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ON $\Sigma^+ \to p + e^+ + e^-$ *AND* $\Sigma^+ \to p + \mu^+ + \mu^-$ *DECAYS*

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An investigation of the process $\Sigma^+ \to p + e^+ + e^-$ is of interest in connection with the search for neutral currents in weak interactions. According to the theory of universal charged currents [1] this decay cannot be induced by weak interaction alone, but it can proceed under the influence of strong and electromagnetic interaction. The total decay probability and recoil-proton energy spectrum are computed by taking into account this circumstance. An expression for the probability of the decay $\Sigma^+ \to p + \mu^+ + \mu^-$ is also obtained and the ratio of the two decay probabilities is estimated.

THREE cases of hyperon decay into a proton and a γ quantum

$$\Sigma^+ \to p + \gamma \tag{1}$$

were observed recently.

Decays of this type, and particularly the structure of the matrix element corresponding to the decay (1), were discussed by Behrends.[2] These decays are due to a combination of strong, weak, and electromagnetic interactions, which can be represented, for example, by the Feynman diagram of Fig. 1, where the weak, strong, and electromagnetic interactions correspond to vertices a, b, and c respectively. However, since there is no theory of strong interactions at present, the process represented by this diagram cannot be calculated, decays such as (1) are examined phenomenologically, and diagrams such as this are replaced by a block A representing the totality of all the diagrams that contribute to the process under investigation (see Fig. 2). The general form of the matrix corresponding to the vertex $(\bar{p}\Sigma\gamma)$ in the matrix element will be

$$\Gamma^\alpha = (a_1 + a_2\gamma_5)\gamma^\alpha + (b_1 + b_2\gamma_5)(\gamma^\alpha\hat{q} - \hat{q}\gamma^\alpha) + (c_1 + c_2\gamma_5)q^\alpha, \tag{2}$$

where γ^α are Feynman matrices, q is the four-momentum of the photon,* and $a_1, \ldots, c_2$ are certain functions of q^2. In view of the gauge invariance we should have

$$g^{\alpha\alpha}q^\alpha\bar{u}_p\Gamma^\alpha u_\Sigma = 0,$$

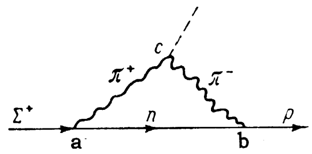

FIG. 1

thus bringing about a connection between a and c:

$$a_1 = c_1q^2/(m_\Sigma - m_p), \qquad a_2 = c_2q^2/(m_\Sigma + m_p).$$

Since $q^2 = 0$ and $qe = 0$ for a real photon (e is the photon polarization vector), the amplitude of the decay (1)

$$M_\gamma = \frac{G}{\sqrt{2}} g^{\alpha\alpha}\bar{u}_p\Gamma^\alpha u_\Sigma e^\alpha$$

depends only on b_1 and b_2, and the decay probability is*

$$w_\gamma = 2G^2(b_1^2 + b_2^2)\Delta^3/\pi, \tag{3}$$

where G is the weak-interaction constant, $\Delta = (m_\Sigma^2 - m_p^2)/2m_\Sigma$ is the maximum photon energy, while m_Σ and m_p are respectively the masses of the Σ^+ hyperon and the proton. Thus, an investigation of this decay does not yield any information on the values of a and c.

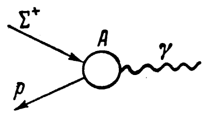

FIG. 2

*We use henceforth the matrices and symbols given in the book by Bogolyubov and Shirkov,[3] and also a system of units in which $\hbar = c = 1$.

*As usual, we assume in first approximation that b_1 and b_2 are constants.

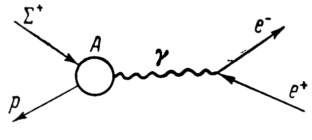

FIG. 3

To obtain effects in which a and c play a role, it is necessary to consider the hitherto unobserved decay with internal conversion of a γ quantum and production of an electron-positron pair

$$\Sigma^+ \to p + e^+ + e^-. \tag{4}$$

This decay corresponds to the diagram shown in Fig. 3 and to the matrix elements

$$M_{e^+ + e^-} = \frac{G}{\sqrt{2}} g^{\alpha\alpha} \bar{u}_p \Gamma^\alpha u_\Sigma q^{-2} \bar{u}_{e^-} \gamma^\alpha u_{e^+}, \tag{5}$$

and in this case the decay amplitude is already dependent on the values of a and c. An investigation of this process can thus yield some information on these constants. Standard calculations yield for the differential probability the following expression

$$dw = 2\pi \frac{G^2 e^2}{4} g^{\alpha\alpha} g^{\beta\beta} M^{\alpha\beta} \frac{1}{q^4} \{K_-^\alpha K_+^\beta + K_-^\beta K_+^\alpha - g^{\alpha\beta}(K_- K_+ + m^2)\} \, \delta(-K_\Sigma + K_p + K_- + K_+) \, d\mathbf{K}_p d\mathbf{K}_- d\mathbf{K}_+ / (2\pi)^6 K_-^0 K_+^0, \tag{6}$$

where m is the electron mass, K_Σ, K_p, K_-, and K_+ are the respective 4-momenta of the hyperon, proton, electron, and positron, and

$$M^{\alpha\beta} = \mathrm{Sp}\,\{(\hat{K}_p + m_p)\Gamma^\alpha (K_\Sigma + m_\Sigma)\bar{\Gamma}^\beta\} / 4K_\Sigma^0 K_p^0$$

(we average over the variables of the Σ^+ hyperon and sum over the variables of the electron and positron).

In the integration with respect to $d\mathbf{K}_- d\mathbf{K}_+$ we change to new variables:[4]

$$q = K_- + K_+, \qquad r = K_- - K_+.$$

We thus obtain

$$\int \{K_-^\alpha K_+^\beta + K_-^\beta K_+^\alpha - g^{\alpha\beta}(K_- K_+ + m^2)\} \frac{d\mathbf{K}_- d\mathbf{K}_+}{K_-^0 K_+^0} = \frac{1}{2} \int (q^\alpha q^\beta - r^\alpha r^\beta - g^{\alpha\beta} q^2)\, \delta(q^2 + r^2 - 4m^2)\, \delta(qr)\, dq dr.$$

The integral with respect to dr has the following tensor structure

$$\int (q^\alpha q^\beta - r^\alpha r^\beta - g^{\alpha\beta} q^2)\, \delta(q^2 + r^2 - 4m^2)\, \delta(qr)\, dr = A g^{\alpha\beta} + B q^\alpha q^\beta,$$

where A and B are certain functions of q^2. To determine these functions we multiply both halves of this identity first by $g^{\alpha\beta}$ and then by $g^{\alpha\beta} q^\alpha q^\beta$, obtaining after a summing over the double index the following two equations

$$\int (-r^2 - 3q^2)\, \delta(q^2 + r^2 - 4m^2)\, \delta(qr)\, dr = 4A + Bq^2,$$
$$0 = Aq^2 + Bq^4.$$

The integral in the left half of the first equation we calculate in a system in which $\mathbf{q} = 0$, and upon changing to the invariant form we obtain

$$A = -Bq^2, \qquad B = \frac{4\pi}{3} \frac{q^2 + 2m^2}{q^2} \sqrt{\frac{q^2 - 4m^2}{q^2}}. \tag{7}$$

We thus have from (6) and (7)

$$dw = \frac{1}{3(2\pi)^6} G^2 e^2 (q^2 + 2m^2) \sqrt{\frac{q^2 - 4m^2}{q^2}} \frac{1}{q^4} \times g^{\alpha\alpha} g^{\beta\beta} M^{\alpha\beta} (-g^{\alpha\beta} + q^\alpha q^\beta q^{-2})\, \delta(-K_\Sigma + K_p + q)\, dq\, d\mathbf{K}_p. \tag{8}$$

After summing over the double index and eliminating the δ function, using the conservation law $m_\Sigma - m_p = q$, we make the following substitution in (8):

$$\frac{d\mathbf{K}_p}{K_p^0} = -4\pi \frac{qdq}{m_\Sigma} |\mathbf{K}_p| = -4\pi \left[\left(\frac{\Delta^2 - q^2}{2m_\Sigma}\right)^2 + \frac{m_p}{m_\Sigma}(\Delta^2 - q^2)\right]^{1/2} \frac{qdq}{m_\Sigma}, \tag{9}$$

where $\Delta = m_\Sigma - m_p$, and the factor 4π is obtained by integration over the angles.

To obtain an expression for the total probability we integrate (8) with respect to dq from $q_{min} = 2m$ to $q_{max} = \Delta$. In the limiting case of small values of Δ/m_Σ (in our case $\sim 1/5$) we obtain, accurate to terms proportional to Δ^2/m_Σ^2 inclusive,

$$\begin{aligned} w = \frac{1}{3} G^2 e^2 \frac{\Delta^3}{\pi^3} \Big\{ &(b_1^2 + b_2^2)\Big[\ln\frac{2\Delta}{m} - \frac{23}{12} - \frac{3}{2}\frac{\Delta}{m_\Sigma}\Big(\ln\frac{2\Delta}{m} - \frac{23}{12}\Big) + \frac{7}{8}\frac{\Delta^2}{m_\Sigma^2}\Big(\ln\frac{2\Delta}{m} - \frac{847}{420}\Big)\Big] - \frac{1}{4}(b_1^2 - b_2^2)\Big[1 - \frac{3}{2}\frac{\Delta}{m_\Sigma} \\ &+ \frac{23}{40}\frac{\Delta^2}{m_\Sigma^2}\Big] + \frac{1}{8}\Big(c_1^2 + \frac{\Delta^2 c^2}{4m_\Sigma(1 - \Delta/2m_\Sigma)^2}\Big)\Big[\ln\frac{2\Delta}{m} - 2 \\ &- \frac{3}{2}\frac{\Delta}{m_\Sigma}\Big(\ln\frac{2\Delta}{m} - 2\Big) + \frac{7}{8}\frac{\Delta^2}{m_\Sigma^2}\Big(\ln\frac{2\Delta}{m} - \frac{221}{105}\Big)\Big] \\ &- \frac{1}{40}\Big(c_1^2 - \frac{\Delta^2 c_2^2}{4m_\Sigma^2(1 - \Delta/2m_\Sigma)^2}\Big)\Big[1 - \frac{1}{2}\frac{\Delta}{m_\Sigma} - \frac{1}{56}\frac{\Delta^2}{m_\Sigma^2}\Big] \\ &+ \frac{1}{4}\Big(b_1 c_1 - \frac{\Delta b_2 c_2}{2m_\Sigma(1 - \Delta/2m_\Sigma)}\Big)\Big[1 - \frac{6}{5}\frac{\Delta}{m_\Sigma} - \frac{1}{8}\frac{\Delta^2}{m_\Sigma^2}\Big] \\ &- \frac{1}{4}\Big(b_1 c_1 + \frac{\Delta b_2 c_2}{2m_\Sigma(1 - \Delta/2m_\Sigma)}\Big)\Big[1 - \frac{9}{5}\frac{\Delta}{m_\Sigma} - \frac{3}{8}\frac{\Delta^2}{m_\Sigma^2}\Big]\Big\}. \end{aligned} \tag{10}$$

From (3) and (10) we can find the internal-conversion coefficient

$$\rho = w(\Sigma^+ \to p + e^+ + e^-)/w(\Sigma^+ \to p + \gamma).$$

To estimate the order of magnitude of ρ we limit ourselves in (10) to the first term, and get

$$\rho = \frac{2\alpha}{3\pi}\Big(\ln\frac{2\Delta}{m} - \frac{23}{12}\Big) \approx \frac{1}{130}. \tag{11}$$

Deviations from this value can be due to corrections for the structure of the matrix element and for the dependence of the form factors on q^2. Thus, for example, if we take into account the second term in (10), which is proportional to $b_1^2 - b_2^2$, we obtain for the variation of ρ the following estimate

$$\frac{2\alpha}{3\pi}\ln\frac{2\Delta}{m} - \delta < \rho < \frac{2\alpha}{3\pi}\ln\frac{2\Delta}{m} + \delta, \quad \delta = \frac{1}{4}\frac{2\alpha}{3\pi}.$$

To estimate the influence of the dependence of the form factors on q^2 we put, for example,

$$b^2(q^2) = b_0^2\left(1 - \tfrac{1}{6}q^2r^2\right),$$

where $r \approx m_\pi/2 \approx 1/\Delta$ is the interaction radius (m_π is the pion mass). We then obtain

$$w = \frac{1}{3}G^2e^2\frac{\Delta^3}{\pi^3}(b_{01}^2 + b_{02}^2)\left(\ln\frac{2\Delta}{m} - \frac{179}{90}\right).$$

From this we conclude that this influence is small.

We note that expression (11) is in the nature of the transverse part of the conversion coefficient, since the longitudinal part due to the term $q^\alpha q^\beta/q^2$ in (8) is independent of $(b_1^2 + b_2^2)$ and $(b_1^2 - b_2^2)$ (see [5]).

Let us consider also the recoil-proton momentum distribution obtained from (6) after integrating over the electron and positron momenta. Neglecting in this expression terms that result in a small correction, we obtain

$$dw = C'\sqrt{\Delta^2 - q^2}\sqrt{q^2 - 4m^2}\,q^{-2}\,dq.$$

The factor C′ depends here on the form factors. From the conservation law we obtain (accurate to Δ/m_Σ)

$$dw = C''\mathbf{K}_p^2\sqrt{\Delta^2 - \mathbf{K}_p^2 - 4m^2}\,(\Delta^2 - \mathbf{K}_p^2)^{-3/2}d\,|\mathbf{K}_p|.$$

Let us put $x^2 = \mathbf{K}_p^2/\Delta^2$ and $a = 2m/\Delta$. Then

$$dw = Cf(x)\,dx,$$

$$f(x) = x^2\sqrt{1 - a^2 - x^2}\,(1 - x^2)^{-3/2}, \qquad 0 \leqslant x^2 \leqslant 1 - a^2.$$

$f(x)$ has a sharp maximum at $x_{max} = (2 - 2a^2)/(2 + a^2)$, and $f(x_{max}) \approx 2(\Delta/2m)^2$ (see Fig. 4). This means that the decays occur for the most part with formation of "narrow" pairs, wherein the proton acquires a maximum momentum.

From (8) and (9) we can obtain, under the same assumptions with respect to the structure of the matrix element, an expression for the total decay probability

$$\Sigma^+ \to p + \mu^+ + \mu^-. \qquad (12)$$

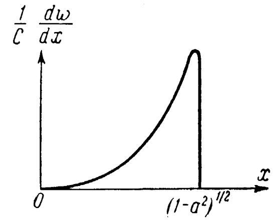

FIG. 4

Recognizing that the muon mass is large (compared with the electron mass), we can modify slightly the method of integrating with respect to $d\mathbf{K}_p$ in (8) and confine ourselves to terms proportional to $(b_1^2 + b_2^2)$ and $(b_1^2 - b_2^2)$. We then obtain, with accuracy to $m_\mu^2/\Delta^2 \approx 0.17$ inclusive,

$$w = \frac{1}{3}G^2e^2\frac{\Delta^3}{\pi^3}\cdot\frac{\pi}{2}\left\{(b_1^2 + b_2^2)\left(\frac{13}{8}\frac{m_\mu^2}{\Delta^2} - \frac{1}{16}\right) - \frac{3}{4}(b_1^2 - b_2^2)\left(\frac{1}{4} - \frac{m_\mu^2}{\Delta^2}\right)\right\},$$

where m_μ is the muon mass. For the ratio of the probabilities of decays (4) and (12) we obtain the estimate

$$\frac{w(\Sigma^+ \to p + e^+ + e^-)}{w(\Sigma^+ \to p + \mu^+ + \mu^-)} \approx \frac{\ln(2\Delta/m) - {}^{23}/_{12}}{\pi\,(13m_\mu^2/8\Delta^2 - {}^1/_{16})/2} \approx 14.5.$$

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