"METALLIC" REFLECTION OF NEUTRONS

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The reflection of neutrons from strongly absorbing media is treated. A formula is found for the reflection coefficient for neutrons in the limit of zero energy. We show that the absorption cross section in the medium tends toward saturation for sufficiently small neutron energies.

THERE are a variety of ways of constructing neutron mirrors.[1,2] We are proposing the use of metallic reflection of neutrons, i.e., their reflection from strongly absorbing media. The theory of the reflection coefficient for neutrons is well known,[3,4] but one usually assumes that the reflecting medium has weak absorption and that the wave vector of the neutron is real.

In a strongly absorbing medium, we cannot neglect the imaginary part of the wave number $k = k_1 + ik_2$, and the wave function for a neutron propagating along the z axis has the form

$$\psi = (2k_2/k_1)^{1/2} e^{i(k_1+ik_2)z}. \tag{1}$$

The normalization has been chosen so that the total number of particles in the medium, which occupies the halfspace $z > 0$, is equal to $1/k_1$. We expand (1) in a Fourier integral:

$$\psi = \int_{-\infty}^{+\infty} c(p)\, e^{ipz} dp. \tag{2}$$

Then

$$c(p) = \left(\frac{k_2}{\pi k_1}\right)^{1/2} \frac{1}{k_2 - i(p-k_1)}. \tag{3}$$

For an undamped plane wave with wave vector p we can write the usual formulas for the absorption and scattering cross sections [cf. [4], formulas (39.6)]:

$$\sigma_c = 4\pi p^{-2}\eta_i (1 - 2\eta_i), \qquad \sigma_s = 4\pi p^{-2}(\eta_r^2 + \eta_i^2), \tag{4}$$

where $\eta = \eta_r + i\eta_i$ is the complex phase of the scattered wave. The total cross sections are given by the following integrals:

$$\bar{\sigma}_c = \int_{-\infty}^{+\infty} \sigma_c(p)\, |c(p)|^2\, p\,dp, \qquad \bar{\sigma}_s = \int_{-\infty}^{+\infty} \sigma_s(p)\, |c(p)|^2\, p\,dp \tag{5}$$

(for our normalization, $\int_{-\infty}^{\infty} |c(p)|^2\, p\,dp = 1$). The quantity η is expressed in terms of the quasipotential $u = A\delta(r)$ for the interaction of the nucleon and the nucleus by the formula

$$\eta = -2m\hbar^{-2}pA/4\pi.$$

If $E < 0.025$ ev, A is independent of energy, so that for small p the phase η is proportional to p. If in addition $\eta_i \ll 1$, i.e., $\eta_i^2 \ll \eta_i$, then $p\sigma_c$ is independent of energy, while $p\sigma_s \sim p$. Then

$$\bar{\sigma}_c = -\frac{4\pi}{k_1}\frac{2m}{\hbar^2}\frac{\operatorname{Im} A}{4\pi},$$

$$\bar{\sigma}_s = 4\pi\left(\frac{2m}{\hbar^2}\right)^2 \left[\left(\frac{\operatorname{Re} A}{4\pi}\right)^2 + \left(\frac{\operatorname{Im} A}{4\pi}\right)^2\right]. \tag{6}$$

The additivity of the potentials from individual nuclei gives

$$U = V + iW = \rho A,$$

where ρ is the number of nuclei per unit volume in the medium, and U is the potential in the medium. Then

$$W = -\frac{\hbar^2}{2m} k_1 \rho \bar{\sigma}_c, \quad V = \pm \frac{\hbar^2}{2m} \rho\, [4\pi\bar{\sigma}_s - (k_1\bar{\sigma}_c)^2]^{1/2}. \tag{7}$$

If a beam of neutrons with wave number k_0 enters the medium from the space outside, the neutron wave number in the medium is $k^2 = k_0^2 + (\hbar^2/2m)U$, and consequently

$$k^2 = k_0^2 \mp \alpha + ik_1\rho\bar{\sigma}_c, \tag{8}$$

$$\alpha = \rho\, [4\pi\bar{\sigma}_s - (k_1\bar{\sigma}_c)^2]^{1/2}. \tag{9}$$

The upper and lower signs refer to positive and negative scattering amplitudes. Consequently,

$$k_2 = \tfrac{1}{2}\rho\bar{\sigma}_c, \qquad k_1^2 = k_0^2 + \tfrac{1}{4}(\rho\bar{\sigma}_c)^2 \mp \alpha. \tag{10}$$

When the absorption cross section in the medium is small, one obtains the formula for scattering media [cf. [4], formula (43.8)]. But for media with large absorption, at energies of $10^{-6} - 10^{-8}$ ev, the term $k_1\rho\bar{\sigma}_c$ becomes much greater than the other terms in (8) and, in the limit when we neglect the potential scattering, we have

$$k_1 = k_2 = \rho\bar{\sigma}_c/2. \qquad (11)$$

The limiting formula for the refractive index is

$$n = (k_1 + ik_2)/k_0 = (\rho\bar{\sigma}_c/k_0)\,(1 + i). \qquad (12)$$

The reflection coefficient for $k_0 \ll \rho\bar{\sigma}_c$ ($|n| \gg 1$) will be given by the formula

$$R = \left|\frac{(n^2 - \sin^2\vartheta)^{1/2} - \cos\vartheta}{(n^2 - \sin^2\vartheta)^{1/2} + \cos\vartheta}\right|^2 = 1 - \frac{4k_0}{\rho\bar{\sigma}_c}\cos\vartheta, \qquad (13)$$

where ϑ is the angle to the normal.

In the more general case, when the potential scattering cannot be neglected (in the same approximation with respect to k_0),

$$R = 1 - 4k_1k_0\cos\vartheta/(k_1^2 + k_2^2). \qquad (14)$$

For $k_0 \ll \rho\sigma_c$, α the absorption cross section tends toward a limiting value. In fact, according to (6),

$$\sigma_0 = a/k_1. \qquad (15)$$

Then, substituting (15) in (9), we get for $k_0 \rightarrow 0$,

$$k_1^2 = (\rho a)^2/4k_1^2 \mp \alpha. \qquad (16)$$

The expression (16) is an equation for determining the minimum value of k_1 (α does not depend on k_1). Solving this equation, we find

$$k_{1min}^2 = \mp \tfrac{1}{2}\alpha + \tfrac{1}{2}[(\rho a)^2 + \alpha^2]^{1/2}. \qquad (17)$$

Consequently for neutrons with zero energy in the vacuum, the real part of the wave number in the medium is different from zero [where the upper sign in (17) refers to the case of a positive scattering amplitude].

Substituting (17) in (15), we get

$$\sigma_{c\,max} = a\left[\tfrac{1}{2}(\rho^2a^2 + \alpha^2)^{1/2} - \tfrac{1}{2}\alpha\right]^{-1/2}.$$

Thus the absorption cross section in the medium remains finite as $k_0 \rightarrow 0$, since we always have $\sigma_c \le \sigma_{c\,max}$. If $\alpha \ll \rho a$,

$$\sigma_{c\,max} = \sqrt{2a/\rho}. \qquad (18)$$

In this case the reflection coefficient is equal to

$$R = 1 - 4k_0\,(2a\rho)^{-1/2}\cos\vartheta. \qquad (19)$$

From formula (10) it follows that in a medium with absorption total internal reflection is impossible, although the reflection coefficient can become very large if the absorption is small.

Let us consider a specific example—the isotope Gd^{157}, which has the largest known absorption cross section of all the stable nuclei, $\sigma_c = 2 \times 10^5$ barn for E = 0.025 ev. For gadolinium, $\rho = 2.3 \times 10^{22}$ cm^{-3}. According to (18), $\sigma_{c\,max} = 7.7 \times 10^7$ barns. This cross section is reached for a neutron energy $E \le 1.6 \times 10^{-7}$ ev ($T < 2 \times 10^{-3}$ °K). In this case, $R = 1 - 45\,T^{1/2}\cos\vartheta$ (where T is the neutron energy in degrees).

Thus we see that metallic reflection of neutrons is much less effective than other methods for obtaining neutron mirrors.

We note that the normal vibrations of the atoms of the medium do not change relations (18) and (19), since the Doppler effect does not change cross sections which have a 1/v variation, as follows from the normalization of the wave function to unit flux.

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