TWO-ELECTRON CHARGE EXCHANGE OF PROTONS IN HELIUM DURING FAST COLLISIONS

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The integral cross section for capture of two electrons by protons in helium is calculated in the first Born approximation.

SEVERAL recent theoretical and experimental papers are devoted to the capture of electrons in atomic collisions. The purpose of these papers, in addition to a determination of the effective capture cross sections, was to ascertain, by comparison of the theoretical and the experimental results, whether a particular approximate calculation method is applicable.[1] It has become clear, in particular, that the cross section for the capture of one electron by the protons in hydrogen, calculated in the first Born approximation, agrees with the experimental cross section up to the limits of applicability of the Born approximation.

It must be noted that most published papers have been devoted to single-electron processes. In the present communication we determine the cross section for one of the many-electron processes, namely the cross section for the capture of two electrons in a collision between fast protons and helium atoms (when the Born approximation is valid). In this case the Born threshold is about 100 kev (in the laboratory frame). This process has already been investigated theoretically[2] at low colliding-particle velocities, and experimentally[3] at medium velocities.

The differential c.m.s. cross section of the transition of two electrons from the ground state of the helium atom to the ground state of the H^- ion is

$$d\sigma = \frac{k_2}{k_1} | f(\theta) |^2 do;$$

$$f(\theta) = \frac{\mu}{2\pi} \int\int\int e^{-i\mathbf{k}_2\mathbf{r}} \psi(r_1, r_2) V(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_1) e^{i\mathbf{k}_1\mathbf{s}} \varphi(p_1, p_2) \times d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r},$$

$$V(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_1) = 2/p_1 + 2/p_2 - 2/|\mathbf{r}_1 - \mathbf{p}_1|.$$

Here $\mathbf{s}(\mathbf{r})$ is the radius vector of the proton (α particle) relative to the center of gravity of the helium atom (the H^- ion); $\mathbf{r}_1, \mathbf{r}_2 (\mathbf{p}_1, \mathbf{p}_2)$ are the radius vectors of the electrons relative to the proton (α particle); $\mathbf{k}_1(\mathbf{k}_2)$ is the wave vector before (after) scattering, with $k_1^2 = \mu E$, $K_2^2 = \mu(E - E_1 + E_2)$, where E —energy of relative motion, E_1 and E_2 are respectively the energies of the H^- and He ground states, $\mu = {}^4/_5 \epsilon$ —reduced mass, ϵ —the ratio of the mass of the electron to the mass of the proton;

$$\psi = (z^3/\pi)^2 e^{-z(r_1+r_2)}, \qquad \varphi = (z'^3/\pi)^2 e^{-z'(p_1+p_2)}$$

—respectively the wave functions of the ground state of the H^- ion and the helium atom; z = 0.69, z′ = 1.69. These formulas are written in atomic units ($\hbar = e = m = 1$); the cross section is in a_0^2 units ($a_0 = 0.529 \times 10^{-8}\ \mathrm{cm}^2$).

The scattering amplitude can be reduced to a form convenient for numerical integration. Transformations analogous to those used in [4] yield

$$f(\theta) = 48\mu z^4 \alpha^4 \pi^{-2} (J_1 - J_2);$$

$$J_1 = 16 \int_0^1\int_0^1 dx\, dy\, xy (1-y) a^{-3} b^{-5} A^{-5} F_1(x, y),$$

$$J_2 = \int_0^1\int_0^1 dx\, dy\, xy (1-x)(1-y) a^{-5} b^{-5} A^{-5} F_2(x, y).$$

$$F_1 = a^3 A^2 + 2Ab(a+b)^2 (3a^2 - 2ab + b^2) + 16ab^2 (a+b)^4,$$

$$F_2 = 3A^4 + 2A^3 (2a^2 + 3ab + 2b^2) + 8A^2 [(a+b)^4 - ab(2a^2 + 3ab + 2b^2)] + 48Aab(a^2 + b^2)(a+b)^2 + 128a^2b^2(a+b)^4,$$

$$a^2 = 1 + (\alpha^2 - 1)x + \varkappa^2 (1-x)x,$$

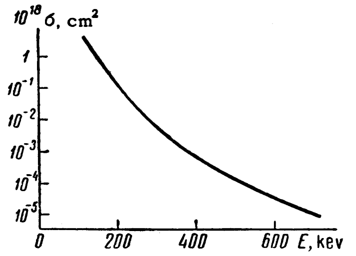
$$b^2 = 1 + (\alpha^2 - 1)y + \varkappa^2(1-y)y,$$

$$A = (a+b)^2 + (\mathbf{g} + \varkappa x + \varkappa y)^2, \qquad \alpha = z/z',$$

$$\mathbf{g} = \frac{1}{z'}\left(\frac{2}{2+\varepsilon}\mathbf{k}_1 - \mathbf{k}_2\right), \qquad \varkappa = \frac{\varepsilon}{z'}\left(\frac{\mathbf{k}_1}{4+2\varepsilon} + \frac{\mathbf{k}_2}{1+2\varepsilon}\right),$$

$$\cos\theta = \frac{\mathbf{k}_1\mathbf{k}_2}{k_1k_2}.$$

The integrals J_1 and J_2 in the cross section are brought about by the interaction between the



electrons and the incoming proton and by the interaction between the nuclei, respectively. At very high energies, they decrease with energy in like fashion. Indeed, as $E \to \infty$ we have

$$J_1 \sim \int_0^1 x\,dx \int_0^1 \frac{y(1-y)}{b^5 A^3}\,dy, \quad J_2 \sim \int_0^1 \frac{x(1-x)}{a^5}\,dx \int_0^1 \frac{y(1-y)}{b^5 A}\,dy.$$

A is proportional to E in the entire range of values of x and y, and hardly depends on x or y; therefore

$$J_1 \sim \frac{1}{E^3} \int_0^1 \frac{y(1-y)}{b^5}\,dy, \qquad J_2 \sim \frac{1}{E} \left\{ \int_0^1 \frac{y(1-y)}{b^5}\,dy \right\}^2,$$

hence, after integration,

$$J_1 \sim E^{-5}, \qquad J_2 \sim E^{-5}.$$

From the physical point of view, it becomes obvious that the contribution to the cross section from the interaction between the nuclei should decrease with increasing E more rapidly than the contribution due to the interaction between the electrons and the incoming particle.

Such a difficulty exists also in the analysis of the capture of a single electron.[5,6] Several authors[7,8] were able to reconcile the calculated cross section with the physical notions regarding the nature of the charge-exchange process. At low energies, however, the single charge exchange cross sections calculated in[5-8] are practically the same. In the present note we calculate the double charge exchange cross section for energies where the corrections proposed in [7,8] are apparently still insignificant.

The calculated integral cross section is shown in the figure as a function of the energy of the incoming protons in the laboratory frame. At the present time there are no experimental data for energies greater than 100 kev. It would be desirable to obtain such data and to compare them with the results of this paper. We note that the negative hydrogen ion apparently has no excited levels. The calculated cross section can therefore be directly compared with experiment.

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