τ DECAY AND $\pi\pi$ COUPLING

J. WOLF and W. ZOELLNER

Joint Institute for Nuclear Research

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Integral equations for the probability of τ decay are derived. An effective range analysis gives s-wave scattering lengths for the τ interaction; from which the experimental τ-decay spectrum can be synthesized. Satisfactory agreement with experiment is obtained for $a_2 \approx 0.2$ and $a_0 \approx 0.3$. In conjunction with the integral equation for $\pi\pi$ scattering, these values speak in favor of the existence of a (T = 2) resonance in $\pi\pi$ interaction.

1. INTRODUCTION

THE τ decay is one of the few processes which enable us to investigate the $\pi\pi$ interaction without the influence of other particles. The small kinetic energy of the emitted pions facilitates the theoretical analysis, and the rather extensive experimental data enable us to compare theory with experiment.

To investigate the τ decay we propose to use the Mandelstam representation. It must be noted, however, that such a representation is in general valid for decay processes only in a certain approximation.[1] Fubini and Stroffolini have shown[2] that the imaginary part of the spectral functions corresponds to processes with three particles in intermediate states.* The imaginary part makes no contribution in the two-particle approximation, which we use. In the present paper we consider the consequences of such an approximation. Analogous considerations were used by Khuri and Treiman.[3]

For the determination of the τ decay we shall henceforth write out integral equations with account of the interaction between pions in p states. It thus becomes possible in principle to verify the solutions of the integral equations of $\pi\pi$ scattering.

To estimate such $\pi\pi$-interaction data as the scattering lengths, we can neglect the p waves. A comparison of the calculated matrix element with the experimental spectrum of the τ decay yields an estimate for the scattering lengths of the s waves of the $\pi\pi$ interaction. Using these scattering lengths and the integral equations from [4] we can obtain further information on the $\pi\pi$ interaction.

*The authors are grateful to G. Bonneva for a preprint in which this problem is also discussed.

2. INTEGRAL EQUATIONS WITH ACCOUNT OF p WAVES

To obtain equations for the amplitudes of the τ^+ and τ'^+ decay into three pions we consider the following reactions:

$$\tau^+ + \pi_1^- \to \pi_2^+ + \pi_3^-, \qquad \text{(I)}$$

$$\tau^+ + \pi_2^- \to \pi_3^- + \pi_1^+, \qquad \text{(II)}$$

$$\tau^+ + \pi_3^+ \to \pi_1^+ + \pi_2^+; \qquad \text{(III)}$$

$$\tau'^+ + \pi_1^0 \to \pi_2^0 + \pi_3^+, \qquad \text{(I')}$$

$$\tau'^+ + \pi_2^0 \to \pi_3^+ + \pi_1^0, \qquad \text{(II')}$$

$$\tau'^+ + \pi_3^- \to \pi_1^0 + \pi_2^0. \qquad \text{(III')}$$

The invariant variables of these processes assume in c.m.s. of reaction (III) the form

$$s_1 = \gamma - 2q_3^2 + 2p_3q_3z_3, \quad s_2 = \gamma - 2q_3^2 - 2p_3q_3z_3,$$
$$s_3 = 4(q_3^2 + \mu^2). \qquad (1)$$

Here m and μ denote respectively the K-meson and pion masses; $\gamma = (m^2 - \mu^2)/2$; p_3 and q_3 are the momenta of the particles before and after collision; z_3 denotes the cosine of the scattering angle. Analogous relations are true also in the c.m.s. of the other reactions.

The variables s_i satisfy the condition

$$s_1 + s_2 + s_3 = m^2 + 3\mu^2. \qquad (2)$$

We shall use also the following invariant combinations:

$$\begin{aligned} 2\eta_1 &= s_2 - s_3 = 4p_1q_1z_1, \\ 2\eta_2 &= s_3 - s_1 = 4p_2q_2z_2, \\ 2\eta_3 &= s_1 - s_2 = 4p_3q_3z_3. \end{aligned} \qquad (3)$$

We need the values of the invariant variables for $z_i = \pm 1$. In this case we obtain

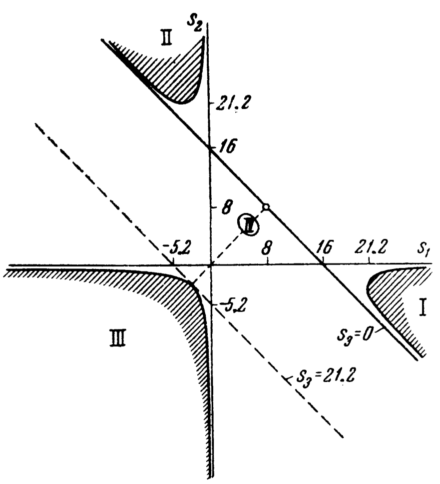

FIG. 1

$$s_1 s_2 s_3 = \mu^2 (m^2 - \mu^2)^2. \quad (4)$$

Figure 1 is a plot of Eq. (4). Here I, II, and III are the physical regions of the corresponding scattering processes, while region IV corresponds to the decay process. In the case of an unreal K-meson mass ($m = 3\mu$) this region contracts to a point. For real $s_i = 4(\nu_i = 1)$* with $m = 3.6\mu$ the product $(p_i q_i)$ becomes complex when

$$-1 < \nu_i < 0, \quad 0.7 < \nu_i < 4.3.$$

Let us assume the following representation

$$A(s_1, s_2, s_3) = \frac{1}{\pi}\int_4^\infty \frac{\rho(s') + \eta_1\sigma(s')}{s' - s_1} ds' + \frac{1}{\pi}\int_4^\infty \frac{\rho(s') - \eta_2\sigma(s')}{s' - s_2} ds' + \frac{1}{\pi}\int_4^\infty \frac{\lambda(s')}{s' - s_3} ds', \quad (5)$$

which for $m \leq 3$ can be obtained, as the Cini-Fubini approximation[5] for p waves, from the ordinary Mandelstam representation. For $\sigma = 0$ in the s-wave approximation, relation (5) was used in several investigations.[3,6]

From (5) and the unitarity conditions for processes I and III we obtain equations that define the functions ρ, λ, and σ. We write the partial-wave expansion of the scattering amplitudes in the form

$$A(\nu_1 z) \approx A_0(\nu) + 3pqzA_1(\nu) \quad (6)$$

and define the s and p waves respectively as

$$A_0(\nu) = \frac{1}{2}\{A(+) + A(-)\},$$
$$A_1(\nu) = \frac{1}{6pq}\{A(+) - A(-)\}. \quad (7)$$

Here $A(\pm) = A(\nu, z = \pm 1)$; we neglect the d waves and higher waves. We denote the amplitudes of the s waves of processes III and I by F_0 and G_0, and the amplitudes of processes III′ and I′ by f_0 and g_0. The amplitudes of the p waves are denoted respectively by G_1 and g_1.

From (5), (7), and (1) we obtain

$$F_0(s) = \Lambda^F + \frac{s-4}{\pi}\int_4^\infty \frac{ds'\lambda(s')}{(s'-4)(s'-s)} + \frac{2}{\pi}\int_4^\infty \frac{ds'\rho(s')}{s'-\gamma}\,\frac{(\alpha-\gamma)(s'-\alpha)+4p^2q^2}{(s'-\alpha)^2-4p^2q^2} - \frac{1}{\pi}\int_4^\infty ds'\sigma(s')\,\frac{(6\nu+4-\gamma)(s'-\alpha)+4p^2q^2}{(s'-\alpha)^2-4p^2q^2} - \frac{\gamma-4}{\pi}\int_4^\infty \frac{ds'\sigma(s')}{s'-\gamma}, \quad (8)$$

$$G_0(s) = \Lambda^G + \frac{s-4}{\pi}\int_4^\infty \frac{ds'\rho(s')}{(s'-4)(s'-s)} + \frac{1}{\pi}\int_4^\infty \frac{ds'}{s'-\gamma}\{\rho(s') + \lambda(s')\}\,\frac{(\alpha-\gamma)(s'-\alpha)+4p^2q^2}{(s'-\alpha)^2-4p^2q^2} + \frac{1}{2\pi}\int_4^\infty ds'\sigma(s')\,\frac{(6\nu+4-\gamma)(s'-\alpha)+4p^2q^2}{(s'-\alpha)^2-4p^2q^2} + \frac{\gamma-4}{2\pi}\int_4^\infty \frac{ds'\sigma(s')}{s'-\gamma}, \quad (9)$$

where $\alpha = (m^2 - 1)/2 - 2\nu = \gamma - 2\nu$. For the amplitude F_0 the subtraction is carried out at the point $s_1 = s_2 = \gamma$, $s_3 = 4$, for G_0 at the point $s_1 = 4$, $s_2 = s_3 = \gamma$ where $\Lambda^F = A(\gamma, \gamma, 4)$ and $\Lambda^G = A(4, \gamma, \gamma)$. These complex constants are related by Eq. (5). We note that (8) and (9) actually contain only the real parts of Λ^F and Λ^G, and therefore only one constant appears. For p waves we obtain an analogous equation.

We note that (8) and (9) contain only even powers of pq, and therefore the imaginary part vanishes in the regions where $(pq)^2$ becomes negative. For ρ, λ, and σ the unitarity conditions yield

$$\rho = \sqrt{\frac{\nu}{\nu+1}}\left\{G_0\Pi_0^{0*} - \frac{1}{6}F_0(\Pi_0^{0*} - \Pi_0^{2*})\right\},$$
$$\lambda = \sqrt{\frac{\nu}{\nu+1}}F_0\Pi_0^{2*}, \quad \frac{2}{3}\sigma = \sqrt{\frac{\nu}{\nu+1}}G_1\Pi_1^{1*}. \quad (10)$$

$\Pi_l^T(\nu)$ are the partial amplitudes of the $\pi\pi$ scattering for the isotopic spin T. In the derivation of (10) we use the selection rule $|\Delta T| = 1/2$, which yields the following relations between the amplitudes that determine the τ and τ' decays:

$$F_0 = 2g_0, \quad G_0 = f_0 + g_0, \quad G_1 = -g_1. \quad (11)$$

Equations (8) and (9) can be written in the form of dispersion relations in the ν plane. It is then

*We put $\nu_i = q_i^2$ and $\mu^2 = 1$.

easy to verify that the crossing amplitudes in the range of the arguments $1 < \nu^c < 4.3$ [where $\nu^c = \alpha(\nu) - 2pq$] make no contribution to the dispersion relations. Therefore the possible resonance of the $\pi\pi$ interaction in the p wave, expected approximately in this region, has practically no effect on the values of F_0 and G_0. It must be remembered that the region of the physical τ decay is limited to $\nu \approx 0.7$, and therefore amplitudes with large values of ν will exert only an insignificant influence on the decay. However, in spite of the weak influence of the p waves, their account may be important for a verification of solutions of the integral equations of the $\pi\pi$ interaction.

3. EQUATIONS FOR s AMPLITUDES AND ANALYSIS IN THE EFFECTIVE-RADIUS APPROXIMATION

Neglecting the terms containing the p wave in (8) and (9), we can obtain a system of integral equations for the s waves only. However, to obtain a numerical estimate it is more convenient to use a system of integral equations obtained from (5) by integrating along the lines $z_i = 0$. We therefore choose instead of (6) and (7) the following definition for the s waves:

$$A(\nu, z=0) \approx A_0(\nu). \tag{12}$$

This leads to the relations

$$\operatorname{Re} F_0(\nu) = \Lambda + \mathrm{P}\,\frac{1}{\pi}\int_0^\infty d\nu'\lambda(\nu')\left\{\frac{1}{\nu'-\nu} - \frac{1}{\nu'-\nu_0}\right\} + \mathrm{P}\,\frac{2}{\pi}\int_0^\infty d\nu'\rho(\nu')\left\{\frac{1}{\nu'-(1-\nu)/2} - \frac{1}{\nu'-\nu_0}\right\}, \tag{13}$$

$$\operatorname{Re} G_0(\nu) = \Lambda + \mathrm{P}\,\frac{1}{\pi}\int_0^\infty d\nu'\rho(\nu')\left\{\frac{1}{\nu'-\nu} - \frac{1}{\nu'-\nu_0}\right\} + \mathrm{P}\,\frac{1}{\pi}\int_0^\infty d\nu'\{\rho(\nu')+\lambda(\nu')\}\left\{\frac{1}{\nu'-(1-\nu)/2} - \frac{1}{\nu'-\nu_0}\right\}. \tag{14}$$

Subtraction is made at the point $\nu_1 = \nu_2 = \nu_3 = \nu_0 = 1/3$, where $\Lambda = \operatorname{Re} A(\nu_0, \nu_0, \nu_0)$. In the region of physical τ decay, we have $(0 \le \nu \le 1)$

$$\operatorname{Im} F_0(\nu) = \lambda(\nu) + 2\rho(1-\nu/2). \tag{15}$$

$$\operatorname{Im} G_0(\nu) = \rho(\nu) + \rho(1-\nu/2) + \lambda(1-\nu/2). \tag{16}$$

Relations (13) and (16) are used for analysis in the effective-radius approximation. This approximation can be quite good in view of the weak energy dependence of the τ decay probability. In this way Khuri and Treiman[3] obtained by comparison with experiment estimates for the difference in the s-scattering lengths in the $\pi\pi$ interaction. Their research, however, must be made more accurate, for example, by taking into account the influence of the imaginary part on the decay probability.

We start from the following approximation:

$$\sqrt{\frac{\nu}{\nu+1}}\,\Pi_0^T(\nu) \approx \frac{a_T\sqrt{\nu}}{1-ia_T\sqrt{\nu}} \approx a_T\sqrt{\nu}, \quad F_0 \approx G_0 \approx \Lambda, \tag{17}$$

where a_T is the scattering length.

Substituting (17) in (15) and (17), we obtain

$$\frac{1}{\Lambda}\operatorname{Re} F_0(\nu) = 1 + \frac{a_2}{\pi} Z_1(\nu) + \frac{5a_0+a_2}{3\pi} Z_2(\nu), \tag{18}$$

$$\frac{1}{\Lambda}\operatorname{Re} G_0(\nu) = 1 + \frac{5a_0+a_2}{6\pi} Z_1(\nu) + \frac{5a_0+7a_2}{6\pi} Z_2(\nu);$$

$$\frac{1}{\Lambda}\operatorname{Im} F_0(\nu) = a_2\sqrt{\nu} + \frac{5a_0+a_2}{3}\sqrt{\frac{1-\nu}{2}} \quad (0 \leqslant \nu \leqslant 1), \tag{19}$$

$$\frac{1}{\Lambda}\operatorname{Im} G_0(\nu) = \frac{5a_0+a_2}{6}\sqrt{\nu} + \frac{5a_0+7a_2}{6}\sqrt{\frac{1-\nu}{2}} \quad (0 \leqslant \nu \leqslant 1),$$

where

$$Z_1(\nu) = \sqrt{\nu}\ln\frac{1-\sqrt{\nu}}{1+\sqrt{\nu}} - \sqrt{\nu_0}\ln\frac{1-\sqrt{\nu_0}}{1+\sqrt{\nu_0}},$$

$$Z_2(\nu) = \sqrt{\frac{1-\nu}{2}}\ln\frac{1-\sqrt{(1-\nu)/2}}{1+\sqrt{(1-\nu)/2}} - \sqrt{\nu_0}\ln\frac{1-\sqrt{\nu_0}}{1+\sqrt{\nu_0}}. \tag{20}$$

Using (18) and (19) as well as the approximation

$$\sqrt{\frac{\nu}{\nu+1}}\,\Pi_0^T(\nu) \approx \sqrt{\nu}a_T + i\nu a_T^2 \tag{17'}$$

for the amplitudes of the $\pi\pi$ interaction, we obtain expressions for ρ and λ, which we substitute in (5). After integration and expansion in powers of ν_i (with account of the fact that $\nu_1 + \nu_2 + \nu_3 = 1$) the real part of the decay amplitude assumes the form

$$\frac{\pi}{\Lambda}\operatorname{Re} A = \text{const} - \frac{\pi}{\alpha} R(a_2, a_0)\nu_3. \tag{21}$$

Here

$$\frac{\pi}{\alpha} R(a_2, a_0) = \frac{5}{3}(a_2 - a_0) + \frac{5}{3}(a_2^2 - a_0^2) - \frac{5}{9\pi}(a_2^2 + a_2a_0 - 2a_0^2) - \frac{5}{3}(a_2^3 - a_0^3) \tag{22}$$

and $\alpha = \frac{1}{4}(m^2 - 1)^2 - 1 = 0.7$ when $m^2 \approx 13$.

For ν_i we have

$$\nu_i = \alpha(1 - t_i); \tag{23}$$

here t_i is the kinetic energy of the π meson with index i in the rest system of the τ meson. Carrying out the subtraction at the symmetrical point $t_i = t_0 = 1/2$, where $\Lambda = \operatorname{Re} A(t_0)$, we obtain

$$\Lambda^{-2}\{\operatorname{Re} A(t_3)\}^2 = 1 + (2t_3 - 1)R(a_2, a_0) + \Gamma^2. \tag{24}$$

Here

a_2	—1	—0.3	—0.333	—0.2	0	0	0	0.1	0.1	0.15
a_0	—0.465	—1	—0.135	—0.27	—0.5	—0.3	0	—0.65	—0.45	—0.4
σ ($\nu_3 = 0$; $\nu_1 = \nu_2 = 0.5$)	0.64	0.45	0.89	1.21	1.23	1.08	1	1.41	1.28	1.28
σ ($\nu_3 = 0.7$; $\nu_1 = \nu_2 = 0.15$)	1.41	0.01	1.05	0.68	0.79	0.85	1	0.71	0.79	0.81
a_2	0.2	0.2	0.25	0.3	0.333	0,333	0.4	0.5	0.6	0.7
a_0	—0.3	0.2	—0.35	—0.5	—0.2	0.135	—0.3	0	—0.8	0.11
σ ($\nu_3 = 0$; $\nu_1 = \nu_2 = 0.5$)	1.22	1.20	1.32	1.13	1.23	1.0	1.36	1.11	1.45	0.95
($\nu_3 = 0.7$; $\nu_1 = \nu_2 = 0.15$)	0.82	0.87	0.78	0.76	0.83	0.78	0.79	0.95	0.49	1.03

$$\Gamma^2 = \tfrac{1}{4}(2t - 1)^2 \{R(a_2, a_0)\}^2. \tag{25}$$

For the imaginary part of the decay amplitude with

$$\operatorname{Im} A(\nu_1, \nu_2, \nu_3) = \rho(\nu_1) + \rho(\nu_2) + \lambda(\nu_3) \tag{26}$$

we obtain from (11), (17′), (18), and (19) the expression

$$\begin{aligned}
\frac{1}{\Lambda}\operatorname{Im} A(\nu_1, \nu_2, \nu_3) \equiv I &= (\sqrt{\nu_1} + \sqrt{\nu_2})\Big\{\frac{1}{6}(5a_0 + a_2) \\
&\quad - \frac{5}{54\pi}(a_2 - a_0)(4a_0 - a_2)\Big\} + \sqrt{\nu_3}a_2\Big\{1 + \frac{5}{9\pi}(a_2 - a_0)\Big\} \\
&\quad + (\nu_1^{3/2} + \nu_2^{3/2})\Big\{\frac{1}{6}(5a_0^2 + a_2^2) + \frac{5}{18\pi}(a_2 - a_0)(4a_0 - a_2)\Big\} \\
&\quad + \nu_3^{3/2}a_2\Big\{a_2^2 - \frac{5}{3\pi}(a_2 - a_0)\Big\} \\
&\quad + \Big(\nu_1\sqrt{\frac{\nu_2 + \nu_3}{2}} + \nu_2\sqrt{\frac{\nu_1 + \nu_3}{2}}\Big) \\
&\quad \times \Big\{\frac{5}{9}a_0^2(a_0 + 2a_2) + \frac{1}{18}a_2^2(5a_0 + a_2)\Big\} \\
&\quad + \nu_3\sqrt{\frac{\nu_1 + \nu_2}{2}}\Big\{\frac{1}{3}a_2^2(5a_0 + a_2)\Big\}.
\end{aligned} \tag{27}$$

With the aid of (23) and (24) we obtain for the square of the Feynman amplitude*

$$\begin{aligned}
\sigma' &\equiv \Lambda^{-2}\{(\operatorname{Re} A)^2 + (\operatorname{Im} A)^2\} \\
&= 1 + (2t_3 - 1)R(a_2 a_0) + K(a_2, a_0; \nu_i),
\end{aligned} \tag{28}$$

where

$$K(a_2, a_0; \nu_i) = \Gamma^2 + I^2. \tag{29}$$

We shall compare this expression with the quantity σ, which adequately describes the experimental data[7]:

$$\sigma \equiv |M|^2 = 1 + 0.2(2t_3 - 1). \tag{30}$$

The quantities σ and σ' are related:

$$\sigma = \sigma' / [1 + I^2(\nu_0)]. \tag{31}$$

We list the data on the dependence of the theoretical value of σ on the scattering lengths a_2, a_0 [by definition $\sigma(\nu_1 = \nu_2 = \nu_3 = 1/3) = 1$].

Agreement with the experimental energy dependence can be obtained, for example, when $a_2 \approx 0.2$ and $a_0 \approx -0.3$. In this case $R(0.2; -0.3) = 0.16$ and

$$\begin{aligned}
&\sigma(\nu_3 = 0,\ \nu_1 = \nu_2 = 0.5) \approx 1.2, \\
&\sigma(\nu_3 = \nu_1 = \nu_2 = 1/3) = 1, \\
&\sigma(\nu_3 = 0.7,\ \nu_1 = \nu_2 = 0.15) \approx 0.8,
\end{aligned}
\qquad
\frac{1}{\Lambda^2}(\operatorname{Im} A)^2 = \begin{cases} 0.07, \\ 0.02, \\ 0.00. \end{cases} \tag{32}$$

The experimental relation (30) for the energies used in (32) yields indeed $\sigma = 1.2$, 1, and 0.8.

Figure 2 is a plot of the equations $R(a_2, a_0) = 0.2$ and $I(\nu_i = 1/3) = 0$. If we allow a 40 percent error in the determination of the energy-dependent part of σ, we find that the values of the area shaded in Fig. 2 duplicate the experimental data with a sufficient degree of accuracy.

We have given values for σ at equal energies $\nu_1 = \nu_2$. We note that when $\nu_1 \neq \nu_2$ we obtain in the shaded region values which differ little from (32):

$$\begin{aligned}
&\sigma(\nu_3 = 0,\ \nu_1 = 0.3,\ \nu_2 = 0.7) = 1.24, \\
&\sigma(\nu_3 = 1/3,\ \nu_1 = 2/3,\ \nu_2 = 0) \approx 1, \\
&\sigma(\nu_3 = 0.7,\ \nu_1 = 0.3,\ \nu_2 = 0) = 0.84.
\end{aligned} \tag{33}$$

With the aid of the selected scattering lengths we can duplicate also the experimental data[8] on π-meson spectra. Thus, when $a_2 = 0.2$ and $a_0 = -0.3$ we obtain for the π^+ spectrum

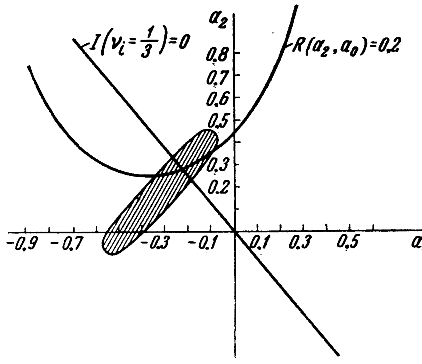

FIG. 2

*The authors have been informed of Gribov's paper[9]. We note that the diagrams which he takes into account arise in our case by iteration of the resultant system of equations.

$$W(t_1) = \begin{cases} 0.92 \\ 1 \\ 1.11 \end{cases} \text{for} \begin{cases} \nu_1 = 0, \ \nu_2 = \nu_3 = 0.5, \\ \nu_i = {}^1/_3, \\ \nu_1 = 0.7, \ \nu_2 = \nu_3 = 0.15. \end{cases} \quad (34)$$

Knowledge of the $\pi\pi$ scattering lengths a_2 and a_0 obtained from the τ decay enable us to draw certain conclusions regarding the relative values of the partial waves of the $\pi\pi$ interaction. From the integral equations for $\pi\pi$ scattering[4] we can derive the following relation between a_2 and a_0:

$$2a_0 - 5a_2 = \frac{1}{\pi} \int_0^\infty \frac{d\nu}{\nu(\nu+1)} \{2 \operatorname{Im} \Pi_0^0(\nu)$$

$$+ 9 \operatorname{Im} \Pi_1^1(\nu) - 5 \operatorname{Im} \Pi_0^2(\nu)\}. \quad (35)$$

For all pairs a_2 and a_0 duplicating the experimental τ-decay spectrum, the combination $(2a_0 - 5a_2)$ is negative. Such a result does not of necessity contradict the possibility of p-wave resonance, but in any case is evidence in favor of the existence of a $(T = 2)$ resonance in $\pi\pi$ interaction.

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145