CORRELATION OF MOTION OF FOUR NUCLEONS IN THE Po^{212} NUCLEUS

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The spectrum of the excited states and the transitions between these states are calculated for a nucleus with two nucleon pairs in excess of the filled shells. Residual pair interaction is taken into account along with the self-consistent potential. The importance of taking np forces in such nuclei into account is demonstrated.

1. INTRODUCTION

THE self-consistent potential acting on an individual nucleon, as well as the residual pair forces in the nucleus, have already been determined in earlier investigations.[1-4] The self-consistent potential consists of two principal parts, central V_C and surface V_S. The central potential is

$$V_C(i) = V(r_i) - \lambda\left(\frac{\hbar}{2Mc}\right)^2 \frac{l_i s_i}{r_i} \frac{\partial V(r_i)}{\partial r_i},$$

$$V(r_i) = -V_0/[1 + e^{\alpha(r_i - r_0)}]. \quad (1)$$

The parameters α, λ, and V_0 were calculated from the energies of the ground and excited levels of nuclei with filled shells ± one nucleon (for more details, see [1]). The surface potential V_S is due to the quadrupole interaction between the outer particle and the core particles, and is usually represented in the form

$$V_S(i) = -\varkappa(r_i)\sqrt{\frac{\hbar\omega}{2C}} \sum_\mu (b_\mu + (-1)^\mu b^+_{-\mu})\, Y_{2\mu}(\vartheta_i, \varphi_i). \quad (2)$$

Here b^+ and b are the phonon creation and annihilation operators. The parameters $\hbar\omega$ and C, which characterize the phonon energy and the deformability of the surface respectively, were determined from the E2-transition probabilities and the quadrupole moments.[4] In the region of heavy nuclei, $\hbar\omega$ = 2—3 Mev and C = (1—2) × 10^3 Mev. For the radial matrix elements we have the following estimate

$$\langle n|\varkappa(r_i)|n'\rangle \approx (-1)^{n+n'}(35-40)\ \text{Mev}$$

where n is the principal quantum number.

In nuclei with two nucleons (holes) in excess of twice-filled shells, in addition to the self-consistent field, pair forces begin to play an important role which must be correctly accounted for. The pair interaction operator was taken in the form [2,3]

$$V_p(ij) = -(v_s\pi_s + v_t\pi_t)\exp\{-|\mathbf{r}_i - \mathbf{r}_j|^2/\rho^2\}. \quad (3)$$

Here v_s and v_t are the parameters of the singlet and triplet interaction, π_s and π_t are the operators of singlet and triplet projection, and ρ is the effective radius of interaction. The parameters v_s, v_t, and ρ were determined separately for the nn, pp, and np pairs in the corresponding nuclei. The results obtained show that v_s is somewhat greater for the pp pair than for the nn pair, but we cannot say that this difference has physical meaning and is not the result of inaccurate knowledge of the energy of the single-particle levels in the Pb^{210} and Po^{210} nuclei. For the np pair in Bi^{209} the main contribution to the interaction energy is made by triplet forces, and the parameter v_t determined for this case is more reliable.

2. SINGULARITIES IN THE FOUR-PARTICLE PROBLEM

The problem of determining the states of four interacting nucleons in the nucleon has its own singularities, which greatly complicate the solution. We consider, for example, as in the case of two nucleons, five neutron and five proton single-particle levels (see Fig. 1). We then obtain for the neutron pair and for the proton pair 225 configurations, each multiply degenerate. Even if we confine ourselves to the zeroth-approximation levels, with energies ≤3.5 Mev, we obtain more than 70 levels with spin $I = 0^+$, more than 270 levels with spin $I = 2^+$, etc. These are only levels without phonons (N = 0); if states with $N \neq 0$ are taken into account, the number of levels increases still more. Naturally, not all these levels must be taken into account if we are interested in excitations not higher than 3 Mev, for after excluding the pair interaction the levels will be approximately 6 Mev apart. But in order to calcu-

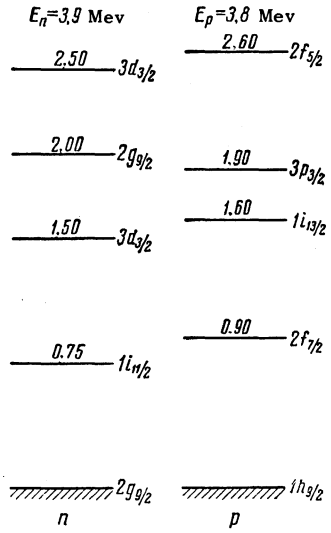

FIG. 1. Single-particle levels of Po^{212} nucleons.

late correctly the configuration mixture, it is necessary to take into account a sufficient number of levels, and this leads to great computational difficulties, which so far could not be overcome even in the simple problem without account of the interaction with the surface. Only programming for a computer resulted in a complete solution of the entire four-particle problem.

On the basis of the data accumulated in the earlier investigations, we can formulate the following assumptions of importance to the four-nucleon problem.

1. The nucleons move in the self-consistent potential (1) and (2).

2. The residual forces are short-range pair forces.

3. The parameters of the pair forces are the same for all particle pairs (isotopic independence of the pair forces). Naturally, small deviations of the parameters of the pair forces or of the self-consistent field are actually possible. But if these deviations are significant, they can be observed and subsequently accounted for.

We chose for the pair-force parameters the values $\rho = 2f$, $v_s = 30-36$ Mev, and $v_t = 15-20$ Mev. These are precisely the forces obtained for Pb^{210}, Bi^{210}, and Po^{210}.

3. METHOD OF CALCULATION

In the present problem we consider a system of four nucleons (2n + 2p) moving in a central field, and a nuclear surface executing quadrupole oscillations. The nuclear surface is needed only to account for the V_S interaction. The single-particle and phonon functions were determined from the equations

$$H_0(i)\,|\,n_i l_i j_i m_i\rangle = E_i^{(0)}\,|\,n_i l_i j_i m_i\rangle, \qquad i = 1, 2, 3, 4; \tag{4}$$

$$H_S\,|\,NRM_R\rangle = (N + {}^5/_2)\hbar\omega\,|\,NRM_R\rangle;$$

$$H_0(i) = p_i^2/2M + V_C(i), \tag{4'}$$

where $n_i l_i j_i m_i$ are the quantum numbers of the i-th nucleon, N is the number of phonons with energy $\hbar\omega$, while R and M_R are the angular momentum of the phonon and its projection on the z axis.

In the zeroth approximation the eigenfunction of the entire system, corresponding to a definite momentum I, projection M, and parity ν, is written in the form

$$|\,[(n_1 l_1 j_1,\ n_2 l_2 j_2)\,J_{12}]_a\,,\ [(n_3 l_3 j_3,\ n_4 l_4 j_4)\,J_{34}]_a,\ J;\ NR;\ IM\nu\rangle. \tag{5}$$

The indices 1 and 2 pertain to neutrons while 3 and 4 pertain to protons. The index a denotes antisymmetrization of the functions. We have in mind here the following connections between the angular momenta:

$$\mathbf{j}_1 + \mathbf{j}_2 = \mathbf{J}_{12}, \quad \mathbf{j}_3 + \mathbf{j}_4 = \mathbf{J}_{34},$$

$$\mathbf{J}_{12} + \mathbf{J}_{34} = \mathbf{J}, \quad \mathbf{J} + \mathbf{R} = \mathbf{I}.$$

The total Hamiltonian of the system is

$$H = \sum_{i=1}^{4} H_0(i) + H_S + \sum_{i=1}^{4} V_S(i) + \sum_{i<j}^{4} V_p(ij). \tag{6}$$

We seek the solution of the Schrödinger equation with Hamiltonian (6) in the form of an expansion in the eigenfunctions (5):

$$|\,IM\nu\rangle = \sum_{\alpha} c^I(\alpha)\,|\,[(j_1 j_2)\,J_{12}]\,,\ [(j_3 j_4)\,J_{34}]_a,\ J;\ NR;\ IM\nu\rangle,$$

$$\alpha \equiv (j_1 j_2 J_{12} j_3 j_4 J_{34} J N R). \tag{7}$$

To abbreviate the notation, the index j_i denotes the set of three quantum numbers $m_i l_i j_i$.

The eigenvalues and the coefficients $C^I(\alpha)$ of expansion in eigenfunctions of the Hamiltonian (6) are obtained by diagonalizing the energy matrix. To set up the energy matrix it is necessary to calculate the matrix elements of the operators $\Sigma V_p(ij)$ and $\Sigma V_S(i)$. The matrix element of the pair interaction has six terms and its value is

$$\begin{aligned}
&\langle[(j_1 j_2)\,J_{12}]_a,\ [(j_3 j_4)\,J_{34}]_a,\ J;\ NR;\ IM\nu\,|\sum_{i<j}^{4} V_p(ij)\,|\\
&\to[(j_1' j_2')J_{12}']_a, [(j_3' j_4')\,J_4']_a,\ J';\ N'R';\ IM\nu\rangle\\
&=\delta_{NN'}\delta_{RR'}\delta_{JJ'}\{\delta_{J_{12}J_{12}'}\delta_{J_{34}J_{34}'}[\delta_{j_3 j_3'}\delta_{j_4 j_4'}\\
&\times\langle[(j_1 j_2)\,J_{12}M_{12}]_a\,|\,V_p(12)\,|\,[(j_1' j_2')\,J_{12}M_{12}]_a\rangle\\
&+\delta_{j_1 j_1'}\delta_{j_2 j_2'}\langle[(j_3 j_4)\,J_{34}M_{34}]_a\,|\,V_p(34)\,|\,[(j_3' j_4')\,J_{34}M_{34}]_a\rangle]\\
&+4\langle[(j_1 j_2)\,J_{12}]_a,\ [(j_3 j_4)\,J_{34}]_a,\\
&\quad JM\,|\,V_p(13)\,|\,[(j_1' j_2')\,J_{12}']_a,\ [(j_3' j_4')\,J_{34}']_a; JM\rangle\}.
\end{aligned} \tag{8}$$

This general expression must be reduced to a form which is simplest and most convenient for machine computation. Let us rewrite (8) in the form of a sum of the matrix elements of the non-antisymmetrized functions

$$\delta_{NN'}\delta_{RR'}\delta_{JJ'}\Big\{\delta_{J_{12}J'_{12}}\delta_{J_{34}J'_{34}}\Big[\delta_{j_3j'_3}\delta_{j_4j'_4}N_{j_1j_2}N_{j'_1j'_2}$$
$$\times\sum_P(-1)^pP\langle(j_1j_2)\,J_{12}M_{12}\,|\,V_p(12)\,|\,(j'_1j'_2)J_{12}M_{12}\rangle$$
$$+\delta_{j_1j'_1}\delta_{j_2j'_2}N_{j_3j_4}N_{j'_3j'_4}$$
$$\times\sum_P(-1)^pP\langle(j_3j_4)\,J_{34}M_{34}\,|V_p(34)\,|\,(j'_3j'_4)\,J_{34}M_{34}\rangle\Big]$$
$$+4N_{j_1j_2}N_{j'_1j'_2}N_{j_3j_4}N_{j'_3j'_4}\sum_P(-1)^pP\,\langle(j_1j_2)\,J_{12},\,(j_3j_4)\,J_{34};$$
$$\to JM\,|V_p(13)\,|\,(j'_1j'_2)\,J'_{12},\,(j'_3j'_4)\,J'_{34};\,JM\rangle\Big\}. \qquad (9)$$

Here P is the operator which permutes the indices inside each of the pairs j_1j_2, j_3j_4, $j'_1j'_2$, and $j'_3j'_4$. The summation extends over all the different permutations which actually change the arrangement of the indices. Each pair of indices makes its own contribution to the power p of the factor (-1). If the permutation j_ij_k is even the corresponding term vanishes, and if it is odd its value is $(j_i+j_k-J_{ik}+1)$. The normalizing factors of the functions of the particle pair j_ij_k have the form

$$N_{j_ij_k}=1/\sqrt{2}+(1-1/\sqrt{2})\,\delta_{j_ij_k}. \qquad (10)$$

In order to calculate the matrix element of the np interaction, it is necessary to go over in the last term of (9) to a different coupling scheme, in which the first particle is connected with the third and the second with the fourth. Then

$$\langle(j_1j_2)\,J_{12},\,(j_3j_4)\,J_{34};\,JM\,|V_p(13)\,|\,(j'_1j'_2)\,J'_{12},\,(j'_3j'_4)\,J'_{34};\,JM\rangle$$
$$=\delta_{j_2j'_2}\delta_{j_4j'_4}\sum_{J_{13}J_{24}}A\begin{Bmatrix}j_1 & j_2 & J_{12}\\ j_3 & j_4 & J_{34}\\ J_{13} & J_{24} & J\end{Bmatrix}A\begin{Bmatrix}j'_1 & j_2 & J'_{12}\\ j'_3 & j_4 & J'_{34}\\ J_{13} & J_{24} & J\end{Bmatrix}$$
$$\times\langle(j_1j_3)\,J_{13}M_{13}\,|V_p(13)\,|\,(j'_1j'_3)\,J_{13}M_{13}\rangle. \qquad (11)$$

Here $A\{\ \}$ are normalized generalized Racah coefficients.

This formula is difficult to compute, for it calls for double summation of a rather complicated expression. We make therefore the following transformations. We represent the pair interaction operator in the form of two terms

$$V_p(13)=V(|\mathbf{r}_1-\mathbf{r}_3|)\,[a+b(\sigma_1\sigma_3)]$$
$$=V_p^{(a)}(13)+V_p^{(b)}(13); \qquad (12)$$
$$V_p^{(a)}(13)=a\sum_k f_k(\mathbf{T}^k(1)\,\mathbf{T}^k(3)), \qquad (12')$$
$$V_p^{(b)}(13)=b\sum_k f_k(\mathbf{T}^k(1)\,\mathbf{T}^k(3))(\sigma_1\sigma_3)$$
$$=b\sum_{kk'}(-1)^{k+k'+1}f_k(\mathbf{U}^{k'}(1)\,\mathbf{U}^{k'}(3)). \qquad (12'')$$

Here $\mathbf{U}^{k'}$ is the tensor product of the tensor operators $\mathbf{T}^k$ and σ:

$$\mathbf{U}^{k'}_{q'}=[\mathbf{T}^k\times\sigma]^{k'}_{q'}=\sum_{q\mu}C^{k'q'}_{kq1\mu}\mathbf{T}^k_q\sigma_\mu, \qquad (13)$$
$$=k-1,\,k,\,k+1;\ q'=-k',\,-k'+1,\,\ldots,\,k';$$
$$f_k=\int\exp\left\{-\left(\frac{\mathbf{r}_1-\mathbf{r}_3}{\rho}\right)^2\right\}P_k\cos\omega_{13})\,d\Omega, \qquad (14)$$
$$a=(v_s+3v_t)/4,\qquad b=-(v_s-v_t)/4. \qquad (15)$$

Then

$$\langle(j_1j_3)\,J_{13}M_{13}\,|V_p(13)\,|\,(j'_1j'_3)\,J_{13}M_{13}\rangle=(-1)^{j_1+j'_3-J_{13}}$$
$$\times2\sum_{kk'}F^k\langle l_1\|\mathbf{T}^k\|l'_1\rangle\langle l_3\|\mathbf{T}^k\|l'_3\rangle\,W\,[j_1j'_1j_3j'_3;\,k'J_{13}]$$
$$\times\left[a\delta_{kk'}A\begin{Bmatrix}l_1 & 1/2 & j_1\\ l'_1 & 1/2 & j'_1\\ k & 0 & k\end{Bmatrix}A\begin{Bmatrix}l_3 & 1/2 & j_3\\ l'_3 & 1/2 & j'_3\\ k & 0 & k\end{Bmatrix}\right.$$
$$\left.-b(-1)^{k+k'}\left(\frac{2k'+1}{2k+1}\right)A\begin{Bmatrix}l_1 & 1/2 & j_1\\ l'_1 & 1/2 & j'_1\\ k & 1 & k'\end{Bmatrix}A\begin{Bmatrix}l_3 & 1/2 & j_3\\ l'_3 & 1/2 & j'_3\\ k & 1 & k'\end{Bmatrix}\right], \qquad (16)$$

where F^k are radial integrals of the Slater type.

In each term of (16) the dependence of J_{13} enters only in the form $(-1)^{-J_{13}}W[j_1j'_1j_3j'_3;\,k'J_{13}]$. This enables us to sum over J_{13} and J_{24} in (11), using the relations

$$\sum_{J_{13}J_{24}}(-1)^{-J_{13}}W\,[j_1j'_1j_3j'_3;\,k'J_{13}]\,A\begin{Bmatrix}j_1 & j_2 & J'_{12}\\ j_3 & j_4 & J_{34}\\ J_{13} & J_{24} & J\end{Bmatrix}A\begin{Bmatrix}j'_1 & j_2 & J'_{12}\\ j'_3 & j_4 & J'_{34}\\ J_{13} & J_{24} & J\end{Bmatrix}$$
$$=(-1)^{j'_1+j_2+j'_3+j_4+j_1+j'_3+J_{34}+J'_{34}+J}[(2J_{12}+1)\,(2J'_{12}+1)$$
$$(2J_{34}+1)\,(2J'_{34}+1)]^{1/2}\,W\,[j_1j'_1J_{12}J'_{12};\,k'j_2]$$
$$W\,[j_3j'_3J_{34}J'_{34};\,k'j_4]\,W\,[J_{12}J'_{12}J_{34}J'_{34};\,k'J]. \qquad (17)$$

Finally, the matrix element of the operator $V_p(13)$ becomes

$$\langle(j_1j_2)\,J_{12},\,(j_3j_4)\,J_{34},\,JM\,|V_p(13)\,|\,(j'_1j'_2)\,J'_{12},\,(j'_3j'_4)$$
$$\times\,J'_{34};\,JM\rangle=(-1)^{j'_1+j_2+j'_3+j_4+J_{34}+J'_{34}+J}\,\delta_{j_2j'_2}\delta_{j_4j'_4}$$
$$\times[(2J_{12}+1)\,(2J'_{12}+1)\,(2J_{34}+1)\,(2J'_{34}+1)]^{1/2}\,2\sum_{kk'}F^k$$
$$\times\langle l_1\|\mathbf{T}^k\|l'_1\rangle\langle l_3\|\mathbf{T}^k\|l'_3\rangle W[j_1j'_1J_{12}J'_{12};k'j_2]\,W[j_3j'_3J_{34}J'_{34};\,k'j_1]$$
$$\times W\,[J_{12}J'_{12}J_{34}J'_{34};\,k'J]\left[a\delta_{kk'}A\begin{Bmatrix}l_1 & 1/2 & j_1\\ l'_1 & 1/2 & j'_1\\ k & 0 & k\end{Bmatrix}A\begin{Bmatrix}l_3 & 1/2 & j_3\\ l'_3 & 1/2 & j'_3\\ k & 0 & k\end{Bmatrix}\right.$$
$$\left.-b(-1)^{k+k'}\left(\frac{2k'+1}{2k+1}\right)A\begin{Bmatrix}l_1 & 1/2 & j_1\\ l'_1 & 1/2 & j'_1\\ k & 1 & k'\end{Bmatrix}A\begin{Bmatrix}l_3 & 1/2 & j_3\\ l'_3 & 1/2 & j'_3\\ k & 1 & k'\end{Bmatrix}\right]. \qquad (18)$$

Formulas (16) and (18), apart from a factor, differ only in the fact that one Racah coefficient in the former is replaced by a product of three Racah coefficients in the latter. Inasmuch as (16) and (18) are of the same type, it is convenient to use formula (16) for the calculation of the matrix elements of the nn and pp interaction.

The matrix elements of the V_S interaction were calculated with the formula

$$\langle [(j_1 j_2) J_{12}]_a [(j_3 j_4) J_{34}]_a, J, NR;\ IMv | V_S | [(j_1' j_2')$$
$$\times J_{12}']_a [(j_3' j_4') J_{34}']_a, J'; N'R'; IMv\rangle$$
$$= 2\delta_{N,N\pm 1} (-1)^{1+\sum_{i=1}^{4}(n_i+n_i')} |\langle n | \varkappa(r_i) | n'\rangle| \sqrt{\frac{5}{4\pi}\frac{\hbar\omega}{2C}}$$
$$\times \langle NR \| b \| N'R'\rangle \sqrt{(2J+1)(2J'+1)}\, W[IRJ'2; JR']$$
$$\times \Big\{ \delta_{j_3 j_3'} \delta_{j_4 j_4'} \delta_{J_{34} J_{34}'} (-1)^{J_{34}-J_{12}'-J-J_{12}} \sqrt{(2J_{12}+1)(2J_{12}'+1)}$$
$$\times W[J_{12} J J_{12}' J'; J_{34} 2]\ N_{j_1 j_2} N_{j_1' j_2'} \sum_P P \delta_{j_2 j_2'} (-1)^{p+j_2-j_1'}$$
$$\times \langle l_1 j_1 \| \mathbf{T}^2 \| l_1' j_1'\rangle\, W[j_1 J_{12} j_1' J_{12}'; j_2 2]$$
$$+ \delta_{j_1 j_1'} \delta_{j_2 j_2'} \delta_{J_{12} J_{12}'} (-1)^{J_{12}-J'} \sqrt{(2J_{34}+1)(2J_{34}'+1)}$$
$$\times W[J_{34} J J_{34}' J'; J_{12} 2]\ N_{j_3 j_4} N_{j_3' j_4'} \sum_P P \delta_{j_4 j_4'} (-1)^{p+j_4-j_3'}$$
$$\times \langle l_3 j_3 \| \mathbf{T}^2 \| l_3' j_3'\rangle\, W[j_3 J_{34} j_3' J_{34}'; j_4 2] \Big\}. \qquad (19)$$

As noted earlier, even if we confine ourselves to single-particle levels lying within 3 Mev, the energy matrices are still found to be of rather large order. For comparison with experiment, it is sufficient to calculate accurately the positions of the first three levels with given I, which are the most sensitive to the parameters of the pair interaction. We stipulate that the error in the determination of the energies of these levels be less than 0.1 Mev. Then, to obtain this accuracy, there is no need to operate with high-order matrices. It was established that if we cross out from the matrix with given spin I the columns in which the ratio of the non-diagonal matrix elements a_{ik} to the difference between diagonal elements, namely $a_{ik}/(a_{ii} - a_{kk})$, does not exceed 0.05 (i = 1, 2, 3; k > 3), then each of the levels i = 1, 2, 3 will shift by not more than 0.1 Mev. This was carefully investigated by testing many matrices.

In the present four-particle problem for I = 0 we first calculated and investigated in detail a 46-th order matrix (with N = 0). For I = 2 we calculated the first three rows of a 200-th order matrix and checked which of the high levels can be neglected. It was thus established that the 31-th order matrices,* with which we subsequently operated, guarantee the required accuracy in the determination of the energy and the level functions.

4. SPECTRUM OF Po^{212}

Let us trace the formation and shifts of the levels, by successively turning on the interaction in parts. The single-particle levels of the four particles (Fig. 1) will be multiply degenerate. When the pair interaction diagonal matrix elements are added, the degeneracy is lifted. The matrix elements of the nn and pp interaction are strongly dependent on the intermediate momenta J_{12} and J_{34} (see [2]) and give rise to momentum level splits. The matrix elements of the np interaction depend weakly on either the intermediate momenta or on the total momentum I, and therefore produce a parallel downward shift of all the levels without an appreciable distortion of the preceding splitting. The result is the level scheme shown in Fig. 2.

We then take account of the non-diagonal matrix elements, in other words the interaction between the levels. We first turn on only the matrix ele-

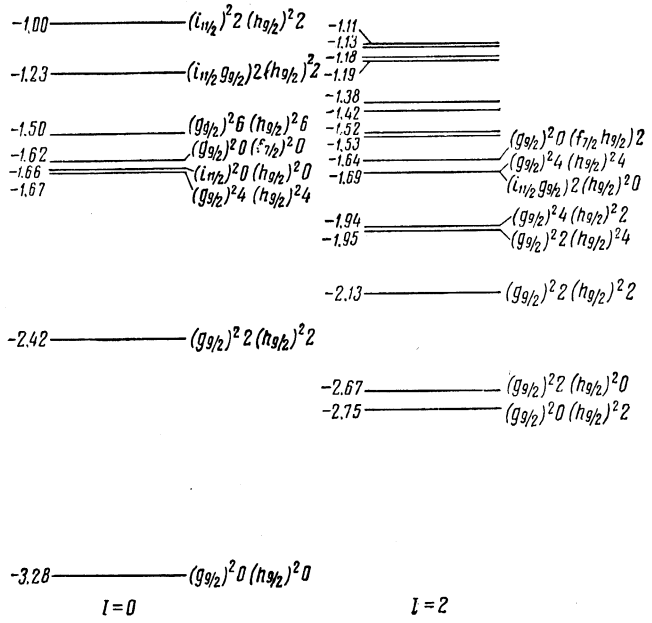

FIG. 2. Lower levels of four Po^{212} nucleons with account of the single-particle energy and the diagonal matrix elements of the pair interaction. The configuration is indicated on the right and the energy on the left. The energy is referred to the single-particle ground level.

*Including levels with N = 0, 1, and 2.

ments of the nn and pp forces. The first level* $I = 0_1^+$ will interact both via the nn forces and the pp forces [with levels 4, 7, etc. in downward order on Fig. 2; see formula (8)], and will drop more than twice as far as in the case of a single pair.

On the other hand, the first level $I = 2_1$ will interact only via the nn forces (with levels 3, 5, 9 ... on Fig. 2), while the second level $I = 2_2$ will interact only via the pp forces (with levels 3, 4, etc), and consequently both will drop lower than the level $I = 0_1$. We then find that the two levels with $I = 2$ will lie ~ 1 Mev above the $I = 0_1$ level and will be close to each other. Such a level scheme disagrees with experiment. We now take into account the level interaction due to the np forces. It must be noted that the nondiagonal np matrix elements between the levels of the main configuration are approximately four times greater than for a single particle pair, and are comparable in magnitude with the matrix elements of the nn and pp forces in the case $I = 0$, when the latter are large. The np interaction of all the lower levels will therefore be strong and the first levels with given I will move down owing to repulsion. But the 2_1 level will drop farther than the 0_1 level because the $I = 2$ levels are denser than the $I = 0$ levels. Thus, a certain sequence of equidistant levels 0_1–2_1–2_2 is formed, with properties similar to those of the vibrational levels, but actually entirely different in nature. An account of the np forces is essential not only for the levels $I = 2$, but also for all levels of the four-particle system.

Finally, let us take into account the interaction between the external particles and the surface and let us calculate the entire spectrum. Figure 3 shows the calculated level scheme of Po^{212}, obtained with parameters $\rho = 2f$, $v_i = 15$ Mev, $v_s = 35$ Mev, $\hbar\omega = 3$ Mev, and $C = 1000$ Mev. For comparison, the experimental level scheme is shown on the right. The Po^{212} nucleus was experimentally investigated by many authors [5-9] and the results obtained in different laboratories are quite close to each other. It is also important to verify the correctness of the calculated binding energy for the ground state. The measured energy required to remove two neutrons and two protons from Po^{212} is 19.4 Mev. The single-particle energy of the four particles in potential (1) is $-(7.8 + 7.6) = -15.4$ Mev (see [1]), the energy of interaction with the surface is -0.7 Mev (for C = 1000 Mev), and the energy of Coulomb repulsion between two protons is + 0.4 Mev. Consequently

*The parity sign will henceforth be omitted where self-evident.

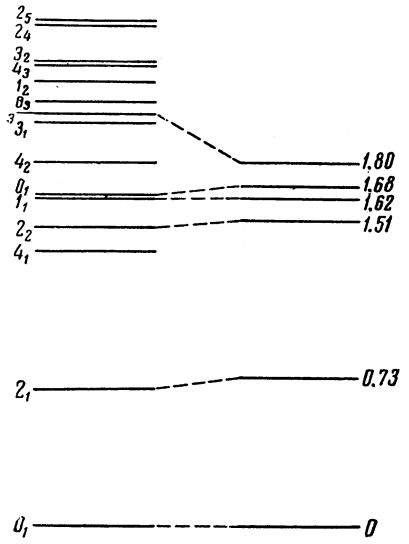

FIG. 3. Level scheme of Po^{212} after diagonalization of the matrices. The experimental level scheme is shown on the right.

the pair energy of the four particles is -3.7 Mev. The calculated total pair energy for the chosen parameters is -3.8 Mev.

To illustrate the sensitivity of the level scheme and of the binding energy to the choice of parameters, Table I lists the energies of the first three levels with spins $I = 0$ and 2 for different parameter combinations. By plotting different curves we can determine the dependence of the energy, of the distance between levels, etc. on the choice of each parameter separately.

Table I. Energies of the first three levels with spins $I = 0$ and 2 as a function of the potential parameters

Parameters*				$I = 0$	$I = 2$
v_t	v_s	$\hbar\omega$	C		
15	35	0		−3.75 −2.42 −1.94	−3.22 −2.69 −2.21
15	35	3	1000	−4.50 −2.87 −2.40	−3.82 −3.02 −2.48
15	35	3	1500	−4.26 −2.73 −2.24	−3.67 −2.93 −2.30
15	30	3	1000	−4.10 −2.64 −2.12	−3.56 −2.73 −2.20
20	35	3	1000	−4.88 −3.20 −2.77	−4.28 −3.34 −2.66
20	40	3	1000	−5.24 −3.45 −3.06	−4.54 −3.54 −2.88
15	35	3	2000	−4.14 −2.68 −2.20	−3.58 −2.85 −2.36

*All values are in Mev. $\rho = 2f$.

Table II. Fundamental coefficients $c^I(\alpha)$ of the eigenfunction expansion of the Po^{212} levels

$I=0$			$I=2$			
$(j_1j_2)\,J_{12},\ (j_3j_4)\,J_{34};\ J;\ NR$	0_1	0_2	$(j_1j_2)\,J_{12},\ (j_3j_4)\,J_{34};\ J;\ NR$	2_1	2_2	2_3
$(g_{9/2})^2\,0\,(h_{9/2})^2\,0;\ 0;\ 00$	0.80	—0.39	$(g_{9/2})^2\,0\,(h_{9/2})^2\,2;\ 2;\ 00$	0.63	—0.60	0.18
$(g_{9/2})^2\,2\,(h_{9/2})^2\,2;\ 0;\ 00$	0.36	0.77	$(g_{9/2})^2\,2\,(h_{9/2})^2\,0;\ 2;\ 00$	0.55	0.66	0.20
$(g_{9/2})^2\,4\,(h_{9/2})^2\,4;\ 0;\ 00$	0.12	0.26	$(g_{9/2})^2\,2\,(h_{9/2})^2\,2;\ 2;\ 00$	—0.26	—0.04	0.88
$(g_{9/2})^2\,6\,(h_{9/2})^2\,6;\ 0;\ 00$	0.04	0.05	$(g_{9/2})^2\,2\,(h_{9/2})^2\,4;\ 2;\ 00$	0.18	—0.18	—0.01
$(i_{11/2})^2\,0\,(h_{9/2})^2\,0;\ 0;\ 00$	0.10	—0.17	$(g_{9/2})^2\,4\,(h_{9/2})^2\,2;\ 2;\ 00$	0.17	0.21	0.02
$(g_{9/2})^2\,0\,(f_{7/2})^2\,0;\ 0;\ 00$	0.09	—0.12	$(g_{9/2})^2\,4\,(h_{9/2})^2\,4;\ 2;\ 00$	—0.04	—0.01	0.29
$(g_{7/2})^2\,0\,(h_{9/2})^2\,0;\ 0;\ 00$	0.12	—0.09	$(g_{7/2})^2\,0\,(h_{9/2})^2\,2;\ 2;\ 00$	0.10	—0.11	0.04
$(g_{9/2})^2\,0\,(i_{13/2})^2\,0;\ 0;\ 00$	—0.01	0.09	$(g_{9/2})^2\,2\,(i_{13/2})^2\,0;\ 2;\ 00$	—0.08	—0.12	—0.05
$(g_{9/2})^2\,0\,(h_{9/2})^2\,2;\ 2;\ 12$	—0.24	—0.03	$(i_{11/2})^2\,0\,(h_{9/2})^2\,2;\ 2;\ 00$	0.07	—0.09	0.05
$(g_{9/2})^2\,2\,(h_{9/2})^2\,0;\ 2;\ 12$	—0.28	—0.03	$(g_{9/2})^2\,0\,(h_{9/2})^2\,0;\ 0;\ 12$	—0.17	—0.01	—0.07
$(g_{9/2})^2\,2\,(h_{9/2})^2\,2;\ 2;\ 12$	0.08	0.21	$(g_{9/2})^2\,0\,(h_{9/2})^2\,2;\ 2;\ 12$	0.11	—0.07	—0.13
$(g_{9/2})^2\,2\,(h_{9/2})^2\,4;\ 2;\ 12$	—0.07	—0.17	$(g_{9/2})^2\,0\,(h_{9/2})^2\,4;\ 4;\ 12$	—0.12	0.13	—0.09
$(g_{9/2})^2\,4\,(h_{9/2})^2\,2;\ 2;\ 12$	—0.08	—0.17	$(g_{9/2})^2\,4\,(h_{9/2})^2\,0;\ 4;\ 12$	—0.10	—0.14	—0.05
$(d_{5/2}g_{9/2})\,2\,(h_{9/2})^2\,0;\ 2;\ 12$	—0.11	0.06	$(g_{9/2})^2\,2\,(h_{9/2})^2\,0;\ 2;\ 12$	0.10	0.09	—0.12
$(g_{9/2})^2\,0\,(h_{9/2})^2\,0;\ 0;\ 20$	0.06	—	$(g_{9/2})^2\,2\,(h_{9/2})^2\,2;\ 4;\ 12$	—0.18	—0.01	0.02
$(g_{9/2})^2\,4\,(h_{9/2})^2\,0;\ 4;\ 24$	0.04	0.02	$(g_{9/2})^2\,0\,(h_{9/2})^2\,2;\ 2;\ 24$	0.03	0.01	—

Table II lists the coefficients $C^I(\alpha)$ needed to construct the lower-state functions [see formula (7)]. All the functions are characterized by a large configuration mixture and this greatly influences the properties of the levels.

5. γ-TRANSITION PROBABILITIES

The γ-transition probability is sensitive to the structure of the initial and final state functions. This can be traced by calculating the reduced transition probability $B(\lambda)$, which is connected with the total transition probability $T(\lambda)$ by the relation

$$T(\lambda;\ I\to I') = B(\lambda; I\to I')\,(\Delta E)^{2\lambda+1}, \tag{20}$$

$$B(\lambda;\ I\to I') = \frac{8\pi(\lambda+1)}{\lambda\,[(2\lambda+1)!!]^2\,\hbar\,(\hbar c)^{2\lambda+1}\,(2I+1)} \times \Big| \sum_{p\alpha\alpha'} c^I(\alpha)\,c^I(\alpha')\,\langle \alpha I\nu \| \mathfrak{M}_p(\lambda) \| \alpha' I'\nu'\rangle + \sum_{\alpha\alpha'} c^I(\alpha)\,c^I(\alpha')\,\langle \alpha I\nu \| \mathfrak{M}_s(\lambda) \| \alpha' I'\nu'\rangle^2 \Big|. \tag{21}$$

$\mathfrak{M}_p(\lambda)$ and $\mathfrak{M}_s(\lambda)$ are the operators of the nucleon and collective transitions with multipolarity λ.

The sum in (21) breaks up into two parts; one corresponds to partial phononless transitions, and the other is connected with the change in the number of phonons. The contribution from the second part of the sum is essential for E1 transitions.

Table III lists the calculated values of $B(\lambda)$ for transitions between levels up to an excitation energy ~ 2 Mev. The value of $B(\lambda)$ varies within wide limits and is an individual characteristic of each transition separately. A distinguishing feature of the transition $2_1 \to 0_1$ is that all the members of the sum (21) have the same sign. Both parts of the sum give approximately the same contribution when C = 1000 Mev. On the whole the probability of the $2_1 \to 0_1$ transition with C = 1000 Mev is found to be five times larger than the so-called single-particle probability. The $2_2 \to 0_1$ transition differs strongly from the preceding one. The difference in the signs of $c^I(\alpha)$ causes the principal terms of the sum (21) to drop out and

Table III. Reduced probabilities* of γ transitions from Po^{212} levels

Transition	C = 1000 Mev		C = 2000 Mev	
	$B(E2)\cdot10^{-11}$, $sec^{-1}\cdot Mev^{-5}$	$B(M1)\cdot10^{-11}$, $sec^{-1}\cdot Mev^{-3}$	$B(E2)\cdot10^{-11}$, $sec^{-1}\cdot Mev^{-5}$	$B(M1)\cdot10^{-11}$, $sec^{-1}\cdot Mev^{-3}$
$2_1 \to 0_1$	4.30		2.20	
$2_2 \to 0_1$	0.05		0.01	
$2_2 \to 2_1$	0.04	61.00	0.02	68.00
$1_1 \to 0_1$		11.54		11.50
$1_1 \to 2_1$	1.60	5.03	1.04	4.90
$0_2 \to 2_1$	2.70		4.00	
$2_3 \to 0_1$	0.20		0.21	
$2_3 \to 2_1$	2.20	0.01	0.77	0.01
$2_3 \to 4_1$	0.38		0.10	
$2_3 \to 2_2$	0.44	4.90	0.33	5.80
$2_4 \to 2_1$	0.14	1.20	0.03	7.60
$4_1 \to 2_1$	7.20		3.30	

*The eigenfunctions were calculated with parameters $\rho = 2f$, $v_t = 15$ Mev, $v_S = 35$ Mev, $\hbar\omega = 3$ Mev, and C = 1000 and 2000 Mev.

$B(E_2)$ becomes two or three orders smaller than the first transition. This transition is particularly sensitive to the parameters of the forces.

The $2_2 \rightarrow 2_1$ transition has high intensity. In this case the electric component of the transition is small because of the strong quenching in the sum (21), while the magnetic component is, to the contrary, large. The $2_3 \rightarrow 2_1$ transition differs from the preceding one in that the magnetic component is small and the electric one increases sharply. The direct transition $2_3 \rightarrow 0_1$ has a noticeable intensity and can be observed. From the 1_1 level there is an intense M_1 transition to the ground level 0_1 and a mixed $(E_2 + M_1)$ transition to the excited level 2_1.

Most transitions in Table III have been experimentally observed, but only their multipolarities and relative intensities have been determined (see, for example, [7]). All the known experimental data on γ transitions are in good agreement with the theoretical calculations. This shows that an account of the mixture of configurations makes it possible to calculate the γ-transition picture sufficiently completely. As to the absolute values of the transition probabilities, these have been determined earlier from a comparison of the intensity of the γ line and the long-range α particles from a given level. In this case the α-decay probability was found from a semi-empirical formula verified against the transitions from the ground levels. The data on $B(\lambda)$ in Table III show that this method yields too high a probability. For example, the probability of the $2_1 \rightarrow 0_1$ transition is estimated in [7] to be 2.5×10^{-12} sec, while from Table III we get for probability not more than 1.2×10^{-11} sec.

The theoretical spectrum contains levels with spins $I = 3$ and 4, the 4_1 level being rather low. Since however, the β decay of Bi^{212} to these levels is strongly forbidden and the γ transitions to the 4_1 level have a low probability, it is quite difficult to observe these levels.

In the discussion of the spectra of Bi^{210} and Po^{210}, a suggestion has been made that the single-particle levels $i_{11/2}$ and $i_{13/2}$ move approximately 0.5 Mev below their positions in Pb^{209} and Bi^{209}, because the boundary of the potential V_C is less diffuse. This raises the question of the positions of these levels in Po^{212}. All the previous calculations were made without lowering the $i_{11/2}$ and $i_{13/2}$ levels. To verify the effect of the change in the positions of these single-particle levels, we calculated the spectrum with $i_{11/2}$ and $i_{13/2}$ dropped 0.5 Mev. It turned out that all the levels previously-present up to 1.8 Mev barely shifted, but new levels appeared, which we denote by 0_i, 1_i, and 2_i^-. The 0_i level lies near 2_1, 1_i lies near 1_1, and 2_i^- is higher than 2.2 Mev. It is very difficult to detect these levels by the γ rays,* nor are any other means clear. Since the investigation has shown that a shift of the $i_{11/2}$ and $i_{13/2}$ levels in Po^{212} within the indicated limits does not greatly affect all the results obtained, the question loses its urgency.

6. CONCLUSIONS

1. The previously obtained self-consistent and pair potentials were applied to a nucleus with two particle pairs in excess of the filled shells. The results show that an account of these potentials is sufficient to determine and describe in detail the pattern of excited states of such a nucleus.

2. We calculated the spectrum of Po^{212}, the binding energy, and the γ-transition probabilities. All the known levels have been found and their properties explained. The additional levels that should exist are indicated.

3. The theoretical results cannot be reconciled with the experimental data without account of the np interactions.

4. The method developed can be readily generalized to include the case of 8 nucleons (4n + 4p) in excess of the filled shells.

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*Thus, $B(E2; 2_2 \rightarrow 0_i) = 4 \times 10^7$; $B(E2; 2_3 \rightarrow 0_i) = 6 \times 10^7$; $B(E2; 1_i \rightarrow 2_1) = 7 \times 10^8$; $B(MI; 1_i = 0_i) = 10^{10}$. All are two or three orders smaller than for those observed.

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