PROTON-PROTON SCATTERING AT 8.5 Bev

DO IN SEB, L. F. KIRILLOVA, P. K. MARKOV, L. G. POPOVA, I. N. SILIN, É. N. TSYGANOV, M. G. SHAFRANOVA, B. A. SHAKHBAZYAN, and A. A. YULDASHEV*

Joint Institute for Nuclear Research

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Elastic pp scattering at 8.5 Bev was studied with the aid of emulsion pellicles exposed perpendicularly to the primary proton beam. Altogether 480 elastic scattering events have been found. The total elastic scattering cross section is (8.74 ± 0.40) mb. The differential cross section is investigated in the c.m.s. angular interval from 1.5° to 20.5°. The experimental data are not in agreement with the simple model in which the real part of the phase shifts and the dependence of the interaction cross section on the spin state are neglected. The total pp interaction cross section computed from the experimental data under these assumptions exceeds the experimental value by more than three standard deviations of the error. It can be said that the real part of the scattering amplitude does not exceed half of the imaginary part. The rms pp interaction range is found to be 1.15 ± 0.05 f.

1. EXPERIMENTAL ARRANGEMENT, ANALYSIS OF EVENTS AND RESULTS

SOME of our data on proton-proton elastic scattering at 8.5 Bev have been published earlier.[1] We now present results based on improved statistics.

Two emulsion stacks (hereafter referred to as stacks Nos. 1 and 2) were used in the experiment. Stack No. 1 consisted of 400 NIKFI-BR emulsion pellicles $10 \times 10 \times 2$ cm exposed to the 8.5-Bev internal proton beam of the proton synchrotron of the Joint Institute for Nuclear Research. The beam entered the stack perpendicularly to the plane of the emulsion. The emulsion contained $(2.90 \pm 0.06) \times 10^{22}$ hydrogen atoms per cm^3.

The emulsion was scanned under magnifications of $\times$ 630 and $\times$ 450 over an area 3×3 cm in the central region of the pellicle. The mean beam-proton density in this area was $(2.01 \pm 0.05) \times 10^5$ particles/cm^2. The total volume of emulsion scanned was 8.03 cm^3.

In order to determine the efficiency for finding the events and to increase the reliability of the results, the volume was scanned twice. From the two-prong stars found we selected stars whose external appearance resembled pp elastic scattering. Upon examination, part of them could be rejected as obviously not conforming with the criteria for pp elastic scattering (events classified as "not-to-be-measured"), the remaining part was measured carefully. The events in the latter group ("to-be-measured") were used to determine the scanning efficiency in the c.m.s. angular interval 0 — 12.5°. The efficiency for finding events in the region 0 — 2.5° was investigated very carefully. Events which proved not to be elastic scattering were segregated according to the angular intervals as a function of the "recoil-proton" range.

In order to improve the statistical accuracy in the determination of the scanning efficiency in the angular interval 12.5 — 20.5°, we also used the events of the "not-to-be-measured" type for which the scanning efficiency did not differ from elastic scattering. These events were also sepa-

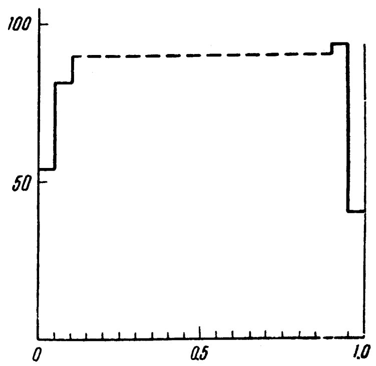

FIG. 1. Depth distribution of the recorded two-prong elastic-like stars. The abscissa axis represents the distance from the glass in fractions of the total pellicle thickness, the ordinate axis gives the number of events.

*Physico-technical Institute, Acad. Sci. Uzbek S.S.R.

Table I

$\theta_{c.m.s.}$, deg	Scanning efficiency Stack 1	Scanning efficiency Stack 2	Differential cross sections, mb/sr Stack 1	Stack 2	Combined data
1.5 — 2.5	0.916 ± 0.030	1.000 — 0.000	153.6 ± 33	142^{+49}_{-41}	149 ± 27
2.5 — 4.5	0.970 ± 0.008	0.918 ± 0.046	124.0 ± 15	103^{+32}_{-26}	120 ± 13
4.5 — 6.5	0.968 ± 0.010	0.914 ± 0.035	93.0 ± 11	92^{+21}_{-15}	93 ± 9.6
6.5 — 8.5	0.945 ± 0.015	0.868 ± 0.049	63.3 ± 7.7	51^{+13}_{-10}	59.5 ± 6.3
8.5 — 10.5	0.845 ± 0.036	—	35.9 ± 5.5	—	35.9 ± 5.5
10.5 — 12.5	0.890 ± 0.040	—	13.3 ± 2.9	—	13.3 ± 2.9
12.5 — 14.5	0.700 ± 0.055	—	6.5 ± 2.1	—	6.5 ± 2.1
14.5 — 16.5		—	4.0 ± 1.5	—	4.0 ± 1.5
16.5 — 18.5		—	1.0 ± 0.7	—	1.0 ± 0.7
18.5 — 20.5		—	0.5 ± 0.5	—	0.5 ± 0.5

rated into angular intervals as a function of the range of the slow proton or its ionization found from gap measurements.

If N_1 is the number of events of a given type found in one scanning and N_2 is the number of events of the same type found in the second scanning, while N_{12} is the number of events which were found in the first scanning that were also found in the second scanning, then, if the scanning efficiency is constant for the entire volume, the efficiency of the first, second, and double scannings are

$$\varepsilon_1 = N_{12}/N_2, \qquad \varepsilon_2 = N_{12}/N_1,$$
$$\varepsilon = [1-(1-\varepsilon_1)(1-\varepsilon_2)],$$

respectively. The statistical error in the determination of the scanning efficiency is given by the expressions* [2]:

$$(\overline{(\Delta\varepsilon_1)^2})^{1/2} = (\varepsilon_1(1-\varepsilon_1)/N_2)^{1/2}, \quad (\overline{(\Delta\varepsilon_2)^2})^{1/2} = (\varepsilon_2(1-\varepsilon_2)/N_1)^{1/2},$$
$$(\overline{(\Delta\varepsilon)^2})^{1/2} = N_{12}\left\{\left(\frac{1-\varepsilon_1}{N_1}\right)^3 + \left(\frac{1-\varepsilon_2}{N_2}\right)^3 + \left(\frac{1-\varepsilon_1}{N_1} + \frac{1-\varepsilon_2}{N_2}\right)^2\left(\frac{1-\varepsilon_1\varepsilon_2}{N_{12}}\right) - 2\left(\frac{1-\varepsilon_1}{N_1} + \frac{1-\varepsilon_2}{N_2}\right)\left[\left(\frac{1-\varepsilon_1}{N_1}\right)^2 + \left(\frac{1-\varepsilon_2}{N_2}\right)^2\right]\right\}^{1/2}.$$

If the conditions given above are not fulfilled, the calculated value of the efficiency is overestimated. However, if the scanning efficiency is high (90 — 97%), this systematic error cannot be appreciable. Upon examination of the events it was important to discard events situated at distances less than 20 μ from the free surface and from the glass in unprocessed emulsion, since such events were missed very frequently (Fig. 1).

The calculated scanning efficiency for stack No. 1 is shown in Fig. 2 and in Table I as a function of the scattering angle.

To separate cases of elastic scattering on a free proton we used the same criteria given earlier in [1]. The range of the recoil proton was measured with an error not exceeding 5%. The angle of emission of the recoil proton was measured to an accuracy of 1.5 — 2°. The scattering angle of the primary proton was measured by means of the method described earlier in [1] to an accuracy of 3 — 4′. Such an accuracy of measurement made it possible to reduce the contribution from background to (0.55 ± 0.15)% (method of estimation of background is described in [1]).

Altogether, 354 cases (including 145 cases reported earlier[1]) satisfying the elastic scattering criteria within the limits of three standard deviations were found. The measured differential cross sections are shown in Table I.

In order to improve the statistics in the region of small scattering angles we used the water-soaked stack No. 2.[3] The stack was exposed to an 8.2-Bev internal proton beam in the proton synchrotron of the Joint Institute for Nuclear Research also perpendicularly to the plane of the emulsion pellicles. The beam density at the time of exposure was 1.8×10^5 protons/cm^2. The emulsion was also scanned twice with an immersion

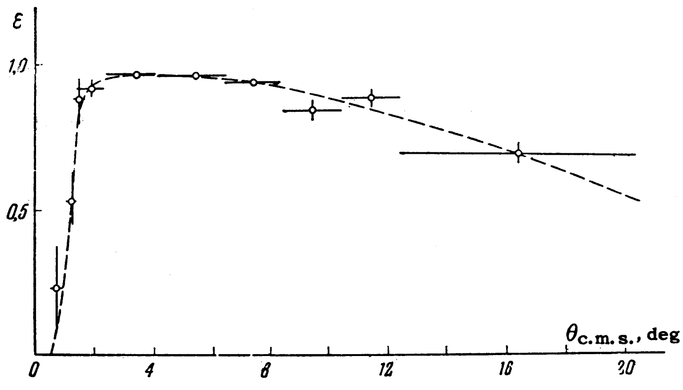

FIG. 2. Variation of the scanning efficiency for pp elastic scattering for a double scanning as a function of the c.m.s. scattering angle.

*In[2] and[10] it was shown that the formulas for $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$ in[11] are not valid. In[2], moreover, it was shown that the formula for $\Delta\varepsilon$ in[11] is also not valid.

objective under a magnification $\times$ 630. The events were analyzed in the manner described previously in [1].

To determine the angle of emission of the scattered proton θ, we carried out, as a rule, the "coarse" measurements described in [1]. The standard deviation of the beam divergence was 5′. The mean thickness of the pellicles was 1100 μ. In this case the accuracy of the measurement of θ was about 6′. Events with range R $\lesssim$ 200 μ, doubtful cases, and 12 cases of different range for establishing the range—energy curve were measured accurately[1] on a base of 3300 μ. The contribution from background events to the number of separated cases is (1.0 — 1.3)%. The scanning efficiency was determined from cases of scattering and elastic-like events of the "to-be-measured" type with the recoil-proton ranges lying in the same interval. The efficiencies are also shown in Table I.

The water-soaked emulsion contained (5.40 ± 0.13) $\times 10^{22}$ hydrogen nuclei per cm^3. The use of the water-soaked stack made it possible to increase the speed for finding elastic scattering events in the small-angle region two- to threefold. We found in this stack 126 cases of elastic scattering, of which 107 were in the angular interval 1.5 — 8.5 (c.m.s.). To determine the differential cross section we introduced a correction for the loss of cases at the glass and free surface of the emulsion. The data for stack No. 2 are shown in Table I along with the combined data for stacks Nos. 1 and 2.

The elastic scattering cross section turned out to be 8.74 ± 0.40 mb.

2. DISCUSSION OF RESULTS

We have shown earlier that the measured values of the differential cross sections at small angles greatly exceed the differential cross sections at 0° calculated from the optical theorem under the assumption of a spin-independent interaction. The total cross section for pp interactions σ_{tot} was taken there as 30 mb. Subsequently, it was found that the total cross section considerably exceeded this value;[4,5] the mean value from the two measurements is $\sigma_{tot} = 41.5 \pm 1.0$ mb. According to our data, the differential cross section at 2° is 149 ± 27 mb/sr, while the optical theorem leads to the value 111 ± 5 mb/sr.

As was stressed earlier,[1] the discrepancy between the experimental data on the differential cross sections in the small-angle region and the value calculated from the spin-independent model of a purely absorbing proton can be explained only by the existence of a real part in the scattering amplitude or a difference in the total pp interaction cross section in the singlet and triplet states, or both factors simultaneously. The study of the interference between Coulomb and nuclear scattering can clarify this matter. If the amplitude of the nuclear scattering has a real part comparable to the imaginary part, then we should observe interference between the nuclear and Coulomb scattering, depending on the sign of the real part. Conversely, if the real part of the scattering amplitude is small, then the increase in the differential cross section close to the angle zero to a value greater than that given by the optical theorem can be explained only by the dependence of the cross section on the spin states.

To study these possibilities we carried out calculations according to the following schemes.

A. We considered a complex potential[6] varying with the distance by a Gaussian law. It was assumed for simplicity that the interactions involving the proton spins σ_1 and σ_2 can be due either to spin-orbital or spin-spin forces (tensor forces are not considered). We assumed that at large energies E the quasi-classical approximation is valid (in our case the wagelength $\lambdabar$ is 0.99 $\times 10^{-14}$ cm, which is much smaller than the proton radius) and we calculated the nuclear phase shift from the formula

$$\delta_l = -\frac{E}{\hbar^2 c^2 k} \int\limits_{(l+1/2)/k}^{\infty} \frac{V(r, \sigma_1, \sigma_2)\, dr}{\sqrt{r^2 - k^{-2}(l+1/2)^2}}.$$

The Coulomb phase-shift calculations followed the method given by Stapp, Ypsilantis, and Metropolis.[7]

From the well-known expression for the M matrix of identical particles with spin ½ and with allowance for the Coulomb interaction, we calculated, by the method of least squares, the best fit for the differential cross sections and the corresponding parameters for the potential. In a number of variants we also used the experimentally determined total cross sections for pp interactions. At the same time, we calculated the total and inelastic pp cross sections from the identical-particle formulas.

Silin and Shakhbazyan showed[6]* that the spin-orbit interaction, at least in the generally adopted form, cannot cause a strong difference in the

*As a result of errors made in the calculation and in the program for the calculation of model 5 in[6] the conclusion in[6] that only one variant can occur when the real part of the potential has a plus sign and the singlet state predominates over the triplet state is not valid.

Table II*

Calculation A		Calculation B	
Given parameters	Results of calculation	Given parameters	Results of calculation
σ_{tot} not fixed $\varkappa \equiv 1$ $u \equiv 0$	$\sqrt{\overline{r^2}}=(1.19\pm0.04)\cdot10^{-13}$cm $w = 53.1 \pm 5.2$ Mev $\sigma_{tot} = 48.3 \pm 1.8$ mb $\chi^2 = 3.87$	σ_{tot} not fixed $\varkappa \equiv 1$ $A \equiv 0$	$\sqrt{\overline{r^2}}= (1.15 \pm 0.04)\cdot10^{-13}$cm $B=(0,554 \pm 0.056)\cdot10^{14}$ cm^{-1} $\sigma_{tot} = 47.6 \pm 1.6$ mb
$\varkappa \equiv 1$ $u_{\text{initial}} > 0$	$\sqrt{\overline{r^2}}= (1.22 \pm 0,05)\cdot10^{-13}$cm $u = 32,5 \pm 3$ Mev $w = 34.6 \pm 5.6$ Mev $\chi^2 = 7.6$	$\varkappa \equiv 0.8$ $A_{\text{initial}} > 0$	$\sqrt{\overline{r^2}}= (1.15 \pm 0.04)\cdot10^{-13}$cm $A = (0.348 \pm 0.049)\cdot10^{14}$cm^{-1} $B = (0.623 \pm 0.092)\cdot10^{14}$ cm^{-1} $\chi^2 = 5,76$
$\varkappa \equiv 1$ $u_{\text{initial}} < 0$	$\sqrt{\overline{r^2}}= (1.15 \pm 0.04)\cdot10^{-13}$cm $u = -26.1 \pm 4.2$ Mev $w = 46.3 \pm 6.7$ Mev $\chi^2 = 6.06$	$\varkappa \equiv 1$ $A_{\text{initial}} < 0$	$\sqrt{\overline{r^2}}= (1.23 \pm 0.04)\cdot10^{-13}$cm $A=(-0.350\pm0.029)\cdot10^{14}$cm^{-1} $B = (0.398 \pm 0.062)\cdot10^{14}$cm^{-1} $\chi^2 = 6.96$
$\varkappa_{\text{initial}} < 1$ $u_{\text{initial}} > 0$	$\varkappa = 0.24 \pm 0.11$ $\sqrt{\overline{r^2}}= (1,11 \pm 0.10)\cdot10^{-13}$cm $u = 41,5 \pm 71,8$ Mev $w = 138,8 \pm 89,0$ Mev $\chi^2 = 6.29$	$\varkappa_{\text{initial}} < 1$ $A_{\text{initial}} > 0$	$\varkappa = 0,28 \pm 0.18$ $\sqrt{\overline{r^2}}= (1.14 \pm 0.11)\cdot10^{-13}$ cm $A= (0,490 \pm 0.560)\cdot10^{14}$ cm^{-1} $B= (1.36 \pm 1.09)\cdot10^{14}$ cm^{-1} $\chi^2 = 5.85$
$\varkappa_{\text{initial}} < 1$ $u_{\text{initial}} < 0$	$\varkappa = 0.29 \pm 0.28$ $\sqrt{\overline{r^2}}= (1,11 \pm 0.08)\cdot10^{-13}$cm $u = -48.7 \pm 19,5$ Mev $w = 108.6 \pm 98$ Mev $\chi^2 = 5.85$	$\varkappa_{\text{initial}} < 1$ $A_{\text{initial}} < 0$	$\varkappa = 0.34 \pm 0.29$ $\sqrt{\overline{r^2}}= (1.12 \pm 0.07)\cdot10^{-13}$ cm $A= (-0,401\pm0.33)\cdot10^{14}$ cm^{-1} $B = (1.25 \pm 0.99)\cdot10^{14}$ cm^{-1} $\chi^2 = 5.85$
$\varkappa_{\text{initial}} < 1$ $u \equiv 0$	$\varkappa = 0.25 \pm 0.07$ $\sqrt{\overline{r^2}}= (1.09 \pm 0.04)\cdot10^{-13}$cm $w = 144.8 \pm 28.8$ Mev $\chi^2 = 6.15$		
$\varkappa_{\text{initial}} > 1$ $u \equiv 0$	$\varkappa = 16.5 \pm 67.8$ $\sqrt{\overline{r^2}}= (1.13 \pm 0.06)\cdot10^{-13}$cm $w = 4.3 \pm 18.0$ Mev $\chi^2 = 6.55$		

*Everywhere except for the first row we took $\sigma_{tot} = 41.5 \pm 1.0$ mb.

scattering cross sections of the singlet and triplet states at high energies. We therefore considered a complex potential of form

$$V(r, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2) = -\{(u_1 + iw_1) + (-1)^{S+1}(u_2 + iw_2)\overline{(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)}\}\exp\{-\gamma^2 r^2\},$$

where S is the total spin of the system of two protons, $\overline{(\sigma_1\sigma_2)}$ are the eigenvalues of the operator $(\sigma_1\sigma_2)$, and the parameter γ is connected with the rms radius of the interaction by the relation

$$(\overline{r^2})^{1/2} = \sqrt{3/2}/\gamma.$$

The following elements of the scattering matrix are different from zero: M_{SS}, $M_{1,1} \equiv M_{-1-1} \equiv M_{00} = M_t$. If the equality of the particle masses are taken into account, we obtain a factor 2 in the expressions for the total cross sections. For simplicity, we determined the parameters of the singlet potential and the value of the ratio of the triplet to the singlet potential, which is assumed to be the same for the real and imaginary parts, i.e., we used the expressions

$$V_S = -(u + iw)e^{-\gamma^2 r^2}, \qquad V_t = \varkappa V_S.$$

The transition from u, w, κ to the quantities u_1, w_1, u_2, w_2 is given by the relations

$$u_1 = \tfrac{1}{2}u(1-\varkappa), \quad w_1 = \tfrac{1}{2}w(1-\varkappa),$$
$$u_2 = \tfrac{1}{2}u(3\varkappa-1), \quad w_2 = \tfrac{1}{2}w(3\varkappa-1).$$

B. We used the optical model in which we considered the dependence of the complex refractive index on the spin states. The Coulomb interaction was taken into account by the method of Bethe.[8] Here, however, the nuclear scattering amplitude was not written in the Born approximation, but in the quasi-classical approximation and was different for the singlet and triplet states. In order to limit ourselves to the least number of parameters, we also used a simplified dependence of the complex refractive index on the spin states. The refractive indices for the singlet s and triplet t states were taken with a Gaussian dependence on the distance:

$$K_s = (A + iB)e^{-r^2/a^2}, \qquad K_t = \varkappa K_s.$$

The quantity a is connected with the rms radius of interaction by the relation $(\overline{r^2})^{1/2} = \sqrt{3/2}\,a$.

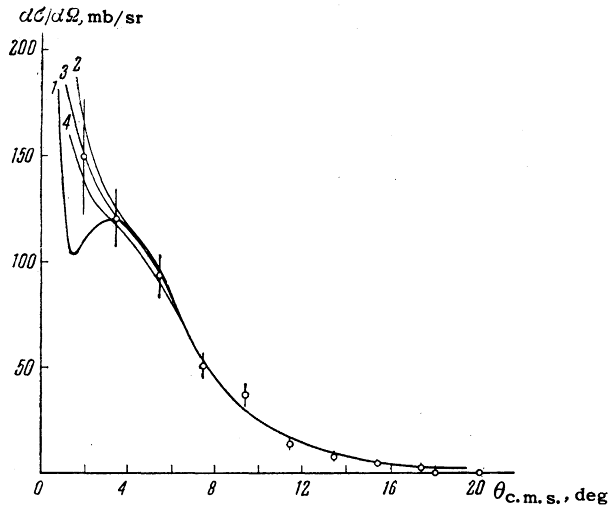

FIG. 3. Experimentally measured differential cross sections and the best fits calculated for variants: 1) $\kappa = 1$, $u > 0$; 2) $\kappa = 1$, $u < 0$; 3) $\kappa = 1$, $u = 0$; 4) $\kappa = 0.24$, $u > 0$.

The nuclear and electromagnetic form factors were taken to be the same. It should be noted that, under the conditions of applicability of the optical model, the amplitudes for identical and nonidentical particles reduce to the same expression. We used in the calculation the experimental value of the total cross section and the method of least squares for the calculation of the model parameters.

To characterize the deviation of the calculated curve from the experimental points in both schemes of calculation, we used the quantity χ^2, whose mean value is $\overline{\chi^2} = n - m$, where n is the number of experimental points, and m is the number of unfixed parameters of the model.

The errors in the determination of the parameters calculated from the error matrix by a linearization method are not valid if the function is already strongly nonlinear in these parameters within the limits of error (of course, if the errors are small, the linearity condition can be used). Hence we employed the quantity χ^2 to estimate the errors. If the selected function is linear with respect to the parameters, then if one of the parameters is changed from its value at the minimum by one standard deviation and if all the remaining parameters are minimized, χ^2 increases to unity; for a change by two standard deviations, χ^2 increases to 4, etc. In the general case, such an estimate can be invalid; however, it can frequently be used in the nonlinear case, too, since here the linearity condition is not a necessary one. A sufficient condition is the possibility of obtaining a good approximation of the second derivatives of the selected function with respect to its parameters in terms of the first derivatives. This condition is also necessary for good convergence of the linearization method used in our case.

The results of the calculations by the two schemes of calculation are shown in Table II. In the most general case when none of the four parameters are fixed, solutions exist for $u > 0$ ($A > 0$) and $u < 0$ ($A < 0$) for both $\kappa < 1$ and $\kappa > 1$. In the latter case the parameters cannot be estimated with any reasonable accuracy and therefore this solution is not shown. The variant with $\kappa = 0$ (the absence of triplet states) is rejected by the χ^2 criterion, since $\chi^2 = 59.5$ with $\overline{\chi^2} = 8$.

It follows from the calculations that within the framework of the models employed, proton-proton scattering cannot be described without spin and a real part of the potential. Indeed, if σ_{tot} is not fixed in the initial data, then its subsequent calculation from the parameters for the best-fit curve leads to a calculated value greater than the experimental one, the difference $\sigma_{tot\,calc} - \sigma_{tot\,exptl}$ is approximately three full standard deviations. In this case the differential cross sections for purely nuclear scattering at 0° are 151 ± 11 and 146 ± 9 mb/sr, respectively, for calculations A and B.

The variants $\kappa = 1$ and $\kappa < 1$ for $u < 0$ ($A < 0$) do not differ from each other according to the χ^2 criterion; the values of χ^2 in these variants differ by less than unity. In the variants $\kappa = 1$ and $\kappa < 1$ for $u > 0$ ($A > 0$) the values of χ^2 differ by the quantity ~ 1.3. Hence, if we initially assume that the true value of κ lies in the region $0 < \kappa < 1$, then in the case $u > 0$ ($A > 0$), we obtain

$$\varkappa = 0.24^{+0.76}_{-0.11}.$$

With good statistics it should, perhaps, be possible to distinguish this case from the case $\kappa = 1$, $u > 0$ ($A > 0$). However, with our statistical accuracy this difference lies within the limits of one standard deviation, which is illustrated in Fig. 3 (see curves 1 and 4).

The calculation shows that the experimental results can also be explained without the assumption of the existence of a real part of the potential ($u = 0$). As seen from Table II, if it is assumed that $\kappa < 1$, then we obtain $\kappa = 0.25 \pm 0.07$, and for $\kappa > 1$ the best value is $\kappa = 16.5$, but with a large error.

We also carried out an analysis based on the assumptions made by Grishin et al[9]. Here it was assumed that the real part of the scattering amplitude and its dependence on the spin can be neglected if the scattering amplitude is considered not to change sign in the angular intervals 0 — 90° and the identity of the particles is neglected. With

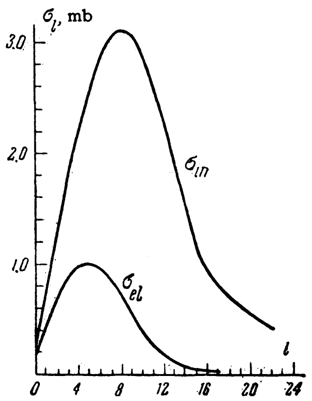

FIG. 4. Partial cross sections for pp elastic σ_{el} and inelastic σ_{in} interactions obtained under the assumptions formulated by Grishin et al.[9]

such an approach we calculated from the experimental data the values $\beta_l = \exp(2i\delta_l)$, where δ_l is the phase shift.

From the unitarity conditions we have $0 \leq \beta_l \leq 1$. We calculated the values of β_l for all values of l up to $l_{max} = 22$. The smallest value should be obtained for β_0. The calculated value was $\beta_0 = +0.27$ and in this sense such a view of the obtained experimental data does not contradict the unitarity condition, although such an approach is not in agreement with the results of the foregoing calculations. This is due to the fact that β_0 is determined with a large error, since the basic contribution to β_0 comes from scattering at large angles. It thus follows that the unitarity criterion can only be used in the case of much greater accuracy.

The pp total interaction cross section is found to be $\sigma_{tot} = 47.3$ mb, which is in agreement with the foregoing calculations. In view of the smallness of the contribution of the first phases, the uncertainty in the total cross section is small.

The partial cross sections for elastic and inelastic interactions obtained in the calculation are shown in Fig. 4. It is seen that the maximum of the partial contributions is observed at $l = 5$ for the elastic interaction and at $l = 8$ for the inelastic interaction. As has already been indicated, the basic contribution to the first phase shifts comes from large-angle scatterings, which were not measured by us, and thus the errors in the calculated first phase shifts are large. However, for β_7, for example, the errors are less than 15%.

3. CONCLUSIONS

1. The rms pp interaction range is independent of all the models discussed above and turns out to be $(1.15 \pm 0.05) \times 10^{-13}$ cm.

2. A difference of three standard deviations is observed between the experimental data and the results of the calculations if it is assumed that the scattering amplitude does not depend on the proton spins and does not contain a real part. An attempt was made to explain this disparity by the existence of a spin-spin interaction and the presence of a real part in the scattering amplitude. If the scattering amplitude does not depend on the spins, then the real part of the scattering amplitude does not exceed 0.5 of the imaginary part and takes on its maximum value.

If it is assumed that the scattering amplitude has no real part, there must be a difference between the interactions in the singlet and triplet states. If it is assumed that the interaction in the singlet state predominates over the interaction in the triplet state ($\kappa < 1$), then $\kappa = 0.25 \pm 0.07$. If the reverse occurs ($\kappa > 1$), then the best value is $\kappa = 16.5$ and as κ approaches unity the value of χ^2 increases and passes through the value 1 when $\kappa = 6.5$. The statistics obtained in this experiment do not permit us to establish which is the cause of the observed discrepancy—the real part of the scattering amplitude or the spin-spin interaction.

In conclusion, the authors express their sincere gratitude to V. I. Veksler for his constant interest and to K. D. Tolstov for assistance in this work.

[1] Lyubimov, Markov, Tsyganov, Cheng, and Shafranova, JETP **37**, 910 (1959), Soviet Phys. JETP **10**, 651 (1960); Markov, Tsyganov, Shafranova, and Shakhbazyan, JETP **38**, 1471 (1960), Soviet Phys. JETP **11**, 1063 (1960).

[2] M. I. Podgoretskiĭ and E. N. Tsyganov, Preprint, Joint Institute for Nuclear Research (in press).

[3] Do, Kriventsova, Lyubomilov, and Shafranova, Materialy III Mezhd. konf. po yadern. fotografii, (Proceedings of Third International Conf. on Nuclear Photography), Moscow, 1960.

[4] von Dardel, Frisch, Mermod, Milburn, Piroué, Vivargent, Weber, and Winter, Phys. Rev. Lett. **5**, 333 (1960).

[5] Ashmore, Cocconi, Diddens, and Wetherell, Phys. Rev. Lett. **5**, 576 (1960).

[6] I. N. Silin and B. A. Shakhbazyan, Preprint D-616, Joint Institute for Nuclear Research, Dubna, 1960.

[7] Stapp, Ypsilantis, and Metropolis, Phys. Rev. **105**, 302 (1957).

[8] H. Bethe, Ann. Phys. **3**, 190 (1958).

[9] Grishin, Saitov, and Chuvilo, JETP **34**, 1221 (1958), Soviet Phys. JETP **7**, 844 (1958); Blokhintsev, Barashenkov, and Grishin, JETP **35**, 311 (1958), Soviet Phys. JETP **8**, 215 (1959).

[10] K. D. Tolstov, Preprint, Joint Institute for Nuclear Research (in press).

[11] Lim, Laby, and Hopper, Nuovo cimento, Suppl. **15**, 382 (1960).

Translated by E. Marquit
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