PHOTOPRODUCTION OF NEUTRINOS ON ELECTRONS AND NEUTRINO RADIATION FROM STARS

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Photoproduction of a neutrino-antineutrino pair on an electron is considered. The cross section for this process is calculated, and the photoneutrino emission by a degenerate or nondegenerate electron gas at high temperature and density is determined in the nonrelativistic approximation. The photoneutrino emission from an electron gas of density 10^5 g/cm^3 at temperatures $\kappa T \geq 40$ kev (κ is Boltzmann's constant) exceeds the neutrino bremstrahlung radiation under the same conditions [4] by two orders of magnitude. In stars with high temperature and density (such as novae between bursts, etc.) the energy radiated in the form of photoneutrinos exceeds the usual photon luminosity.

1. INTRODUCTION

ACCORDING to the theory of Feynman and Gell-Mann [1] there is a direct neutrino-electron scattering process $\nu + e \rightarrow \nu' + e'$ with a matrix element

$$G2^{-1/2}(\bar{e}'\gamma_\mu(1+\gamma_5)\nu)(\bar{\nu}\gamma_\mu(1+\gamma_5)e) \qquad (1)$$

which is of first order in the weak interaction coupling constant. Direct experimental study of this process is extremely difficult at the present time; however, its existence can lead to macroscopic effects which are important in astrophysics. In 1941 Gamow and Schoenberg [2] showed that at the high temperatures and densities which exist in the interiors of stars in the last stages of evolution the process of nuclear electron capture and subsequent beta decay

$$N_Z^A + e^- \rightarrow N_{Z-1}^A + \nu, \qquad N_{Z-1}^A \rightarrow N_Z^A + e^- + \bar{\nu}, \qquad (2)$$

with the emission of two neutrinos, becomes possible. Although the cross section for this process is very small, the energy carried out of the star by neutrinos from this process can exceed the energy emitted in the form of photons. This is because neutrinos travel out freely from the star's interior, while photons have a very short free path and are therefore radiated only by the external envelope of the star. It should be noted, however, that process (2) has an energy threshold, and therefore stellar neutrino emission from this process depends on the presence of nuclei with a low threshold.

Pontecorvo [3] pointed out that if the direct electron-neutrino interaction (1) occurs then neutrino pair bremstrahlung can occur in the scattering of an electron by a nucleus:

$$e + A \rightarrow \nu + \bar{\nu} + e' + A'. \qquad (3)$$

This process is unlike process (2) in that it does not have a threshold; in stars with high temperatures and densities and also high Z it can be an important energy radiation mechanism. [3] Gandel'man and Pinaev [4] showed that in stars with Z about 10, temperature $\kappa T \geq 30$ kev, and density $\rho > 10^5$ g/cm^3, the energy carried off by neutrinos formed in process (3) exceeds the energy radiated in the form of photons.

We wish to point out that the existence of a direct electron-neutrino interaction also leads to the photoproduction of neutrino pairs on electrons:

$$\gamma + e \rightarrow e' + \nu + \bar{\nu}. \qquad (4)$$

This process is first order in the weak interaction and electromagnetic coupling constants. Since the photon number density increases very rapidly with temperature and can be comparable with and even exceed the density of electrons and nuclei at high temperatures, process (4), like process (3), can be an important energy radiation mechanism in stars with high temperature and density.

In this work we calculate the neutrino pair photoproduction cross section on electrons and the photoneutrino radiation by a degenerate or nondegenerate electron gas as a function of temperature

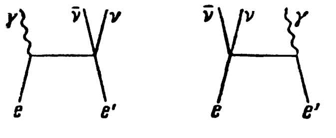

FIG. 1

and density. We show that the photoneutrino radiation by an electron gas is two orders of magnitude greater than the neutrino bremstrahlung radiation under the same conditions. This is mainly due to the fact that the neutrino pair photoproduction cross section increases rapidly with photon energy and the neutrino bremstrahlung cross section increases with increasing energy, while the photon spectrum is shifted to higher energies as compared to the electron spectrum at the same temperature.

2. CROSS SECTION FOR NEUTRINO PAIR PHOTOPRODUCTION OF ELECTRONS

We let $p = (\mathbf{p}, E)$ and $p' = (\mathbf{p}', E')$ be the initial and final electron four-momenta and $k = (\mathbf{k}, \omega)$, $q_1 = (\mathbf{q}_1, \epsilon_1)$, and $q_2 = (\mathbf{q}_2, \epsilon_2)$ be the four-momenta of the photon, neutrino, and antineutrino. The formation of a neutrino-antineutrino pair in the collision of a photon and an electron is described by the two Feynman diagrams shown in Fig. 1. The matrix element for this process is

$$M = \frac{eG}{2\sqrt{\omega}}\{(\bar{v}_n\gamma_\mu(1+\gamma_5)(i\hat{p}_1+m)^{-1}\hat{e}e)(\bar{e}'\gamma_\mu(1+\gamma_5)v_a)$$
$$+(\bar{e}'\hat{e}(i\hat{p}_2+m)^{-1}\gamma_\mu(1+\gamma_5)v_a)(\bar{v}_n\gamma_\mu(1+\gamma_5)e)\}, \quad (5)$$

where e_μ is the photon polarization four vector and

$$p_1 = p + k, \quad p_2 = p' - k, \quad p_f = p' + q_1 + q_2,$$
$$p_i = p + k.$$

For calculation, it is convenient to use a Fierz transformation to obtain M in the form $(\bar{e}'Oe)(\bar{\nu}O\nu)$. Then

$$M = \frac{eG}{2\sqrt{\omega}}(\bar{e}'[\gamma_\mu(1+\gamma_5)(i\hat{p}_1+m)^{-1}\hat{e}$$
$$+\hat{e}(i\hat{p}_2+m)^{-1}\gamma_\mu(1+\gamma_5)]e)$$
$$\times(\bar{v}_n\gamma_\mu(1+\gamma_5)v_a). \quad (6)$$

We then obtain for the differential cross section averaged over initial electron spin direction and photon polarization and summed over final electron spin directions

$$d\sigma = -\frac{1}{(2\pi)^5(pk)}\int\frac{1}{4}\sum|F|^2\delta^4(p_f-p_i)\frac{d^3p'\,d^3q_1\,d^3q_2}{E'\varepsilon_1\varepsilon_2}, \quad (7)$$

where F is the invariant amplitude, $M = (\omega EE'\epsilon_1\epsilon_2)^{-1/2}F$ and

$$-(4e^2G^2)^{-1}\Sigma|F|^2 = [m^2(p'q_1)(p_1q_2)$$
$$+(pk)(p'q_1)(kq_2)](pk)^{-2}+[(pp')[(p_2q_1)(pq_2)$$
$$+(p'q_1)(p_1q_2)]-(kp)(p'q_1)(p+p',q_2)+(kp')$$
$$\times(p+p',q_1)(pq_2)]/(pk)(p'k)+[m^2(p_2q_1)(pq_2)$$
$$+(p'k)(kq_1)(pq_2)](p'k)^{-2}. \quad (8)$$

Integrating over the neutrino and antineutrino momenta with the aid of the formula

$$\int(aq_1)(bq_2)\delta^4(q-q_1-q_2)\frac{d^3q_1\,d^3q_2}{\varepsilon_1\varepsilon_2}$$
$$=\frac{\pi}{6}[2(aq)(bq)+(ab)q^2],$$

we obtain

$$d\sigma = \frac{e^2G^2}{12(2\pi)^4(pk)}\Big\{-q^2(q^2+m^2)\Big[\frac{m^2}{(kp)^2}+\frac{m^2}{(kp')^2}+\frac{2(pp')}{(kp)(kp')}\Big]$$
$$+4q^2+\frac{2(q^2-m^2)(kq)^2}{(kp)(kp')}\Big\}\frac{d^3p'}{E'}, \quad (9)$$

where $q = q_1 + q_2 = p + k - p'$.

In the following we make the nonrelativistic approximation that ω/m, $|\mathbf{p}|/m$, and $|\mathbf{p}'|/m$ are all much less than one. Then the differential cross section (9) in the center-of-mass system is

$$d\sigma = \frac{e^2G^2}{(2\pi)^4m^2\omega}\Big\{\frac{3\omega^2-\mathbf{p}'^2}{6}-\frac{\omega^2+\mathbf{p}'^2}{12\omega^4}[\mathbf{kp}']^2+\frac{1}{3}\,\mathbf{kp}'\Big\}d^3p'. \quad (10)^*$$

In order to write the differential cross section (10) in the laboratory system, it is sufficient to make the replacements $\mathbf{p}' \to \mathbf{p}' - \mathbf{p} - \mathbf{k}$, $\mathbf{k} \to \mathbf{k}$, $d^3\mathbf{p}' \to d^3\mathbf{p}'$.

We note that in general the number of final states of an electron with momentum $\mathbf{p}'$ is given by $(1-n_{\mathbf{p}'})d^3p'/(2\pi)^3$, where $n_{\mathbf{p}'}$ is the occupation number of the state with momentum $\mathbf{p}'$. Thus the differential cross section (10) has an additional factor $(1-n_{\mathbf{p}'})$ in the general case. In a medium with temperature T

$$n_{\mathbf{p}'} = \Big[\exp\Big(\frac{E'-\mu}{\varkappa T}\Big)+1\Big]^{-1},$$

where μ is the chemical potential; in the center of mass system $n_{\mathbf{p}'}$ must be replaced by $n_{\mathbf{p}'+\mathbf{p}+\mathbf{k}}$ in the nonrelativistic case. The integration over $\mathbf{p}'$ of the differential cross section (10) with the weight function $1-n_{\mathbf{p}'+\mathbf{p}+\mathbf{k}}$ cannot be done analytically. Therefore we consider two extreme cases.

1. Nondegenerate electrons. In this case $n_{\mathbf{p}'} \ll 1$, and the total cross section for photoproduction of neutrino pairs on nondegenerate electrons obtained by integrating (9) over $\mathbf{p}'$ is

$$\sigma = \frac{\alpha g^2}{12\pi^2}\Big(\frac{\hbar}{mc}\Big)^2\frac{1}{x(x+\sqrt{x^2+1})}\Big\{\Big[\frac{8}{3}x^4+\frac{10}{3}x^2+\frac{2}{3}-\frac{5}{4x^2}$$
$$+\Big(\frac{8}{3}x^3+2x-\frac{25}{6x}\Big)\sqrt{x^2+1}\Big]\ln\big(x+\sqrt{x^2+1}\big)$$
$$-\Big[\frac{35}{9}x^4+\frac{11}{18}x^2-\frac{25}{6}+\Big(\frac{20}{9}x^3+\frac{3}{2}x-\frac{5}{4x}\Big)\sqrt{x^2+1}\Big]\Big\}. \quad (11)$$

*$[\mathbf{kp}'] = \mathbf{k}\times\mathbf{p}'$.

Here $\alpha = e^2/4\pi = 1/137$ and $g = m^2G = 3.0 \times 10^{-12}$ are dimensionless constants and $x = \omega/m$, where ω is the photon energy in the center-of-mass system. In the nonrelativistic approximation

$$\sigma(\mathbf{k}, \mathbf{p}) = \frac{4\alpha g^2}{35\pi^2}\left(\frac{\hbar}{mc}\right)^2\left(\frac{\omega}{mc^2}\right)^4 = 1.13\cdot 10^{-48}\left(\frac{\omega}{mc^2}\right)^4 \text{ cm}^2. \quad (11')$$

2. Completely degenerate electrons. In this case $n_{\mathbf{p}'+\mathbf{p}+\mathbf{k}} = 1$ if $|\mathbf{p}' + \mathbf{p} + \mathbf{k}| < p_0$ and $n_{\mathbf{p}'+\mathbf{p}+\mathbf{k}} = 0$ if $|\mathbf{p}' + \mathbf{p} + \mathbf{k}| > p_0$, where p_0 is the Fermi momentum. By using the properties of the dependence of the differential cross section on $\mathbf{p}'$ and the fact that $|\mathbf{p}| \leq p_0$, the nonrelativistic total cross section for photoproduction of neutrino pairs on degenerate electrons can be written in the form

$$\sigma(\mathbf{k}, \mathbf{p}) = \frac{e^2G^2}{(2\pi)^4 m^2\omega}\int_{p_0-P}^{\omega} \mathbf{p}'^2\, dp' \int_{\frac{p_0^2-\mathbf{p}'^2-P^2}{2p'P}}^{1} d\cos\theta' \int_0^{2\pi} d\varphi' \quad (12)$$
$$\times\left\{\frac{3\omega^2-\mathbf{p}'^2}{6} - \frac{\omega^2+\mathbf{p}'^2}{12\omega^4}\right.$$
$$\left.\times\,[\mathbf{k}\mathbf{p}']^2 + \frac{1}{3}\mathbf{k}\mathbf{p}'\right\}\theta(P + \omega - p_0),$$

where $\mathbf{P} = \mathbf{p} + \mathbf{k}$, θ' is the angle between $\mathbf{p}'$ and $\mathbf{P}$, φ' is the angle between the planes $(\mathbf{p}', \mathbf{P})$ and $(\mathbf{k}, \mathbf{P})$, and $\theta(x)$ is one for $x > 0$ and zero for $x < 0$. The explicit expression for $\sigma(\mathbf{p}, \mathbf{k})$ is cumbersome and therefore we do not give it here; in the following only the value of this integral at the point $|\mathbf{p}| = p_0$, $P = p_0$ will be needed.

3. PHOTONEUTRINO RADIATION POWER OF AN ELECTRON GAS

The energy carried off by the neutrino pair in a single photoproduction event is $\epsilon_1 + \epsilon_2 = E + \omega - E'$. Therefore the energy carried off by neutrinos from a unit volume of electron gas per unit time (the photoneutrino radiation power per unit volume of medium) is

$$Q_\nu = \iint \frac{2d^3k}{(2\pi)^3}\frac{2d^3p}{(2\pi)^3} n_\gamma(\mathbf{k})\, n_e(\mathbf{p})\, v_{\text{rel}} \int (E + \omega - E')$$
$$\times\, d\sigma(\mathbf{k}, \mathbf{p}, \mathbf{p}'), \quad (13)$$

where $n_\gamma(\mathbf{k})$ and $n_e(\mathbf{p})$ are the momentum distributions of photons and electrons:

$$n_\gamma(\mathbf{k}) = \frac{1}{e^{\omega/\varkappa T} - 1}, \quad n_e(\mathbf{p}) = \frac{1}{e^{(E-\mu)/\varkappa T} + 1}, \quad (14)$$

and $v_{\text{rel}} = 1 - \mathbf{p}\cdot\mathbf{k}/E\omega$ is the relative velocity of the photon and electron.

From energy and momentum conservation it follows that $E - E' \ll \omega$ in the nonrelativistic case. Therefore the energy carried off by the neutrino pair in a single photoproduction is equal to the photon energy $\epsilon_1 + \epsilon_2 = \omega$ and the last integral in (13), which is usually called the effective deceleration, is simply

$$\int (E + \omega - E')\, d\sigma(\mathbf{k}, \mathbf{p}, \mathbf{p}') = \omega\sigma(\mathbf{k}, \mathbf{p}). \quad (15)$$

We consider two cases in which Q_ν can be evaluated analytically.

1. Nondegenerate electrons. In this case $n_e(\mathbf{p}) = C\exp(-E/\kappa T) \ll 1$ and $\sigma(\mathbf{k}, \mathbf{p})$ is given by (11′). After integrating, we obtain

$$Q_\nu = \frac{4\cdot 7!\,\zeta(8)}{35\pi^4}\alpha g^2 mc^2\,\frac{mc^2}{\hbar}\left(\frac{\varkappa T}{mc^2}\right)^8 n_e, \quad (16)$$

Substituting numerical values into (16) and assuming that the medium is almost completely ionized, so that the electron density is related to the matter density ρ by $n_e = 6 \times 10^{23}\,\rho/\mu_e$, where $\mu_e^{-1} = \Sigma_i C_i Z_i/A_i$ with C_i the weight concentration of element with atomic number A_i and charge Z_i, we obtain (with T in kev)

$$Q_\nu = 3.32\cdot 10^{-8} T^8\,(\rho/\mu_e)\ \text{erg/sec-cm}^3 \quad (17)$$

2. Strongly degenerate electrons. In this case $p_0^2/2m \gg \kappa T$ and $n_e(\mathbf{p}) = 1$ if $|\mathbf{p}| < p_0$, $n_e(\mathbf{p}) = 0$ if $|\mathbf{p}| > p_0$; p_0 is the Fermi momentum, which is related to the electron density by

$$p_0/mc = (3\pi^2 n_e)^{1/3}(\hbar/mc) = 1.01\cdot 10^{-2}(\rho/\mu_e)^{1/3}; \quad (18)$$

$\sigma(\mathbf{p}, \mathbf{k})$ is given by Eq. (12). We note that $\sigma(\mathbf{k}, \mathbf{p})$ depends on k and p only through ω, $|\mathbf{p}|$, and $P = |\mathbf{p} + \mathbf{k}|$; namely, $\sigma(\mathbf{k}, \mathbf{p}) = \sigma(\omega, p, P) \times \theta(P + \omega - p_0)$. Therefore the integration over the direction of $\mathbf{p}$ in (13) can be replaced by an integration over P between the limits $|p - \omega|$ and $p + \omega$. We then obtain

$$\int \sigma(\mathbf{k}, \mathbf{p})\, d\Omega_{\mathbf{p}} = \frac{2\pi}{\omega p}\left\{\theta(p - p_0 + 2\omega)\int_{p_0-\omega}^{p+\omega} \sigma(\omega, p, P)\, PdP\right.$$
$$\left. - \theta(|p - \omega| - p_0 + \omega)\int_{p_0-\omega}^{|p-\omega|} \sigma(\omega, p, P)\, PdP\right\}.$$

Then integrating over $|\mathbf{p}|$ between zero and p_0, we obtain

$$\int \sigma(\mathbf{k}, \mathbf{p})\, d^3p = \frac{2\pi}{\omega}\left\{\int_{p_0-2\omega}^{p_0} p\,dp \int_{p_0-\omega}^{p_0+\omega} \sigma(\omega, p, P)\, PdP + \ldots\right\}, \quad (19)$$

where the dots stand for terms proportional to $\theta(2\omega - p_0)$ and $\theta(\omega - p_0)$. In the strongly degenerate case $p_0 \gg \kappa T$; in the integration over ω these terms will therefore be exponentially small [of order $\exp(-p_0/2\kappa T)$ and $\exp(-p_0/\kappa T)$] in comparison with the integral over ω of the term written out in (19); hence, they will be neglected. Furthermore, since the main contribution of the first term in (19) to the integral over ω comes from frequencies ω near $\kappa T \ll p_0$, we can write

	T, kev	Q_ν erg/sec·cm³	q_ν erg/sec·cm³
Degenerate gas	1	$2.08\cdot10^{-4}$	$1.41\cdot10^{-1}$
	5	$4.06\cdot10^{2}$	$1.17\cdot10^{3}$
	10	$2.08\cdot10^{5}$	$4.66\cdot10^{4}$
	20	$1.06\cdot10^{8}$	$1.20\cdot10^{6}$
Nondegenerate gas	30	$1.09\cdot10^{9}$	$3.05\cdot10^{7}$
	40	$1.08\cdot10^{10}$	$1.10\cdot10^{8}$
	50	$6.50\cdot10^{10}$	$3.05\cdot10^{8}$
	70	$9.55\cdot10^{11}$	$1.38\cdot10^{9}$
	100	$1.66\cdot10^{13}$	$6.87\cdot10^{9}$

$$\int_{p_0-2\omega}^{p_0} p\,dp \int_{p_0-\omega}^{p+\omega} \sigma(\omega, p, P) P\,dP \approx 2\omega p_0 \int_{p_0-\omega}^{p_0+\omega} \sigma(\omega, p_0, P) P\,dP$$

$$\approx (2\omega p_0)^2 \sigma(\omega, p_0, p_0). \qquad (20)$$

From Eq. (12) we obtain to lowest order in ω/p_0

$$\sigma(\omega, p_0, p_0) = (2\pi)^{-3} \tfrac{4}{35} (e^2 G^2/m^2)\,\omega^4.$$

Then, to lowest order in $\kappa T/p_0$, we obtain

$$Q_\nu = \frac{12\cdot 8!\,\zeta(9)}{35\,\pi^4} \alpha g^2 mc^2 \frac{mc^2}{\hbar} \left(\frac{\kappa T}{mc^2}\right)^9 n_e \frac{mc}{p_0}. \qquad (21)$$

Using (18) and substituting numerical values, we obtain

$$Q_\nu = 1.5\cdot10^{-7} T^9 (\rho/\mu_e)^{2/3} \text{ erg/sec-cm}^3. \qquad (22)$$

The table lists the photoneutrino radiation power Q_ν per unit volume of electron gas (degenerate and nondegenerate) in erg/sec-cm^3 as a function of temperature for a density $\rho = 10^5$ g/cm^3. The corresponding values of the neutrino bremstrahlung radiation power q_ν found from the formula of Gandel'man and Pinaev[4] are also shown.

4. NEUTRINO RADIATION BY STARS

If the distributions of temperature and density in a star are known, then Eqs. (17) and (22) enable us to find the energy emitted by the star in the form of neutrinos. It is advantageous to compare this energy L_ν, which we will call the neutrino "luminosity" of the star, with the usual photon luminosity L_γ.

1. Nondegenerate star configurations. If the energy emission in the star occurs uniformly over the whole volume (uniform source model) or if all the star's energy is emitted at the center (point source model), then it follows from general properties of the equation for equilibrium of the star that the liminosity L_γ is related to the temperature T_c and density ρ_c at the star's center by[5]

$$L_\gamma = C\mu^{-0.5}\rho_c^{-2.5} b T_c^8, \qquad (23)$$

where b is the constant in Kramer's formula for the photon mean free path:

$$l = b\rho^{-2} T^{3.5},$$
$$\mu^{-1} = \sum_i C_i (Z_i + 1)/A_i. \qquad (24)$$

The value of the constant C and the temperature and density distributions are different in the two models and must be found by numerical integration of the equilibrium equations.

In the following we consider the point source model, since it corresponds more nearly to reality. The temperature and density distributions in this model were found by Cowling[6] by numerical integration of the equilibrium equations. Since the temperature falls rapidly, and the density even more rapidly, with distance from the star's center, the central values of the temperature and density do not characterize the temperature and density of the main stellar mass. We therefore introduce the mean temperature $\bar{T}$ and mean density $\bar{\rho}$ defined by

$$\bar{T} = \frac{1}{M}\int \rho T\,dv, \qquad \bar{\rho} = \frac{1}{v}\int \rho\,dv. \qquad (25)$$

By using the distributions $\rho(r)$ and $T(r)$, it can be shown[5,6] that the central temperature and density are related to the mean temperature and density and also to the mass and radius of the star by

$$T_c = 1.85\,\bar{T} = 6.28\cdot10^{-23}\mu M R^{-1},$$
$$\rho_c = 37.0\bar{\rho} = 8.84 M R^{-3}. \qquad (26)$$

Here and in the following temperatures are in kev and densities in g/cm^3. With the values found by Cowling for the constants, we obtain*

$$L_\gamma = 7.22\cdot10^{35}\mu^{-0.5}\rho_c^{-2.5} b T_c^8 = 1.19\cdot10^{34}\mu^{-0.5}\bar{\rho}^{-2.5} b \bar{T}^8. \qquad (27)$$

We integrate Q_ν over the volume of the star, using the distributions[6] $\rho(r)$ and $T(r)$ and Eq. (26), which relates R, T_c, and ρ_c. We obtain the result†

$$L_\nu = 1.45\cdot10^{25}\mu_e^{-1}\mu^{-1.5}\rho_c^{-0.5} T_c^{9.5}$$
$$= 0.822\cdot10^{27}\mu_e^{-1}\mu^{-1.5}\bar{\rho}^{-0.5}\bar{T}^{9.5}. \qquad (28)$$

*We note that in the corresponding equation [Eq. (16)] in the paper of Gandel'man and Pinaev,[4] the value of the constant is too large by a factor of 5.1. This also affects their Eq. (19).

†We note that in Eq. (18) in the paper of Gandel'man and Pinaev[4] the value of the constant in the expression for the neutrino bremstrahlung luminosity is two or three times too large due to the use of a rough approximation for the temperature and density instead of the numerical values given by Cowling.

The ratio of the photoneutrino and photon luminosities is

$$L_\nu/L_\gamma = 2.01\cdot10^{-11}T_c^{1.5}\rho_c^2/b\mu_e\mu = 0.69\cdot10^{-7}\bar{T}^{1.5}\bar{\rho}^2/b\mu_e\mu. \quad (29)$$

This ratio is of the order of unity for $\bar{\rho} = 3\times10^2$ and $\bar{T} = 10$ kev, for example, which apparently prevail in the stars like subdwarfs which flare up from time to time as novae.[7-9] In stars which are evolving into white dwarfs, $T_c \approx 40$ kev, $\rho_c \approx 5\times10^4$ g/cm^3 (see Öpick[10]); then $L_\nu/L_\gamma \approx 10$.

2. Degenerate star configurations. Since the thermal conductivity of a degenerate electron gas is very high (soft photons cannot be absorbed by the electrons), the temperature is constant in the interior of a star that consists of a degenerate gas. On the other hand, the equation of state of the degenerate gas depends weakly on the temperature. If we neglect this dependence, the density distribution in the star depends on just one parameter, for example, the Fermi momentum at the center of the star $x_c = (p_0/mc)_c$. In a nonrelativistic electron gas x_c^2 is small, and in this case the density distribution is determined by the Lane-Emden polytrope with index $^3/_2$, namely, $\rho(r) = \rho\, f_{3/2}^{3/2}(3.654\, r/R)$, where $f_{3/2}(\xi)$ is the Lane-Emden function with index $^3/_2$ (see [11], Sec. 105, or [5], Ch. 11).

We integrate the photoneutrino radiation power of a degenerate electron gas over the stellar volume, using

$$\int\rho^{2/3}dv = M\bar{\rho}^{-1/3}\,3\left(\frac{\rho_c}{\bar{\rho}}\right)^{2/3}\int_0^1\left(\frac{\rho}{\rho_c}\right)^{2/3}x^2\,dx,$$

where the quantity

$$3\left(\frac{\rho_c}{\bar{\rho}}\right)^{2/3}\int_0^1\left(\frac{\rho}{\rho_c}\right)^{2/3}x^2dx$$

depends weakly on the parameter x_c^2, and is equal to 8.400 for small x_c^2. Then

$$L_\nu = 1.29\cdot10^{-7}MT^9/\mu_e^{2/3}\bar{\rho}^{1/3}. \quad (30)$$

On the other hand, Schatzman[12] showed that the photon luminosity of a degenerate star is related to its temperature and mass by $T = 6.17\times10^7(L_\gamma/M)^{2/7}$ (with T in degrees) or

$$L_\gamma = 2.88\cdot10^{-3}MT^{7/2}, \quad (31)$$

if T is expressed in kev. Thus, the ratio of photoneutrino and photon luminosities is

$$L_\nu/L_\gamma = 4.48\cdot10^{-5}T^{5.5}/\mu_e^{2/3}\bar{\rho}^{1/3}. \quad (32)$$

This ratio is of the order of unity for $\bar{\rho} = 10^5$ g/cm^3 and T = 20 kev. In white dwarfs with $\bar{\rho}\approx10^5$ and $T\approx5$ kev, this gives $L_\nu/L_\gamma \approx 10^{-3}$.

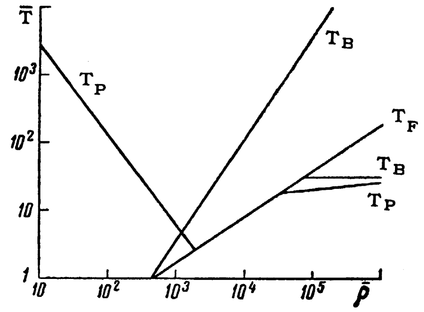

FIG. 2. Mean stellar temperatures and densities for which the photoneutrino and neutrino bremstrahlung luminosities are equal to the photon luminosity.

The dependence of L_ν/L_γ on temperature and density is most conveniently discussed with the aid of a diagram. In Fig. 2 the line labelled T_F separates the degenerate gas region (below T_F) from the nondegenerate gas region (above T_F). The lines T_P in both the degenerate and nondegenerate regions correspond to the mean temperatures and densities for which the photoneutrino luminosity is equal to the usual photon luminosity; above these lines $L_\nu/L_\gamma > 1$ and below them $L_\nu/L_\gamma < 1$. Similarly, the lines T_B correspond to the temperatures and densities for which the neutrino bremstrahlung luminosity is equal to the photon luminosity; The points below T_B in the degenerate region correspond to temperatures and densities for which $L_\nu/L_\gamma > 1$ (and vice versa). As is evident from Fig. 2, the region of stellar temperatures and densities for which the photoneutrino luminosity is greater than or equal to the photon luminosity is much larger than the region in which the neutrino bremstrahlung luminosity exceeds the photon luminosity, and includes the latter as a particular case.

The most interesting region to us is that around $\rho = 5\times10^2$ g/cm^3 and T = 10 kev. These appear to be the densities and temperatures of novae before and after bursts[7-9] (it is known that the same star can flare up as a nova several times and that its mass and luminosity are the same before and after a burst). Stars evolving into white dwarfs have even greater temperatures and densities.[10] Equation (29) and Fig. 2 show that the photoneutrino luminosity is near the photon luminosity or exceeds it in the region of densities greater than 5×10^2 and temperatures greater than 5 kev; therefore neutrino photoproduction must play a significant role in the energy balance in such stars.

What are the greatest stellar neutrino luminosities? It is doubtful that the neutrino current increases strongly during a nova burst, since the energy released during a nova burst is about 10^{45}

erg,[7] which is 10^{-4} of the star's thermal energy. Therefore the mean stellar temperature and consequently the neutrino luminosity are not significantly increased during a nova burst.

A different situation can be observed in a supernova burst. Then the energy release is comparable with the star's thermal energy[7] and amounts to about 10^{50} erg. In such a case, a star with mass near that of the sun $(2 \times 10^{33}\,g)$ is heated up to a temperature of 50 to 100 kev. Since the photoneutrino radiation power per gram of material is $10^6 - 10^8$ erg/g-sec at these temperatures, the neutrino luminosity of a supernova is $10^{39} - 10^{41}$ erg/sec. This is comparable with the usual supernova luminosity $(10^7 - 10^8$ times the sun's). During a burst which lasts 50 to 100 days, the neutrinos carry off $10^{46} - 10^{48}$ erg, which is $10^{-4} - 10^{-2}$ of the total energy released. Thus, although the neutrino luminosity during a stellar supernova burst is colossal, neutrino processes do not play as large a role in the energy balance of supernovae as they can play in novae before and after bursts and in stars evolving into white dwarfs.

Note added in proof (September 19, 1961). After this article was submitted for publication, a paper on the same subject was published by Chiu and Stabler [Phys. Rev. 122, 137 (1961)]. However, they do not consider the photoneutrino radiation by a nonrelativistic degenerate electron gas. Moreover, it should be noted that owing to the use of an unusual relation between probability and cross section [for the customary relation, see C. Møller, Kgl. Danske Videnskab. Selsk., Mat.-fys. Medd. 23, 1 (1945)] their expression for the photoproduction cross section differs from ours by a factor $(pk)/E\omega$ and is not relativistically invariant. We also note that they used Heaviside units, but mistakenly set $e^2/\hbar c = 1/137$, instead of $e^2/4\pi\hbar c = 1/137$, and their results are therefore too small by a factor 4π.

[1] R. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] G. Gamow and M. Schoenberg, Phys. Rev. **59**, 539 (1941).

[3] B. M. Pontecorvo, JETP **36**, 1615 (1959), Soviet Phys. JETP **9**, 1148 (1959).

[4] G. M. Gandel'man and V. S. Pinaev, JETP **37**, 1072 (1959), Soviet Phys. JETP **10**, 764 (1960).

[5] S. Chandrasekhar, An Introduction to the Study of Stellar Structure, University of Chicago Press, Chicago 1939.

[6] T. G. Cowling, Monthly Notices, Royal Astr. Soc. **96**, 42 (1936).

[7] L. H. Aller, Astrophysics, N. Y. 1959.

[8] M. L. Humason, Astrophys. Journ. **88**, 228 (1938).

[9] Z. Kopal, Vistas in Astronomy, v. 2, Pergamon Press, London 1956.

[10] E. I. Öpick, Nuclear Processes in Stars (Russ. Transl.), IIL, Moscow 1957.

[11] L. D. Landau and E. M. Lifshitz, Statistical Physics, Pergamon, 1958.

[12] E. Schatzman, Ann. d'ast. **8**, 143 (1945); **10**, 19 (1947).

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ERRATA

Vol	No	Author	page	col	line	Reads	Should read
13	2	Gofman and Nemets	333	r	Figure	Ordinates of angular distributions for Si, Al, and C should be doubled.	
13	2	Wang et al.	473	r	2nd Eq.	$\sigma_\mu = \frac{e^2 f^2}{4\pi^3}\omega^2 (\ln \frac{2\omega}{m_\mu} - 0.798)$	$\sigma_\mu = \frac{e^2 f^2}{9\pi^3}\omega^2 \left(\ln \frac{2\omega}{m_\mu} - \frac{55}{48}\right)$
			473	r	3rd Eq.	$(e^2f^2/4\pi^3)\,\omega^2 \geqslant \ldots$	$(e^2f^2/9\pi^3)\,\omega^2 \geqslant \ldots$
			473	r	17	242 Bev	292 Bev
14	1	Ivanter	178	r	9	1/73	1.58×10^{-6}
14	1	Laperashvili and Matinyan	196	r	4	statistical	static
14	2	Ustinova	418	r	Eq. (10) 4th line	$[-\frac{1}{4}(3\cos^2\theta - 1)\ldots$	$-\left[\frac{1}{4}(3\cos^2\theta - 1)\ldots\right.$
14	3	Charakhchyan et al.	533	Table II, col. 6 line 1		1.9	0.9
14	3	Malakhov	550	The statement in the first two phrases following Eq. (5) are in error. Equation (5) is meaningful only when s is not too large compared with the threshold for inelastic processes. The last phrase of the abstract is therefore also in error.			
14	3	Kozhushner and Shabalin	677 ff		The right half of Eq. (7) should be multiplied by 2. Consequently, the expressions for the cross sections of processes (1) and (2) should be doubled.		
14	4	Nezlin	725	r	Fig. 6 is upside down, and the description "upward" in its caption should be "downward."		
14	4	Geĭlikman and Kresin	817	r	Eq. (1.5)	$\ldots\left[b^2 \sum_{s=1}^{\infty} K_2(bs)\right]^2$	$\ldots\left[b^2 \sum_{s=1}^{\infty}(-1)^{s+1} K_2(bs)\right]^2$
			817	r	Eq. (1.6)	$\Phi(T) = \ldots$	$\Phi(T) \approx \ldots$
			818	l	Fig. 6, ordinate axis	$\frac{\varkappa_s(T)}{\varkappa_n(T_c)}$	$\frac{\varkappa_s(T)}{\varkappa_n(T)}$
14	4	Ritus	918	r	4 from bottom	two or three	2.3
14	5	Yurasov and Sirotenko	971	l	Eq. (3)	1 < d/2 < 2	1 < d/r < 2
14	5	Shapiro	1154	l	Table	2306	23.6