilon \gg 10^{12}$ ev. For the neutrino energies under consideration,

$\varepsilon = 10^9 - 10^{11}$ ev, the angular distributions of the total neutrino flux from the $\pi \to \mu + \nu$ and $\mu \to e + \nu + \bar{\nu}$ decay are close to the distributions of the neutrino flux from the π-meson decay. Since the neutrinos of energies under consideration traverse the whole earth without absorption (the absorption cross section $\sigma \ll \sigma_{\text{critical}} \sim 10^{-34}$ cm²/nucleon), the angular distribution of neutrino fluxes from the lower hemisphere are identical to the distributions from the upper hemisphere at every point on the earth's surface:

$$P^{\nu}(\varepsilon, \theta) = P^{\nu}(\varepsilon, \pi - \theta).$$

We have assumed in the calculations that all secondary particles conserve the direction of motion of the primary ones. Such an assumption is acceptable since, at low neutrino energies $\varepsilon \sim 1$ Bev, their fluxes (both from π- and μ-meson decays) are distributed in the atmosphere quasi-isotropically, and at high energies, $\varepsilon \gtrsim 10$ Bev, the angle of emission of particles in the l.s. is small.

It should be noted that the results of calculations of the neutrino spectrum are practically independent of the arbitrarily chosen value λ for normalization of the spectra according to the experimental data on the μ-meson spectrum. In fact, the function $f(x, E, \theta) = P^{\pi}(x, E, \theta)/L_{\pi}(E)\rho(x, \theta)$ determines the number of π mesons with energy E which decay per unit path length at the depth x. Since the parameters I_{π} and γ determining the function $f(x, E, \theta)$ are chosen for a match between the calculated and observed μ-meson spectrum, $f(x, E, \theta)$ is fixed and independent of the choice of λ. The neutrino spectra are therefore calculated very accurately, even though the auxiliary function $P^{\pi}(x, E, \theta)$, when substituting λ for λ', is determined with a relative error of the order $\lambda/\lambda'(1 + E_{\pi}/E)$.

The inaccuracy in calculating the neutrino spectrum is thus mainly due to the indeterminacy of the contribution of K mesons to the neutrino flux. K mesons produce neutrinos more efficiently than π mesons since the energy distribution between the neutrino and the μ meson is more favorable for the neutrino ($E_{\nu}/E_{\mu} \approx 0.91$ for the $K \to \mu + \nu$ and 0.30 for the $\pi \to \mu + \nu$ decay), and because of the shorter lifetime and the higher mass of the K mesons. Thus, if the fluxes of the μ mesons with energies ~1 Bev from the $K \to \mu + \nu$ and $\pi \to \mu + \nu$ decays are equal, the neutrino flux of the same energies from K mesons will be roughly six times greater than from π mesons. A neutrino with an energy of ~100 Bev is produced by K mesons roughly 11 times more efficiently. This means that there is a ten percent contribution of K mesons to the μ-meson flux of ~100 Bev energy, so that the neutrino flux of corresponding energies increased by a factor of two. Unfortunately, the experimental data available so far do not permit us to determine the contribution of K mesons to the neutrino flux. The accuracy of the data is such that it permits a contribution of the order of several tens percent to the neutrino flux of K mesons even in the energy range of ~1 Bev, not considering higher energies. At any rate, it is clear that, by assuming the π mesons to be the only source of the μ mesons, we do not overestimate the neutrino flux.

The results obtained show that the total vertical flux of neutrinos with energy greater than 1 Bev from the π and μ meson decays in the atmosphere is five times greater than the estimate of Zheleznykh and Markov,[4] which took only the $\pi \to \mu + \nu$ decay into account. This fact, and an account of the increase of the flux for inclined directions, increases the possibility of detecting μ mesons from reaction (1).

It is necessary to mention here the experimental setup for the detection of events of the type (1). We assume that to observe μ mesons produced by neutrinos [in particular in reactions of type (1)] it is not practical to construct arrays with fixed reaction volume.[3,4] As has been shown by Zheleznykh and Markov, in fixed-volume arrays of reasonable dimensions the main part of the μ mesons come from the outside, i.e., are spurious. It is, therefore, necessary to use an array that detects μ mesons produced only below it and is based on the measurement of the delay in the signals produced by μ mesons traversing three rows of scintillators.

The proposed array is schematically shown in Fig. 5. Here 1, 2, and 3 denote mosaic layers of scintillation counters placed at a sufficient distance from each other, by means of which we can fix the trajectory of the μ meson traversing the array and measure the relative delay times; this enables us to separate the mesons arriving from the lower hemisphere. Absorber a, whose total

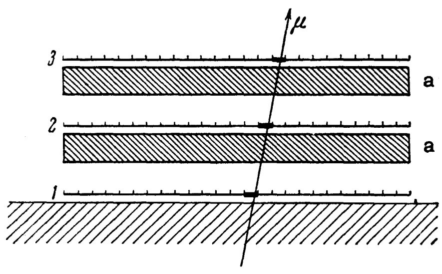

FIG. 5. Array for the detection of events of type (1) produced by neutrinos from the lower hemisphere (explanation in text).

thickness determines the threshold energy of μ-meson detection, is placed between the scintillators. Such an array permits us to determine the statistics of events (1) for different threshold energies and the angular distributions of the detected μ mesons. Both results are sensitive to the behavior of the effective cross section of μ-meson production as a function of neutrino energy, and thus to the particular theoretical assumptions made. The existence of an intermediate meson may, in addition, be observed by the detection of μ meson pairs.

APPENDIX

SPECTRA OF THE PARTICLES FROM THE $\mu \rightarrow e + \nu + \bar{\nu}$ DECAY

Formulas are given below for the spectra of neutrinos and antineutrinos in the $\mu^- \rightarrow e^- + \nu + \bar{\nu}$ decay for the V-A variant of the four-fermion interaction.* The energy spectrum of the neutrinos from the $\mu^- \rightarrow e^- + \nu + \bar{\nu}$ decay in the coordinate system in which the μ meson is at rest (c.m.s.) calculated neglecting the electron mass is

$$N_\nu(x)\,dx = 2x^2(3-2x)\,dx, \tag{14}$$

where $x = E_\nu/E_m$, E_ν is the neutrino energy, and $E_m = m/2$ is the maximum neutrino c.m.s. energy. Because of the symmetry of the matrix element with respect to the permutation of the momenta of the neutrino and of the electron, the neutrino spectrum[14] coincides naturally with the electron spectrum.[15]

The spectrum of antineutrinos in the c.m.s. is given by the formula

$$N_{\bar{\nu}}(x)\,dx = 12x^2(1-x)\,dx, \tag{15}$$

where $x = E_{\bar{\nu}}/E_m$. The mean energies carried away by neutrinos and antineutrinos equal 0.35 and 0.30 m respectively.

In the l.s., the spectra ν and $\bar{\nu}$ are of the form (assuming an unpolarized μ-meson beam)

$$N_{\nu L}(y)\,dy = \begin{cases} \dfrac{16}{(1-\beta^2)^3}\left[3(1-\beta^2) - \dfrac{4}{3}(3+\beta^2)\,y\right]y^2\,dy, & 0 \leqslant y \leqslant \dfrac{1-\beta}{2} \\ \dfrac{1}{\beta}\left\{\dfrac{5}{3} + \dfrac{4}{(1+\beta)^3}\left[\dfrac{8}{3}y - 3(1+\beta)\right]y^2\right\}dy, & \dfrac{1-\beta}{2} \leqslant y \leqslant \dfrac{1+\beta}{2}, \end{cases} \tag{16}$$

$$N_{\bar{\nu} L}(y)\,dy = \begin{cases} \dfrac{32}{(1-\beta^2)^3}\left[3(1-\beta^2) - 2(3+\beta^2)\,y\right]y^2\,dy, & 0 \leqslant y \leqslant \dfrac{1-\beta}{2} \\ \dfrac{2}{\beta}\left\{1 + \dfrac{4}{(1+\beta)^3}\left[4y - 3(1+\beta)\right]y^2\right\}dy, & \dfrac{1-\beta}{2} \leqslant y \leqslant \dfrac{1+\beta}{2}, \end{cases} \tag{17}$$

where β is the velocity of the μ meson in units of the light velocity, $y = \varepsilon/E$, ε is the energy of the neutrino (antineutrino), and E is the energy of the μ meson in the l.s.

In the limit $\beta \rightarrow 0$, the spectra (16) and (17) transform (as they should) into (14) and (15). In practice, $\beta \approx 1$, $(m/E)^2 \ll 1$, and they become $(0 \leq y \leq 1)$

$$N_{\nu L}(y)\,dy \equiv R_{\mu\nu}^{(-)}(E, \varepsilon)\,d\varepsilon = \left(\tfrac{5}{3} - 3y^2 + \tfrac{4}{3}y^3\right)dy, \tag{18}$$

$$N_{\bar{\nu} L}(y)\,dy \equiv R_{\mu\nu}^{(+)}(E, \varepsilon)\,d\varepsilon = 2(1 - 3y^2 + 2y^3)\,dy. \tag{19}$$

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*These formulas have been obtained by Yu. S. Kopysov and V. A. Kuz'min.

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