nium, the [111] axis of which was normal to the plane of the disc, while the normal itself made an angle $\theta = 90°$ with the direction of the magnetic field. The measurements were made at room temperature in a magnetic field of 24,500 oe. The e.m.f. was measured as usual in the direction perpendicular to the magnetic field direction and the direction of illumination. The curve 1b refers to the photomagnetic e.m.f. measured when the angle $\theta \approx 130°$.

By analogy with the anisotropy of the even photomagnetic effect,[2] it was supposed that the measured odd photomagnetic e.m.f. can be regarded as the sum of isotropic and anisotropic components. It is clear that the purely anisotropic component can be obtained by measuring the odd photomagnetic e.m.f. in the direction of the magnetic field or its projection on the plane of the surface. Experiments performed in fact confirmed the presence of an odd photomagnetic e.m.f. when measuring it in the direction of the magnetic field, i.e., in the direction in which there is no isotropic odd photomagnetic effect. The variation of this photomagnetic e.m.f. on the angle φ is given in Fig. 1d (the continuous curve is the function $E = a \sin 6\varphi$). Figure 1c shows the anisotropy curve for the same specimen when $\theta = 75°$ (the continuous curve is the function $E = a \sin 3\varphi + b \sin 6\varphi$).

Curves are given in Fig. 2 showing the variation of the extreme values of the odd photomagnetic e.m.f. with the magnetic field strength. Curves 2a and 2b refer to the photomagnetic e.m.f. measured in the "usual" direction (perpendicular to the magnetic field) for two values of φ: 75°45′ and 22°30′ (see Fig. 1b). The specimen was oriented relative to the magnetic field so that $\theta \approx 130°$. For this value of θ the anisotropic component of the photomagnetic e.m.f. attains a maximum value. Curve 2c shows the variation of the extremal value of the purely anisotropic component of the odd photomagnetic e.m.f. on the magnetic field strength when $\theta \approx 130°$.

The curves presented show that the variation of the odd photomagnetic e.m.f. on the magnetic field is essentially nonlinear. To explain the observed anisotropy of the odd photomagnetic effects in strong magnetic fields, it is apparently necessary to include terms of higher odd degree in the magnetic field in a general phenomenological equation of the Kagan-Smorodinskii[3] type.

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229

RESISTANCE OF THIN SINGLE-CRYSTAL WIRES

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MEASUREMENT of the electrical conductivity of thin metal wires is usually used to determine electron mean free paths.[1] In the standard method for this, the results obtained for the dependence of resistivity on wire diameter are compared with the theoretical curve obtained by Dingle.[2] It must be remembered that Dingle's results were obtained on the assumption of an isotropic, quadratic dispersion law for the electrons. As a result, the ratio ρ/ρ_∞ (ρ_∞ is the resistivity of an infinitely thick wire and ρ that of a wire of diameter d) is expressed as a function of d/λ only (λ is the electron mean free path).

One of us (B. A.) has measured the dependence of the resistivity of tin single crystal wires on diameter. The tin used in the experiments was first subjected to zone refinement.[3] The purity is

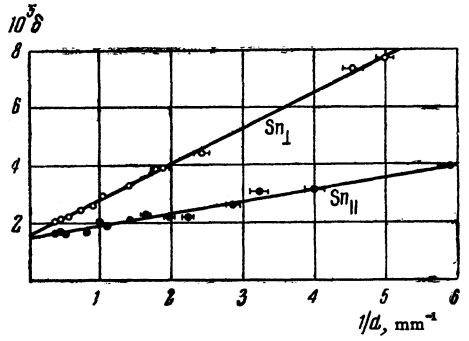

FIG. 1

calculated as 99.99986% ($\delta_{4.2\infty} = 1.5 \times 10^{-5}$). The results obtained are shown in Fig. 1 in the form of the dependence of $\delta = R_{4.2}/R_{293}$ ($R_{4.2}$ is the electrical resistance of a wire at 4.2° K, and R_{293} the resistance of the same wire at 20° C) on the reciprocal of the diameter. The lower curve refers to wires with axis parallel to the principal crystal axis ($Sn_{\parallel}$), and the upper curve to wires with axis perpendicular to the principal axis ($Sn_{\perp}$). In the latter case the wire axis coincides with the [110] direction. These orientations of the single crystals were determined with an accuracy of 2—4°. Since δ was measured with an accuracy not worse than 2—3%, while the error in determining 1/d increased with decreasing diameter, we can say that the slope of the straight lines in Fig. 1 were determined with an accuracy of 1—3%. The slope of the line for $Sn_{\perp}$ is approximately three times greater than for $Sn_{\parallel}$, i.e., the difference in slopes lies definitely outside the experimental errors.*

It is only possible to take a theoretical discussion of the electrical conductivity of single crystal wires to the end (starting from the assumption of an arbitrary dispersion law for the electrons) for the case when the mean free path λ is appreciably greater than the wire diameter. If the axis of the cylindrical wire is perpendicular to the symmetry plane of the crystal, the mean electrical conductivity $\sigma(d)$ is of the form

$$\sigma(d) \approx \frac{8de^2}{3\pi(2\pi\hbar)^3} I, \qquad I = \oint \frac{(\mathfrak{N}\mathbf{b})^2}{\sqrt{1-(\mathfrak{N}\mathbf{b})^2}}\, dS, \tag{1}$$

where $\mathbf{b}$ is a unit vector in the direction of the axis of the wire, $\mathfrak{N}$ is a unit vector normal to the Fermi surface, dS is an element of area on the Fermi surface, and the integration is over the whole surface. We should note that in the derivation of this expression there are no additional restrictions other than those usually assumed to be fulfilled (for example, it was assumed that the temperature is considerably below the Fermi energy). In particular, (1) is also valid for the model of a Fermi liquid. If we introduce the Gaussian curvature of the Fermi surface $K(\theta, \varphi)$ and choose the direction of the vector $\mathbf{b}$ as the polar axis, then (1) can be written in the following form:

$$\sigma(d) \approx \frac{8de^2}{3\pi(2\pi\hbar)^3} \oint \frac{\cos^2\theta\, d\theta\, d\varphi}{K(\theta, \varphi)}. \tag{2}$$

The integrals in (1) and (2) cannot, naturally, be evaluated without an assumption about the dispersion law for the electrons. For a quadratic, anisotropic dispersion law

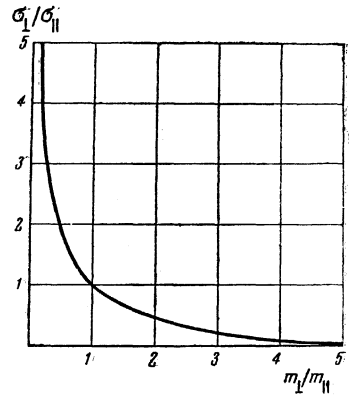

FIG. 2

$$\varepsilon(p) = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3}, \tag{3}$$

and taking the vector $\mathbf{b}$ to be parallel to the first axis, we obtain

$$\sigma(d) = \frac{4de^2}{3\pi\hbar}\left(\frac{3n}{8\pi}\right)^{2/3} (k_2k_3)^{-1/6} [F(k_2, k_3) + F(k_3, k_2)], \tag{4}$$

where n is the number of electrons per unit volume,

$$k_2 = \frac{m_1}{m_2}, \qquad k_3 = \frac{m_1}{m_3},$$

$$F(k_2, k_3) = \int_0^{\pi/2} \frac{d\varphi}{(k_2\cos^2\varphi + k_3\sin^2\varphi)^{1/2}}. \tag{5}$$

For the isotropic case ($k_2 = k_3 = 1$) we obtain the well known Dingle formula:[2]

$$\sigma(d) \approx (4de^2/3\hbar)\,(3n/8\pi)^{2/3} = \sigma_\infty d/\lambda.$$

If $m_2 = m_3 = m_{\perp}$, while $m_1 = m_{\parallel}$, then

$$\sigma(d) \equiv \sigma(d)_{\parallel} = \frac{4de^2}{3\hbar}\left(\frac{3n}{8\pi}\right)^{2/3}\left(\frac{m_{\perp}}{m_{\parallel}}\right)^{2/3}. \tag{6}$$

If $m_1 = m_2 = m_{\perp}$, while $m_3 = m_{\parallel}$ ($k_2 = 1$, $k_3 = k = m_{\perp}/m_{\parallel}$), then

$$\sigma(d) \equiv \sigma(d)_{\perp} = \frac{4de^2}{3\pi\hbar}\left(\frac{3n}{8\pi}\right)^{2/3}\left(\frac{m_{\parallel}}{m_{\perp}}\right)^{1/3} [F(1, k) + F(k, 1)]. \tag{7}$$

From (6) and (7) we obtain

$$\frac{\sigma_{\perp}(d)}{\sigma_{\parallel}(d)} = \frac{1}{\pi k}[F(1, k) + F(k, 1)] \approx \begin{cases} \dfrac{1}{\pi k}\ln\dfrac{1}{k}, & k \ll 1 \\ 1, & k = 1 \\ \dfrac{\ln k}{\pi k^{3/2}}, & k \gg 1 \end{cases} \tag{8}$$

The dependence of $\sigma_{\perp}(d)/\sigma_{\parallel}(d)$ on the ratio of effective masses [Eq. (8)] is shown graphically in Fig. 2.

From the slope of the curves of Fig. 1 and the values $\rho_{\parallel 293} = 14.3 \times 10^{-6}$ ohm cm and $\rho_{\perp 293} = 9.85 \times 10^{-6}$ ohm cm,[4] we determined the products

$[d\Delta\rho(d)]_{\parallel} = d[\rho_{\parallel}(d) - \rho_{\parallel}(\infty)] = 0.61 \cdot 10^{-11}\,\Omega \cdot \mathrm{cm}^2$,

$[d\Delta\rho(d)]_{\perp} = 1.22 \cdot 10^{-11}\,\Omega \cdot \mathrm{cm}^2$.

Using this data we can determine the values of the integrals entering into (1) for the two crystallographic directions:

$$I_{\parallel} = 1.335 \cdot 10^{-48}/[d\Delta\rho(d)]_{\parallel} = 2.2 \cdot 10^{-37} \text{ cgs esu}$$

$$I_{\perp} = 1.335 \cdot 10^{-48}/[d\Delta\rho(d)]_{\perp} = 1.1 \cdot 10^{-37} \text{ cgs esu} \quad (9)$$

The difference in the slopes of the curves of Fig. 1 are thus connected with different values of the surface integrals [Eq. (1)] in different crystallographic directions, i.e., with the shape of the Fermi surface for tin. From this point of view, an experimental study of the difference in the slopes of the straight lines (in δ, 1/d coordinates) for Pb, Cu, Au, Ag and possibly Al in the [100] and [111] directions is of interest; these are the directions in which maximum differences of conductivity of thin wires of cubic crystals are expected (the existence of complicated open Fermi surfaces are assumed for these metals, except Al[5]).

If the crystal anisotropy is described in terms of effective masses, then for tin, for which $\Delta\rho_{\parallel}(d)/\Delta\rho_{\perp}(d) = 0.5$ (at 4.2° K) we must take $m_{\perp}/m_{\parallel} = 1.85$ from Fig. 2 and $n = 4.4 \times 10^{22}$ [calculated according to (6)].

Since the number of atoms per unit volume $n_A = 3.7 \times 10^{22}$ cm^{-3} for tin, there must be $n/n_A = 1.2$ conduction electrons per atom. Naturally, this number cannot be considered the true number of conduction electrons per atom as it was obtained from very simplifying assumptions about a quadratic dispersion law, while from galvanomagnetic and magnetic experiments it is known that the Fermi surface for tin is very complicated.[6]

*A detailed discussion of the experimental method and results will be given separately.

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230